Name:	
Class:	

YEAR 12

TRIAL
2016

MATHEMATICS EXTENSION 1

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 2 hours.
- Write in black or blue pen.
- Board approved calculators & templates may be used
- A Reference Sheet is provided.
- In Question 11 - 14, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 70

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 60 Marks

- Attempt Question 11 – 14
- Answer on blank paper unless otherwise instructed. Start a new page for each new question.
- Allow about 1 hour & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. Let $\alpha,\ \beta$ and γ be the roots of $x^3 + px^2 + q = 0$.

   Express $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$ in terms of p and q.

   (A) pq

   (B) $-pq$

   (C) $-\frac{p}{q}$

   (D) $\frac{p}{q}$

2. If $f(x) = \frac{3 + e^{2x}}{4}$, what is $f^{-1}(x)$ =?

   (A) $\ln(4x - 3)$

   (B) $\frac{1}{2}\ln(4x - 3)$

   (C) $\ln 4x - \ln 3$

   (D) $\frac{1}{2}(\ln 4x - \ln 3)$

3. A family of 6 adults and 3 children are randomly sitting around a circular table.

   What is the number of possible seating arrangements if none of the children sit together?

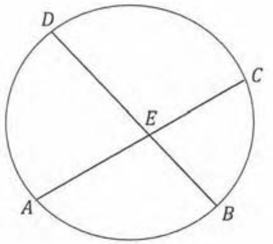
   (A) 720

   (B) 2 400

   (C) 14 400

   (D) 86 400

4. In the diagram below, chords AC and BD of the circle intersect at E.
$AC = 9$ units , $CE = a$ units , $DE = x$ units and $EB = y$ units



NOT TO SCALE

Which of the following statements is true?

(A) $xy = 9a$

(B) $\frac{x}{y} = \frac{9-a}{a}$

(C) $x(x+y) = a(9-a)$

(D) $xy = a(9-a)$

5. For $0 < x < 1$, what is $\frac{d}{dx}\left(\cos^{-1}\frac{1}{x}\right) = ?$

(A) $\frac{-1}{x\sqrt{x^2-1}}$

(B) $\frac{1}{x\sqrt{x^2-1}}$

(C) $\frac{-1}{\sqrt{x^2-1}}$

(D) $\frac{-x}{\sqrt{x^2-1}}$

6. What is the general solution of $\cos 2\theta = \frac{\sqrt{2}}{2}$?

(A) $\theta = \frac{\pi}{8} + n\pi$ or $\theta = \frac{7\pi}{8} + n\pi$, n is an integer

(B) $\theta = \frac{\pi}{8} + 2n\pi$ or $\theta = \frac{7\pi}{8} + 2n\pi$, n is an integer

(C) $\theta = \frac{2\pi}{3} + n\pi$ or $\theta = \frac{4\pi}{3} + n\pi$, n is an integer

(D) $\theta = \frac{\pi}{4} + n\pi$ or $\theta = \frac{3\pi}{4} + n\pi$, n is an integer

7. Which integral is obtained when the substitution $x = \sin\theta$ is applied to $\int \frac{x^2}{\sqrt{1-x^2}}dx$?

(A) $\int \cos^2\theta \, d\theta$

(B) $\int \tan^2\theta \, d\theta$

(C) $\int \sin^2\theta \, d\theta$

(D) $\int \sin^2\theta \, \cos^2\theta \, d\theta$

8. A particle is moving in simple harmonic motion with its velocity given by $v^2 = 16(9 - x^2)$, where x is the displacement.

What is the amplitude A, and the period T, of the motion?

(A) $A = 3$ and $T = \frac{\pi}{4}$

(B) $A = 3$ and $T = \frac{\pi}{2}$

(C) $A = 4$ and $T = \frac{\pi}{4}$

(D) $A = 4$ and $T = \frac{\pi}{2}$

9. What is the term independent of x in the expansion of $\left(x^3 + \frac{2}{x}\right)^{20}$?

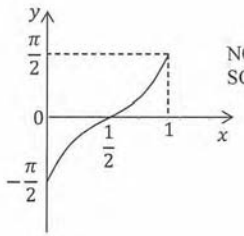
(A) $\binom{20}{10}2^{10}$

(B) $\binom{20}{5}2^{15}$

(C) $\binom{20}{4}2^{16}$

(D) $\binom{20}{5}2^{25}$

10. The diagram shows the graph of a function.



Which function does the graph represent?

(A) $y = -\cos^{-1}(2x-1)$

(B) $y = \sin^{-1}(2x-1)$

(C) $y = \sin^{-1}(x-1)$

(D) $y = -\cos^{-1}(x-1)$

Section II

60 Marks
Attempt questions 11-14
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Use a SEPARATE writing booklet.

(a) Let $P(x) = x^3 - ax^2 + x$ be a polynomial, where a is a real number. When $P(x)$ is divided by $x-3$ the remainder is 12. **2**

Express $P(x)$ as a product of its factors.

(b) Solve $\frac{5}{x-4} \geq 1$ **2**

(c) Find the value of **2**

$$\lim_{x \to 0} \frac{3x \cos 4x}{\sin 2x}$$

Show all necessary working.

(d) Use the substitution $u = 1 + \sqrt{2x}$ to find **3**

$$\int_{\frac{1}{2}}^{2} \frac{1}{1+\sqrt{2x}}\,dx$$

(e) Find the exact value of $\cos(2\cos^{-1}x + \sin^{-1}x)$ when $x = \frac{1}{5}$. **3**

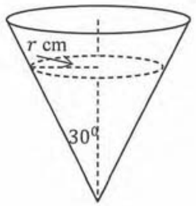
(f) Find $\frac{d}{dx}\left(\tan^{-1}\frac{x}{3}\right)^2$ and hence find the exact value of **3**

$$\int_0^{\sqrt{3}} \frac{\tan^{-1}\frac{x}{3}}{x^2+9}\,dx$$

End of Question 11

QUESTION 12 (15 Marks) Use a SEPARATE writing booklet.

(a) In the figure below the angle between the axis and the slant edge of the right cone is 30^0. Water is poured into the cone at a constant rate of 2 cm^3 per second. At time t seconds, the radius of the water surface is r cm and the volume of water in the cone is $V\text{ cm}^3$.



(i) Show that $V = \frac{\sqrt{3}}{3}\pi r^3$. **1**

(ii) Hence, find the rate at which r is changing when $r = 2$ cm. **2**

(b) The velocity of a particle moving in a straight line is given by

$$v^2 = -9x^2 + 18x + 27\ .$$

(i) Prove that the motion is simple harmonic. **2**

(ii) Hence find the centre of the motion. **1**

(iii) Find the maximum acceleration of the particle and state where it occurs. **2**

(c) Taking $x = -\frac{\pi}{6}$ as a first approximation to the root of the equation $2x + \cos x = 0$, use Newton's method once to show that a second approximation to the root of the equation is $\frac{-\pi - 6\sqrt{3}}{30}$. **2**

(d) An unbiased die is thrown 6 times. Calculate the probabilities that the six scores obtained

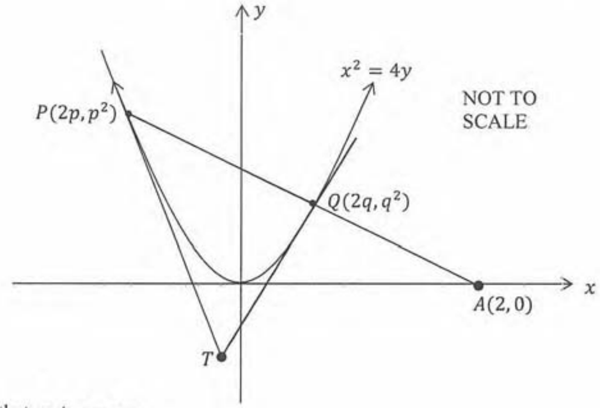
(i) will have a product which is an even number **2**

(ii) will consist of exactly two $6's$ and four odd numbers **3**

End of Question 12

QUESTION 13 (15 Marks) Use a SEPARATE writing booklet.

(a) Let $P(2p, p^2)$ and $Q(2q, q^2)$ be two points on the parabola $x^2 = 4y$. The tangents to the parabola at P and Q intersect at T. The secant PQ passes through the point $A(2, 0)$.



(i) Show that $p + q = pq$. **2**

(ii) Assuming that the equation of the tangents at P and Q are $y = px - p^2$ and $y = qx - q^2$ respectively, determine the cartesian equation of the locus of T, including any restrictions. **3**

(b) The region enclosed by the curve $y = \sin^{-1} x$ and the y –axis between $x = 0$ and $x = \frac{\sqrt{3}}{2}$ is rotated about the y –axis to form a solid. **3**

Find the exact volume of the solid of revolution formed.

(c) A manufacturer of metal pistons finds that on the average, 12% of the pistons are rejected because they are either oversize or undersize.

What is the probability that a batch of 10 pistons will contain no more than 2 rejects? Express your answer to three decimal places. **3**

Question 13 continues on page 9

(d) A household iron is cooling in a room where the temperature of the air is constant at 22^0C.

After t minutes the temperature T of the iron is modelled by the equation $T = 22 + Ae^{-kt}$, where A and k are constants.

The initial temperature of the iron is 80^0C and it cools to 60^0C after 10 minutes.

(i) Find the exact values of the constants A and k. **2**

(ii) How long will it take for the temperature of the iron to cool to 30^0C? Give your answer to the nearest minute. **2**

End of Question 13

QUESTION 14 (15 Marks) Use a SEPARATE writing booklet.

(a) Use mathematical induction to prove that for all positive integers n, **3**

$$\sum_{r=1}^{n} r(r!) = (n+1)! - 1$$

(b) The velocity of a particle moving in a straight line is given by $\dot{x} = e^x + e^{-x}$, where x in metres is the displacement from the origin. Initially the particle is at the origin.

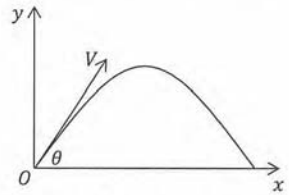
(i) Show that the time taken by the particle as a function of its displacement is given by **2**

$$t = \int \frac{e^x}{e^{2x}+1}\,dx$$

(ii) Hence, by using the substitution $u = e^x$, prove that **2**

$$x = \ln\left(\tan\left(t + \frac{\pi}{4}\right)\right).$$

(c) A particle P is fired from the ground at $t = 0$ with initial velocity V from the origin, at an angle of θ to the horizontal.



Assume that the horizontal and vertical equations of motion for the particle is given by

$$x_P = Vt\cos\theta \quad \text{and}$$

$$y_P = Vt\sin\theta - \frac{1}{2}gt^2$$

(DO NOT PROVE THESE)

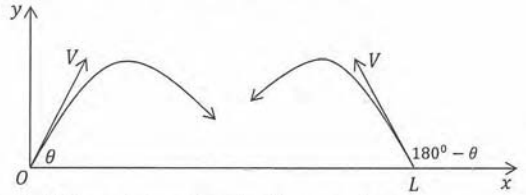
(i) Show that the horizontal range of the projectile, R_P, is given by **3**

$$R_P = \frac{V^2 \sin 2\theta}{g}$$

Question 14 (c) continues on page 11

Question 14 (c) (continued)

A second particle Q, is fired back towards the origin with velocity V from the ground at a distance of L metres to the **right** of the origin at time $t = 0$, with an angle of $(180 - \theta)^0$ to the positive direction of the x – axis.



(ii) Show that if the particles collide, it will occur when **2**

$$t = \frac{L}{2V\cos\theta}$$

(iii) For the particles to collide, it must occur while the particles are still in flight (i.e. above the ground). **3**

Prove that, for the particles to collide in the air,

$$0 < L < \frac{4V^2\cos\theta\sin\theta}{g}$$

End of paper

JRAHS : MATHEMATICS EXTENSION : TRIAL 2016

MCQ :

1. $x^3 + px + q = 0$

$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$

$= \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$

$= \frac{-p}{-q}$

$= p/q$ ∴ D

2. $x = \frac{3+e^{2y}}{4}$

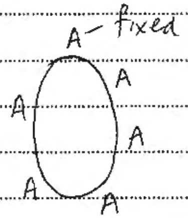
$e^{2y} = 4x - 3$

$2y = \ln(4x-3)$

$y = \frac{1}{2}\ln(4x-3)$

∴ B

3.



The 6 adults can be arranged in 5! ways

If none of the children sit together, they would need to occupy 3 of the 6 positions (between 2 adults)

Hence $^6C_3$ ways.

But the 3 children can be arranged in 3! ways

Hence

$5! \times {^6C_3} \times 3!$

But $5! \times 3! = 6!$

∴ $^6C_3 \times 6!$ ways

$= 14400$

∴ C

4. $DE/EB = EC/EA$

$x/y = a/EA$

∴ $AE = ay/x$

But $DE.EB = AE.EC$

$x.y = (AC-EC).EC$

$= (9-a)a$

$= a(9-a)$

∴ D

5. $\frac{d}{dx}\left(\cos^{-1}\frac{1}{x}\right)$

$= -\frac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot \left(\frac{-1}{x^2}\right)$

$= \frac{1}{\frac{1}{x}\sqrt{x^2-1}} \cdot \frac{1}{x^2}$

$= \frac{1}{x\sqrt{x^2-1}}$

∴ B

6. $\cos 2\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

∴ $2\theta = \frac{\pi}{4} + 2n\pi$

∴ $\theta = \frac{\pi}{8} + n\pi$ or

$2\theta = \frac{7\pi}{4} + 2n\pi$

∴ $\theta = 7\pi/8 + n\pi$

Hence A

7. $x = \sin\theta$

$dx = \cos\theta\, d\theta$

∴ $I = \int \frac{\sin^2\theta \cdot \cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}}$

$= \int \frac{\sin^2\theta \cdot \cos\theta \cdot d\theta}{\cos\theta}$

$= \int \sin^2\theta\, d\theta$

∴ C

8. $v^2 = 16(9-x^2)$

$= n^2(A^2-x^2)$

∴ $n = 4$ and $A = 3$

$\Rightarrow T = 2\pi/4 = \pi/2$

Hence Amplitude $= 3$

Period $= \pi/2$

∴ B

9. $\left(x^3 + \frac{2}{x}\right)^{20}$

$= \sum_{r=1}^{20} {}^{20}C_r \,(x^3)^{20-r}\left(\frac{2}{x}\right)^r$

$= \sum {}^{20}C_r \, x^{60-3r} x^{-r} 2^r$

$= \sum {}^{20}C_r \, x^{60-4r} 2^r$

For term independent of x

$60 - 4r = 0$

$\therefore r = 15$

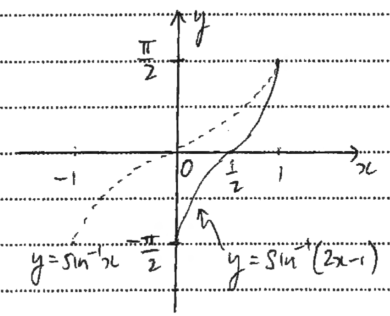
Hence the term is

${}^{20}C_{15}\, x^0 \cdot 2^{15}$

$= {}^{20}C_5\, 2^{15}$

$\therefore$ B

10. $y = \sin^{-1} x$



$\therefore$ B

1. D
2. B
3. C
4. D
5. B
6. A
7. C
8. B
9. B
10. B

1/2

MATHEMATICS Extension 1 : Question 11

Suggested Solutions	Marks	Marker's Comments
11(a) $P(x) = x^3 - ax^2 + x$ $\quad P(3) = 12$		
$\therefore 12 = 27 - 9a + 3$		
$30 - 9a = 12$		
$18 = 9a$		
$a = 2$	1	
$\therefore P(x) = x^3 - 2x^2 + x$		a lot of students wasted time using the factor theorem and long division.
$= x(x^2 - 2x + 1)$		
$= (x)(x-1)^2$	1	
(b) $\frac{5}{x-4} \geq 1$		
$5(x-4) \geq (x-4)^2$ $\quad x \neq 4$		
$5(x-4) - (x-4)^2 \geq 0$		
$(x-4)[5-(x-4)] \geq 0$		
$(x-4)(9-x) \geq 0$ 		
$(x-4)(x-9) \leq 0$	1	*had to factorise correctly to get the first mark.
$4 < x \leq 9$	1	*most students forgot that $x \neq 4$.
(c) $\lim_{x\to 0} \frac{3x}{\sin 2x} \times \lim_{x\to 0} \cos 4x$		
$= \lim_{x\to 0} \frac{2x}{\sin 2x} \times \frac{3}{2} \times \lim_{x\to 0} \cos 4x$	1	*A lot of students used double angles ∴
$= 1 \times \frac{3}{2} \times 1$		*lost a mark if you "lost" $\lim_{x\to 0}$!!!
$= 3/2$	1	
(d) when $x = 1/2$ $\quad u = 2$		
when $x = 2$ $\quad u = 3$	1	both limits correct
$u = 1 + \sqrt{2x}$		
$\therefore u = 1 + (2x)^{1/2}$		
$\frac{du}{dx} = \frac{1}{2}(2x)^{-1/2} \times 2$		
$\therefore \frac{du}{dx} = \frac{1}{\sqrt{2x}}$		
$dx = \sqrt{2x}\, du$		
$= (u-1)\,du$ $\quad$ (as $u = 1 + \sqrt{2x}$)		
$\therefore I = \int_2^3 \frac{1}{u}(u-1)\,du$	1	
$= \int_2^3 \left(1 - \frac{1}{u}\right) du$		
$= \left[u - \ln u\right]_2^3$		

2/2

MATHEMATICS Extension 1 : Question 11 cont.

Suggested Solutions	Marks	Marker's Comments
$= 3 - \ln 3 - (2 - \ln 2)$		
$= 1 - \ln 3 + \ln 2$	1	
$= 1 + \ln \frac{2}{3}$ or $1 - \ln \frac{3}{2}$		
(e) $\cos(2\cos^{-1}x + \sin^{-1}x)$ now $x = \frac{1}{5}$		
$\cos(2\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5})$		
let $\cos^{-1}\frac{1}{5} = m$ and $\sin^{-1}\frac{1}{5} = n$		
$\cos m = \frac{1}{5}$ $0 < m < \frac{\pi}{2}$ $\sin n = \frac{1}{5}$ $0 < n < \frac{\pi}{2}$	1	
		
$\therefore \cos(2m + n)$		
$= \cos 2m \cos n - \sin 2m \sin n$	1	
$= (\cos^2 m - \sin^2 m)\cos n - 2\sin m \cos m \sin n$		
$= \left[\left(\frac{1}{5}\right)^2 - \left(\frac{\sqrt{24}}{5}\right)^2\right]\frac{\sqrt{24}}{5} - 2 \times \frac{\sqrt{24}}{5} \times \frac{1}{5} \times \frac{1}{5}$		
$= \frac{-23}{25} \times \frac{\sqrt{24}}{5} - \frac{2\sqrt{24}}{125}$		
$= \frac{-25\sqrt{24}}{125}$		
$= -\frac{\sqrt{24}}{5}$	1	
$= -\frac{2\sqrt{6}}{5}$		
(f) let $y = \left(\tan^{-1}\frac{x}{3}\right)^2$		a lot of students forgot the "3" or had "9".
$\frac{dy}{dx} = 2\left(\tan^{-1}\frac{x}{3}\right)^1 \times \frac{3}{3^2 + x^2}$		
$= \frac{6\tan^{-1}\frac{x}{3}}{9 + x^2}$	1	
$\therefore \int_0^{\sqrt{3}} \frac{\tan^{-1}\frac{x}{3}}{9 + x^2}\,dx = \frac{1}{6}\left[\left(\tan^{-1}\frac{x}{3}\right)^2\right]_0^{\sqrt{3}}$	1	*Some students forgot to square the $\tan^{-1}\frac{x}{3}$
$= \frac{1}{6}\left(\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)^2 - \left(\tan^{-1}0\right)^2\right)$		
$= \frac{1}{6}\left(\tan^{-1}\frac{1}{\sqrt{3}}\right)^2 - 0$		*sad that some students didn't know their exact values.
$= \frac{1}{6} \times \left(\frac{\pi}{6}\right)^2$		
$= \frac{\pi^2}{216}$	1	

MATHEMATICS: Question 12
Extension ~~2~~ 1

Suggested Solutions	Marks	Marker's Comments
(a)		
(i) $V = \frac{\pi r^2 h}{3}$		
$= \frac{\pi r^2(\sqrt{3}r)}{3}$		
$= \frac{\sqrt{3}\pi r^3}{3}$	1	
(ii) $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$		
$= \frac{dV}{dt} \div \frac{dV}{dr}$		
$= \frac{2}{\frac{\sqrt{3} \times 3\pi r^2}{3}}$	1	
$= \frac{2}{\sqrt{3}\pi r^2}$		
$r = 2 \Rightarrow \frac{dr}{dt} = \frac{2}{4\sqrt{3}\pi}$		
$= \frac{1}{2\sqrt{3}\pi}$ $cm s^{-1}$	1	
$= \frac{\sqrt{3}}{6\pi}$ $cm s^{-1}$		

MATHEMATICS: Question……..
Extension ~~2~~ 1

Suggested Solutions	Marks	Marker's Comments
(b)		
(i) $v^2 = -9x^2 + 18x + 27$		
$= -9(x^2 - 2x - 3)$		
$= -9((x-1)^2 - 4)$		
$= 9(4 - (x-1)^2)$	1	
$\equiv n^2(a^2 - (x-1)^2)$	1	
∴ Motion is simple harmonic		
(ii) x is a maximum when v is 0.		
$0 = (x+1)(x-3)$ from (i)		
$x = -1, 3$		
-1, 3; $-6 \leftarrow \rightarrow 6$; v		
∴ centre is $x = 1$	1	
(iii) acceleration is a maximum at the extremities, $x = 3$, $x = -1$.	1	
$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$		
$= \frac{d}{dx}\left(\frac{-9}{2}x^2 + 9x + \frac{27}{2}\right)$		
$= -9x + 9$		
∴ when $x = 3$, $a = -18$		either solution earned the mark.
$\boxed{\text{or}}$ $x = -1$, $a = 18$	1	$\boxed{\text{BOTH}}$ a and x must be stated together.

MATHEMATICS: Question……..
Extension 2

Suggested Solutions	Marks	Marker's Comments
(c) $2x + \cos x = 0$		
Let $f(x) = 2x + \cos x$		
$f\left(-\frac{\pi}{6}\right) = -\frac{\pi}{3} + \cos\left(-\frac{\pi}{6}\right)$		
$= -\frac{\pi}{3} + \cos\left(\frac{\pi}{6}\right)$		
$= -\frac{\pi}{3} + \frac{\sqrt{3}}{2}$		
$f'(x) = 2 - \sin x$		
$f'\left(-\frac{\pi}{6}\right) = 2 - \sin\left(-\frac{\pi}{6}\right)$		
$= 2 + \frac{1}{2}$		
$= \frac{5}{2}$		
Second application:		
$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$		
$= -\frac{\pi}{6} - \left(\frac{-\frac{\pi}{3} + \frac{\sqrt{3}}{2}}{\frac{5}{2}}\right)$	1	
$= -\frac{\pi}{6} + \frac{2\pi}{15} - \frac{\sqrt{3}}{5}$		
$= \frac{-\pi - 6\sqrt{3}}{30}$	1	
As required		

MATHEMATICS: Question…….. Extension 2		
Suggested Solutions	**Marks**	**Marker's Comments**
(d) (i) $P(\text{even product}) = 1 - P(\text{all odd})$	1	
$= 1 - \left(\frac{1}{2}\right)^6$		
$= \frac{63}{64}$	1	
(ii) $P(4\text{ odds}) = \frac{3}{6} \times \frac{3}{6} \times \frac{3}{6} \times \frac{3}{6}$		
$= \left(\frac{1}{2}\right)^4$		
$P(2\ 6s) = \left(\frac{1}{6}\right)^2$	1	
$P = {}^nC_r\, p^{n-r} q^r$		
$= {}^6C_2 \left(\frac{1}{6}\right)^2 \left(\frac{1}{2}\right)^{6-2}$	1	
$= 15 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{1}{2}\right)^4$		
$= \frac{5}{192}$	1	
Alternative: Let odds be A, evens be B.		
Case 1: $(6\,6\,A\,A\,A\,A) = {}^3C_1 \times \frac{6!}{2! \times 4!} = 45$		One mark for being on the right track, looking for correct cases
Case 2: $(66\ AA\ BB) = {}^3C_2 \times \frac{6!}{2! \times 2! \times 2}$	1	

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Over

MATHEMATICS Extension 1: Question 13		
Suggested Solutions	**Marks**	**Marker's Comments**
a) $m_{PQ} = \frac{p^2 - q^2}{2p - 2q}$		
$= \frac{\cancel{(p-q)}(p+q)}{2\cancel{(p-q)}}$		
$= \frac{p+q}{2}$	1	well done
$y - p^2 = \frac{1}{2}(p+q)(x - 2p)$		
goes through $A(2,0)$		
$0 = \frac{1}{2}(p+q)2 - pq$	1	
$\therefore \underline{p+q = pq}$		
b) $y = px - p^2$ — ①		
$y = qx - q^2$ — ②		
① − ② $(p-q)x - (p^2 - q^2) = 0$		
$(p-q)x = (p-q)(p+q)$		
$\therefore x = p+q \quad p \neq q$		students who substituted $p+q = pq$ into x then had difficulty finding y
$\therefore y = q(p+q) - q^2$ sub in 2	1	
$y = pq - q^2 + q^2$		
$= \underline{pq}$		
$\therefore T(p+q, pq)$		

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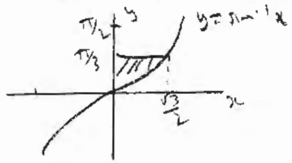
Suggested Solutions	Marks	Marker's Comments
Now $p+q = pq$		
$\therefore\ x = p+q \quad y = pq$	1	
so $y = x$ is locus		
Tangents meet outside of parabola		
sub $y=x$ into $x^2=4y$		equality will not hold $p \neq q$
$x^2 = 4x$		
$x(x-4)=0$		
$\therefore\ x>4$ and $x<0$	1	
lie outside parabola.		
$y=x$ for $x>4$ or $x<0$		

b)



$$V = \pi\int_0^{\pi/3} \sin^2 y\,dy$$
$$= \pi\int_0^{\pi/3} \tfrac12 - \tfrac12\cos 2y\,dy$$
$$= \pi\left[\tfrac12 y - \tfrac14\sin 2y\right]_0^{\pi/3}$$
$$= \pi\left[\tfrac{\pi}{6} + \tfrac{\sqrt3}{8}\right]u^3$$

Marker's comments: • DO NOT NEED TO USE SLICE IN EXT I • Question clearly said y-axis not x-axis. Students found incorrect region had to integrate $\sin^2 y$ to get marks.

Suggested Solutions	Marks	Marker's Comments
c) P(2 or less rejects) in $(0.12+0.88)^{10}$		1 for recognising correct probability required
$P(2)+P(1)+P(0)$	1	
$= {}^{10}C_2(0.12)^2(0.88)^8 + {}^{10}C_1(0.12)(0.88)^9$		1 for one correct term
$+ {}^{10}C_0(0.88)^{10}$	1	1 for final answer.
$= 0.891$	1	
d) i) $T = 22 + Ae^{-kt}$		
$t=0 \quad T=80$		
$80 = 22 + Ae^0$		
$\therefore A = 58 \quad \therefore T = 22+58e^{-kt}$	1	well done.
when $t=10 \quad T=60$		
$\frac{38}{58} = e^{-10k}$		
$-10k = \ln\frac{19}{29}$		
$k = \frac{1}{10}\ln\frac{29}{19}$ OR $-\frac{1}{10}\ln\frac{19}{29}$	1	
(ii) $T = 22 + 58e^{-\frac{1}{10}\ln\frac{29}{19}t}$		
$T = 30$		
$30 = 22 + 58e^{-\frac{1}{10}\ln\frac{29}{19}t}$		
$\frac{8}{58} = e^{-\frac{1}{10}\ln\frac{29}{19}t}$		
$t = \dfrac{10\ln\frac{29}{4}}{\ln\frac{29}{19}}$		
$t = 46.84$		
$\therefore$ takes 47 mins		

MATHEMATICS: Question 14
Extension 1

Suggested Solutions	Marks	Marker's Comments
14. a) Test $n=1$		Note: Students should define k correctly!
$LHS = \sum_{1}^{1} r(r!)$ $= 1 \times 1!$ $= 1$; $RHS = (1+1)! - 1$ $= 2!$ $= 1$		
$\therefore LHS = RHS$		
$\therefore$ The result is true for $n=1$	1	
Assume result is true for $n=k$, $k \in \mathbb{Z}^+$		
$\therefore 1(1!) + 2(2!) + \ldots k(k!) = (k+1)! - 1$		
Test for $n=k+1$		
RTP: $\sum_{1}^{k+1} r(r!) = (k+2)! - 1$		
$LHS = 1(1!) + 2(2!) + \ldots k(k!) + (k+1)(k+1)!$		
$= [(k+1)! - 1] + (k+1)(k+1)!$ (By assumption)	1	
$= (k+1)![1 + (k+1)] - 1$		
$= (k+1)!(k+2) - 1$		
$= (k+2)! - 1$	1	
$\therefore$ If the result is true for $n=k$,		
It is true for $n=k+1$		
$\therefore$ The result is true by Mathematical Induction.		

MATHEMATICS: Question 14
Extension

Suggested Solutions	Marks	Marker's Comments
b) i) $x = e^x + e^{-x}$		
$\frac{dx}{dt} = e^x + e^{-x}$	1	
$= e^x + \frac{1}{e^x}$		
$\frac{dx}{dt} = \frac{e^{2x} + 1}{e^x}$		
$\frac{dt}{dx} = \frac{e^x}{e^{2x}+1}$		
$t = \int \frac{e^x}{e^{2x}+1}\, dx$	1	
ii) let $u = e^x$ $\frac{du}{dx} = e^x$		
$\therefore t = \int \frac{1}{u^2+1}\, du$		
$t = \tan^{-1} u + C$		
$t = \tan^{-1}(e^x) + C$	1	
when $t=0$, $x=0$		
$\therefore \tan^{-1}(1) + C = 0$		
$\therefore C = -\frac{\pi}{4}$		
$\therefore t = \tan^{-1}(e^x) - \frac{\pi}{4}$		
$\therefore \tan^{-1}(e^x) = t + \frac{\pi}{4}$		
$e^x = \tan(t + \frac{\pi}{4})$		
$x = \ln[\tan(t + \frac{\pi}{4})]$	1	

MATHEMATICS: Question 14
Extension 1

Suggested Solutions	Marks	Marker's Comments
c) i) $y_P = Vt\sin\theta - \frac{1}{2}gt^2$		
$= t(V\sin\theta - \frac{1}{2}gt)$		
$y_P = 0 \Rightarrow t = 0$ or $\frac{2V\sin\theta}{g}$	1	
When $t = \frac{2V\sin\theta}{g}$		
$x_P = V\left(\frac{2V\sin\theta}{g}\right)\cos\theta$	1	
$= \frac{V^2 2\sin\theta\cos\theta}{g}$		
$x = \frac{V^2\sin 2\theta}{g}$	1	
ii) $x_L = L - Vt\cos\theta$ $y_L = Vt\sin\theta - \frac{1}{2}gt^2 = y_P$	1	Note: students should explain somehow that the trajectory of P and Q are identical but in opposite directions L units apart.
When $t = \frac{L}{2V\cos\theta}$		
$x_L = L - \frac{V\cos\theta L}{2V\cos\theta}$		
$= L - \frac{L}{2}$		
$= \frac{L}{2}$		
$x_P = Vt\cos\theta$		
$= \frac{VL\cos\theta}{2V\cos\theta}$		
$= \frac{L}{2}$		
$= x_L$	1	
Since $x_L = x_P$ at $t = \frac{L}{2V\cos\theta}$ and $y_L = y_P$ for all t. Then L and P will collide at $t = \frac{L}{2V\cos\theta}$.		

MATHEMATICS: Question 14
Extension

Suggested Solutions	Marks	Marker's Comments
c) iii)		
when $t = \frac{L}{2V\cos\theta}$		
$y_P = y_L = \frac{V\sin\theta L}{2V\cos\theta} - \frac{gL^2}{8V^2\cos^2\theta}$	1	
$= L\left(\frac{\tan\theta}{2} - \frac{gL}{8V^2\cos^2\theta}\right)$		
$y_P > 0 \rightarrow L\left(\frac{\tan\theta}{2} - \frac{gL}{8V^2\cos^2\theta}\right) > 0$	1	
$\therefore 0 < L < \frac{8V^2\cos^2\theta\tan\theta}{2g}$		
$0 < L < \frac{4V^2\sin\theta\cos\theta}{g}$	1	
or		
$0 < t_{flight} < t_{range}$	1	
$0 < \frac{L}{2V\cos\theta} < \frac{2V\sin\theta}{g}$	1	
$0 < L < \frac{4V^2\sin\theta\cos\theta}{g}$	1	
or		
$0 < \frac{L}{2} < x_{range}$	1	
$0 < \frac{L}{2} < \frac{V^2\sin 2\theta}{g}$		
$0 < L < \frac{2V^2\sin 2\theta}{g}$	1	
$0 < L < \frac{4V^2\sin\theta\cos\theta}{g}$	1	