Name: ______________________________________

Class: _________________

2018

Higher School Certificate

Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–10)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I Write your answers on the multiple choice answer sheet provided

1. It is given that $\log_a 8 = 1.893$, correct to 3 decimal places.
   What is the value of $\log_a 4$, correct to 2 decimal places?

   (A) 0.95

   (B) 1.26

   (C) 1.53

   (D) 2.84

2. Which group of three numbers could be the roots of the polynomial equation

$$x^3 + px^2 - 26x + 24 = 0\,?$$

   (A) 1, 3, 4

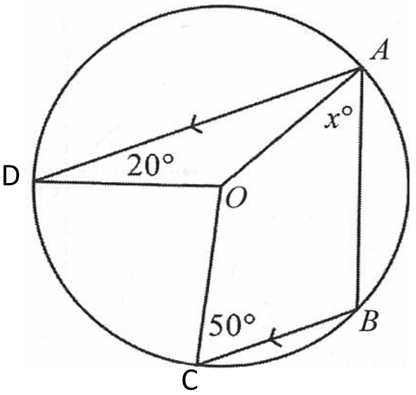
   (B) -1, -3, -8

   (C) -1, -2, 12

   (D) 1, -6, 4

3. A, B, C and D are points on a circle with centre 0. *BC* is parallel to *AD.*
   $\angle ADO$= 20°and $\angle BCO$ = 50°.
   Let $\angle BAO = x^0$

Not to scale



What is the value of *x?*

(A) 15

(B) 35

(C) 40

(D) 55

4. Which expression is equal to $\int cos^2 \frac{2x}{5} dx$?

(A) $\frac{x}{2} + \frac{5}{8} sin \frac{4x}{5} + c$

(B) $\frac{x}{2} - \frac{5}{8} sin \frac{4x}{5} + c$

(C) $\frac{x}{2} + \frac{2}{5} sin \frac{4x}{5} + c$

(D) $\frac{x}{2} - \frac{2}{5} sin \frac{4x}{5} + c$

5. What is the value of $\lim_{x \to \infty} \frac{3x+7}{2-x}$?

(A) 3

(B) -3

(C) $\frac{7}{2}$

(D) $-\frac{7}{2}$

6. A particle is moving in simple harmonic motion along a straight line according to the equation $v^2 = -x^2 + 2x + 8$, where v is its velocity and x its displacement. What is the amplitude of the motion?

(A) 2π metres

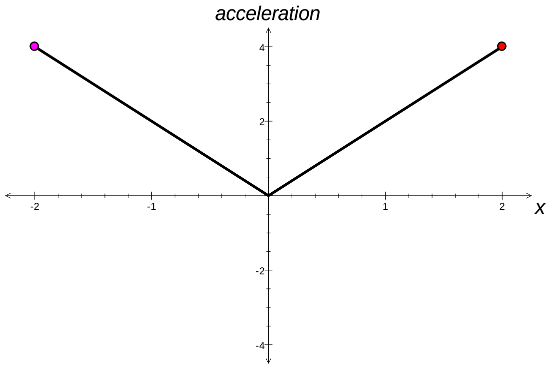
(B) 3 metres

(C) 8 metres

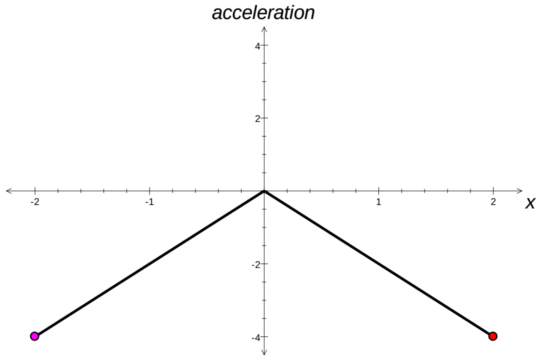
(D) 9 metres

7. A particle is moving along a straight line. The displacement of the particle from a fixed point *O* is given by *x*. The graphs below show acceleration against displacement. Which of the graphs below best represents a particle moving in simple harmonic motion?

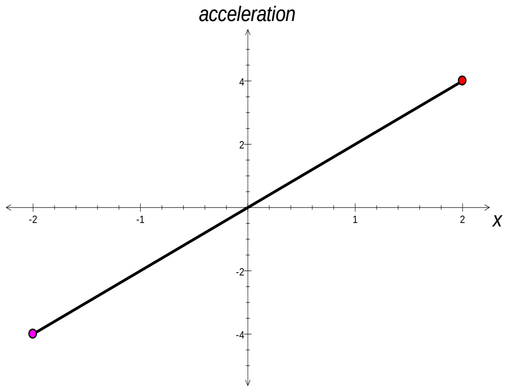
(A)



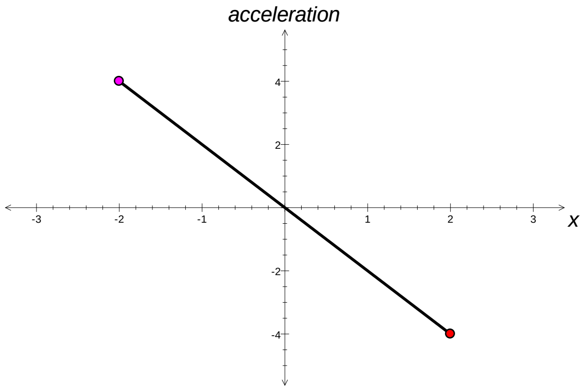
(B)



(C)

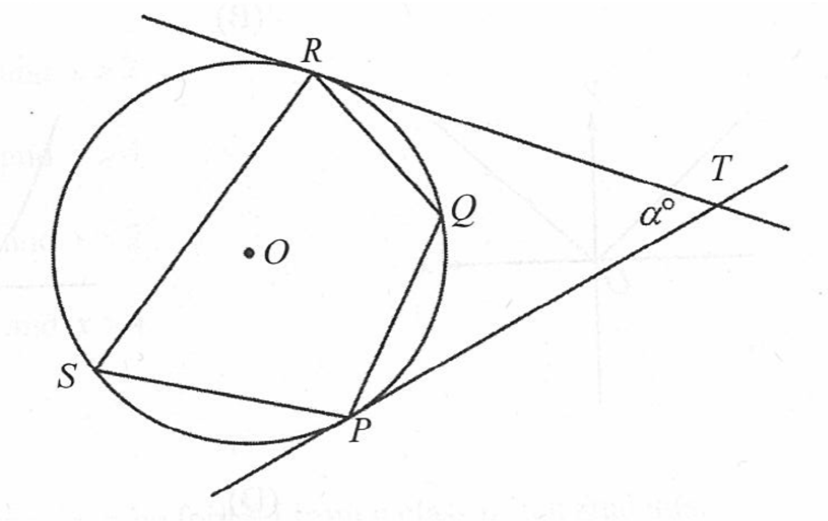


(D)



8. The points P, Q, R and S lie on a circle with centre at O.

The tangents at P and R meet at the point T and ∠RTP=α°



What is the size of ∠PQR in terms of α ?

(A) $(180 - \frac{\alpha}{2})^\circ$

(B) $(180 - \alpha)^\circ$

(C) $(90 + \frac{\alpha}{2})^\circ$

(D) $(90 + \alpha)^\circ$

9. When a dishwasher has completed its cycle, and the contents removed, metallic utensils cool such that the temperature of the utensils is given by $T = 20 + Ae^{-kt}$ where t is the time in minutes after the utensils have been removed from the dishwasher. The utensils cool from $50^\circ$to 35°in 5 minutes.

The exact value of *k* is?

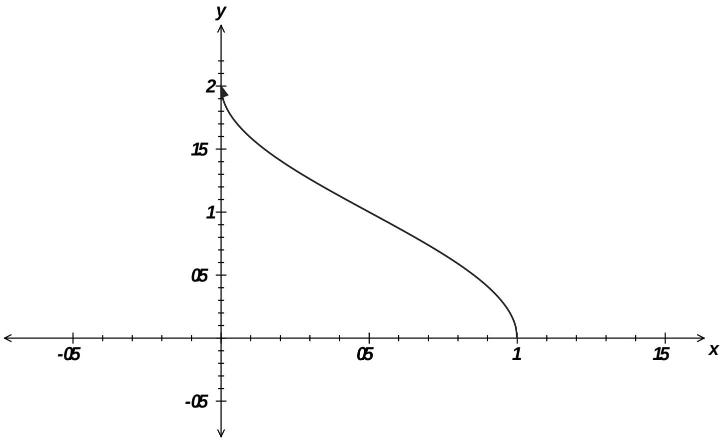
(A) $\frac{1}{5}\log_e 2$

(B) $\frac{1}{5}\log_e \frac{1}{2}$

(C) $\frac{1}{5}\log_e \frac{10}{3}$

(D) $\frac{1}{5}\log_e \frac{3}{10}$

10.



Which of the following could be the equation of the function shown above?

(A) $y = 2\cos^{-1}(2x-1)$

(B) $y = 2\cos^{-1}\left(\dfrac{x-1}{2}\right)$

(C) $y = \dfrac{2}{\pi}\cos^{-1}(2x-1)$

(D) $y = \dfrac{2}{\pi}\cos^{-1}\left(\dfrac{x-1}{2}\right)$

Section II begins on the next page

Section II

60 marks

Attempt Questions 11–14

Allow about 1 hour and 45 minutes for this section

Start each question on a new page. Extra paper is available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/ or calculations.

Question 11 (15 marks) Start a new page.

(a) Differentiate $e^{2x}\log_e 3x$ **2**

(b) Find the primitive function of $\frac{1}{1+9x^2}$ **2**

(c) Solve $\frac{x^2+3}{x} > 4.$ **2**

(d) The equation $\cos 2x = e^{-x}$ has a root near $x = 0.4$
Taking $x = 0.4$ as a first approximation, use Newton's method to find a second approximation to the root of the equation.
Give your answer correct to two significant figures. **2**

(e) Given that sin 18°=$\frac{\sqrt{5}-1}{4}$, use an appropriate expansion of cos 2θ to find the exact value of cos 36°. **3**

(f) A particle is moving such that its displacement (x metres) from a fixed point *O* after *t* seconds is given by $x = 3 - 2\cos 2t$.

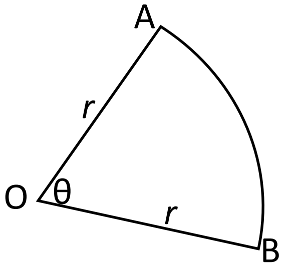
(i) Show that the particle is moving in simple harmonic motion. **2**

(ii) Find the maximum speed of the particle. **2**

Question 12 (15 marks) Start a new page.

(a) The general solution of the equation $\sin 3x = -\frac{1}{2}$ has the form $x = nA + (-1)^n B$, where *n* is an integer. Write down a value for *A* and *B*. **2**

(b) *AOB* is a sector of a circle with centre *O* and radius *r* cm, as shown in the diagram.



$\angle AOB = \theta$ radians

(i) The area of the sector *AOB* is 100 cm^2. Show that $\theta = \frac{200}{r^2}$. **1**

(iii) If the radius is increasing at a constant rate of 0.5 cms^{-1}, find the rate at which $\angle AOB$ is decreasing when *r* = 10 cm. **2**

(c) Find the coefficient of x^3 in the expansion of $(3x - \frac{2}{x})^9$ **3**

(d) Thu plays Garry in a game in which, in any one game, Thu's chances of winning is 0.6 and drawn games are not possible.

(i) If they agree to play 5 games, find the probability that Thu wins by 3 games to 2. (answer to 3 decimal places) **2**

(ii) If they agree to play until one of them wins 3 games, find the probability that Thu wins by 3 games to 2. (leave answer in unexpanded form) **2**

(e) A rocket is fired vertically upwards from the surface of the earth at a velocity of 5 kms^{-1}.

The acceleration of the rocket is given by $\ddot{x} = -\frac{80000}{x^2}$ kms^{-2}, where x is the distance of the rocket from the centre of the Earth. The radius of the Earth is 6400 km

(i) Show that the rocket never comes to rest. **2**

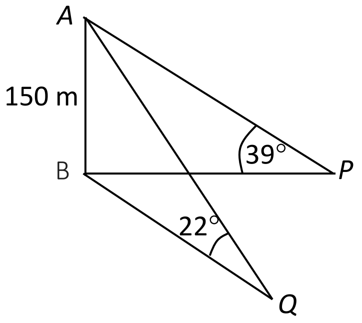
(ii) Find the rocket's velocity when it is 3600 km above the Earth's surface. **1**

Question 13 (15 marks) Start a new page.

(a) A is 150 metres above the horizontal plane BPQ. **3**
AB is vertical. The angle of elevation of A from P is 39°
and the angle of elevation of A from Q is 22°.

P is due East of B and Q is on a bearing of 138°T from B.

Calculate the distance from P to Q, to the nearest 10 metres.



(b) Find the volume of the solid formed when the region enclosed between the curves $y = sin\, x$ and $y = sin 2x$ over the domain $0 \leq x \leq \frac{\pi}{3}$ is rotated about the x axis **3**

(c) Use mathematical induction to prove that

$$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots\ldots\ldots+\frac{1}{n(n+1)(n+2)}=\frac{1}{4}-\frac{1}{2(n+1)(n+2)}$$

for all integers $n\geq1$. **3**

(d) Four couples, each consisting of a male and a female, sit around a circular table. In how many different ways is it possible to seat the four couples so that males and females sit in alternating positions and nobody sits next to his or her partner? **2**

(e) A rocket is fired from a pontoon on the sea. The rocket is aimed to land on a flat 60m high cliff which is 200 m from the pontoon. The angle of projection of the rocket is 45° and its initial velocity is $40\sqrt{2}$ m/s.

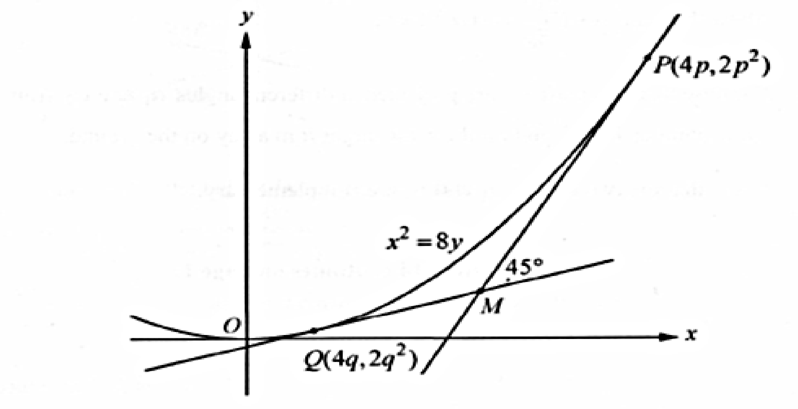
(i) Taking the point of projection as the origin O, derive expressions for the horizontal component *x(t)* and the vertical component *y(t)* of the position of the rocket at time t seconds. (Assume the acceleration due to gravity is 10m/s² and neglect air resistance). **2**

(ii) Find the distance the rocket lands from the edge of the cliff. **2**

Question 14 (15 marks) Start a new page.

(a) Let $f(x) = \frac{1}{\sqrt{1+x^2}}$ for $x \leq 0$.
Find an expression for the inverse function $f^{-1}(x)$ in terms of x **2**

(b) The points $P(4p, 2p^2)$ and $Q(4q, 2q^2)$ lie on the parabola $x^2 = 8y$.
The tangents at P and Q intersect at M.



The acute angle between these tangents is 45˚.

(i) Find the coordinates of M. **2**

(ii) Show that $| p - q | = | 1 + pq |$ **1**

(iii) By eliminating the parameters, show that the equation of the locus of M is

$$x^2 - y^2 - 12y - 4 = 0$$ **2**

(c) Use the substitution $u = t^2 + 2$ to find $\int t^3 \sqrt{t^2 + 2}\, dt$ **2**

(d) Let $f(x) = 2\cos^{-1} x \qquad -1 \leq x \leq 1$
and $g(x) = \sin^{-1}(2x^2 - 1) \qquad -1 \leq x \leq 0$

(i) Sketch the graph of $y = f(x)$ **1**

(ii) Show that $g'(x) = f'(x), \quad -1 < x < 0$ **2**

(iii) Hence, or otherwise, express $g(x)$ in terms of $f(x)$ **2**

(iv) Sketch the graph of $y = g(x)$ **1**

END OF EXAMINATION

Multiple Choice

① $\log_a 8 = 1.893$

$\log_a 2^3 = 3 \log_a 2 = 1.893$

$\cancel{\div 3}\ \log_a 2 = \frac{1.893}{3}$

$\log_a 2^2 = 2 \times \frac{1.893}{3}$

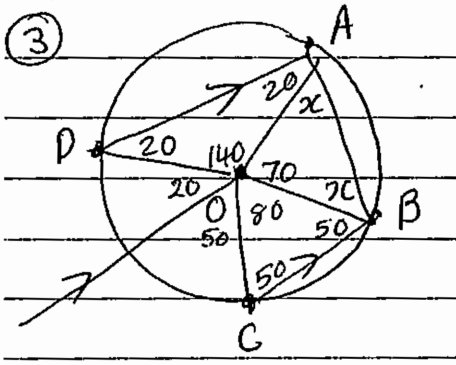
$= 1.262$ B.

② $\alpha + \beta + \gamma = -p$ $\alpha\beta + \beta\gamma + \gamma\alpha = -26$ $\gamma\beta\alpha = -24$

$-6 - 24 + 4 = -26.$ ✓ only B or D

$3 + 8 + 24 \neq -26.$ D

③



Construct OF∥AD

Join O to B

$\angle DOC = 20 + 50 = 70$

$\angle DOA = 180 - 20 - 20 = 140$

$\angle COB = 180 - 50 - 50 = 80$

$\angle BOA = 360 - 80 - 70 - 140 = 70$

$2x + 70 = 180$

$2x = 110$ D

$x = 55$

④ $\int \cos^2 \frac{2x}{5}\, dx = \int \frac{1}{2}\left(1 + \cos \frac{4x}{5}\right) dx$ $\cos 2A = \cos^2 A - \sin^2$

$= \frac{1}{2}\left(x + \frac{5}{4} \sin \frac{4x}{5}\right) + c$ $= 2\cos^2 A - 1$

$\cos^2 A = \frac{1}{2}(1 + \cos 2$

$= \frac{x}{2} + \frac{5}{8} \sin \frac{4x}{5} + C$ $\cos^2 \frac{2x}{5} = \frac{1}{2}(1 + \cos \frac{4}{5}$

A

⑤ $\lim_{x \to \infty} \frac{3x+7}{2-x}$

$= \lim_{x \to \infty} \frac{\frac{3x}{x} + \frac{7}{x}}{\frac{2}{x} - \frac{2}{x}}$

$= \frac{3}{-1} = -3$ B

⑥ $v^2 = -x^2 + 2x + 8$

At endpoints $v=0$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

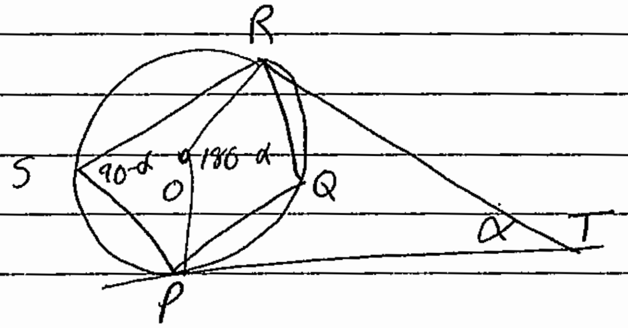
$\therefore$ Endpoints are $x=-2$ & $x=4$

Centre is at $x=1$

$\therefore$ Amplitude $= 3$ ~~A~~ B

⑦ D

⑧ $\angle TRO = \angle PPO = 90^\circ$ (tangent perp to radius at pt of contact).



$\angle ROP + \alpha = 180$ (opp ∠'s cyc quad)

$\angle ROP = 180 - \alpha$

$\angle ROP = 180 - \alpha$

$\angle ROP = 2 \times \angle RSP$ (angle at centre double angle at circum, same arc

$\angle RSP = \frac{1}{2}(180 - \alpha)$

$= 90 - \frac{\alpha}{2}$

SRQP is cyclic quad

$\therefore \angle RQP + \angle RSP = 180$

$\angle RQP = 180 - (90 - \frac{\alpha}{2})$

$= 90 + \frac{\alpha}{2}$

C

⑨ $t=0 \quad T=50$

$50 = 20 + A$

$A = 30$

$t=5, \quad T=35$

$35 = 20 + 30e^{-5k}$

$e^{-5k} = \frac{15}{30}$

$k = -\frac{1}{5}\log_e \frac{1}{2}$

$= -\frac{1}{5}(\log_e 1 - \log_e 2)$

$= \frac{\log_e 2}{5}$ or $\frac{1}{5}\log_e 2$ A

(10) For $y = \frac{2}{\pi}\cos^{-1}(2x-1)$

Range $0 \leq y \leq 2$

Domain $-1 \leq 2x - 1 \leq 1$

$0 \leq x \leq 1$ C

MC B D D A B B D C A C

MATHEMATICS Extension 1 : Question 11

Suggested Solutions | **Marks** | **Marker's Comments**

a) $y = e^{2x} \log_e 3x$

$y' = 2e^{2x} \log_e 3x + \frac{e^{2x}}{x}$

① ①

b) $\int \frac{1}{1+9x^2} \, dx$

① for getting $\tan^{-1}$

$= \frac{1}{3} \int \frac{3}{1+9x^2} \, dx$

① The rest of the function

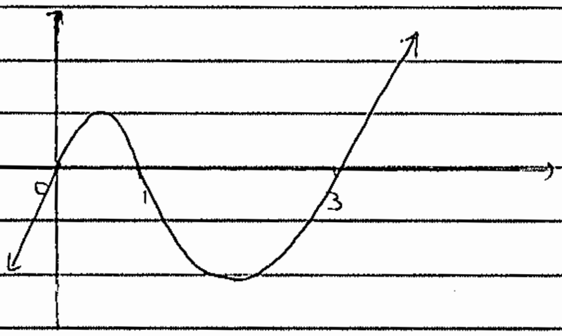
$= \frac{1}{3} \tan^{-1}(3x) + C$

c) $\frac{x^2+3}{x} > 4$

$x(x^2+3) > 4x^2$ ①

$x(x^2-4x+3) > 0$

$x(x-1)(x-3) > 0$



$\therefore 0 < x < 1$ or $x > 3$ ①

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments

d) Solve $f(x) = \cos 2x - e^{-x} = 0$

$$f'(x) = e^{-2x} - 2\sin 2x$$ ①

$$x_2 = 0.4 - \frac{f(0.4)}{f'(0.4)}$$

$= 0.43$ (2 sig fig) ①

No marks were deducted for significant figures.

e) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$ ①

$\therefore \cos 36 = 1 - 2\sin^2 18$

$= 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$

$= 1 - 2\left(\frac{5 - 2\sqrt{5} + 1}{16}\right)$ ①

$= 1 - \left(\frac{6 - 2\sqrt{5}}{8}\right)$

$= \frac{8 - 6 + 2\sqrt{5}}{8}$

$= \frac{2 + 2\sqrt{5}}{8}$ ①

$= \frac{1 + \sqrt{5}}{4}$

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments
f)		Note: SHM is defined to be a motion such that acceleration is proportional to its displacement from a fixed point but in the opposite direction.
i) $x = 3 - 2\cos 2t$		
$\dot{x} = 4\sin 2t$		
$\ddot{x} = 8\cos 2t$	①	
$= -4(-2\cos 2t)$		
$= -4(x-3)$		
$= -2^2(x-3)$		
which is of the form $\ddot{x} = -n^2(x-b)$		
∴ the motion is simple harmonic		
ii) $\dot{x} = 4\sin 2t$	①	
Max of $\sin 2t = 1$		
∴ Max velocity = 4 m/s.	①	
or		
max speed when $x = 3$ (centre of motion)		
∴ $\cos 2t = 0$		
$t = \pi/4$	①	
∴ $V = 4\sin\frac{\pi}{2} = 4$ m/s	①	

MATHEMATICS Extension 1 : Question..12..

Suggested Solutions	Marks	Marker's Comments
a) $\sin 3x = -\frac{1}{2}$		
$3x = n\pi + (-1)^n \sin^{-1}(-\frac{1}{2})$, $n \in Z$		
$3x = n\pi + (-1)^n (-\frac{\pi}{6})$	1	
$\therefore x = n\frac{\pi}{3} + (-1)^n (-\frac{\pi}{16})$		
$\therefore A = \frac{\pi}{3}$, $B = -\frac{\pi}{16}$	1	
b) i) Area of sector $= \frac{1}{2}r^2\theta$		
$\therefore \frac{1}{2}r^2\theta = 100$	1	
$r^2\theta = 200$		
$\theta = \frac{200}{r^2}$		
ii) $\frac{d\theta}{dt} = \frac{d\theta}{dr} \times \frac{dr}{dt}$		
$\frac{dr}{dt} = 0.5\,\text{cm s}^{-1}$		
$\theta = 200r^{-2}$		
$\frac{d\theta}{dr} = -400r^{-3}$		
$= \frac{-400}{r^3}$	1	
$\therefore \frac{d\theta}{dt} = \frac{-400}{r^3} \times 0.5$		
when $r = 10$		
$\frac{d\theta}{dt} = \frac{-400}{10^3} \times 0.5$		
$= -0.2$	1	
$\therefore \angle AOB$ is decreasing at a rate		
of 0.2 radians per second.		
c) $(3x - \frac{2}{x})^9$		
$T_{k+1} = {}^9C_k (3x)^{9-k} (-\frac{2}{x})^k$	1	
$= {}^9C_k\ 3^{9-k} (-2)^k x^{9-k-k}$		
for coefficient of x^3: $9 - 2k = 3$		
$2k = 6$		

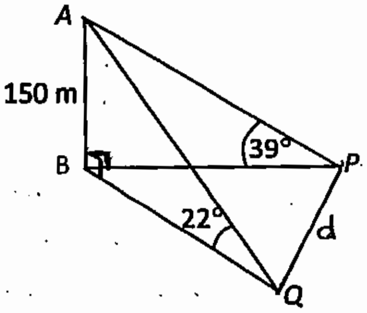
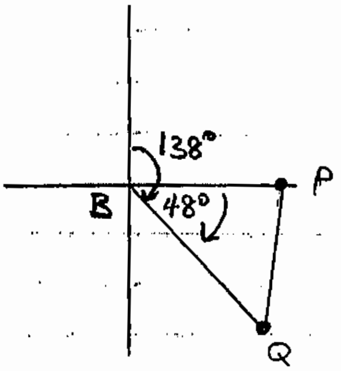
MATHEMATICS Extension 1 : Question..12...

Suggested Solutions	Marks	Marker's Comments
$\therefore k = 3$	1	
$\therefore T_4 = {}^9C_3\ 3^{9-3}(-2)^3\ x^3$		
$= -489\,888\ x^3$		
$\therefore$ coefficient of x^3 is $-489\,888$	1	
d) i) P(Thu wins by 3 to 2) $= {}^5C_3(0.6)^3(0.4)^2$	1	
$= 0.3456$		
$= 0.346$ (3dp)	1	
ii) Thu must win last game.		
Ordering first four games =		
${}^4C_2(0.6)^2(0.4)^2$		1 for explanation
last game: 0.6		
$\therefore P = {}^4C_2(0.6)^2(0.4)^2(0.6)$	1	
e) i) $\ddot{x} = \dfrac{-80000}{x^2}$		
$\dfrac{d}{dx}\left(\frac{1}{2}v^2\right) = -80000x^{-2}$		
$\int d\left(\frac{1}{2}v^2\right) = \int -80000x^{-2}\,dx$		
$\frac{1}{2}v^2 = \dfrac{80000}{x} + C$	2	
when $v = 5$, $x = 6400$		
$\therefore \frac{1}{2}(5)^2 = \dfrac{80000}{6400} + C$		
$\dfrac{25}{2} = \dfrac{25}{2} + C$		
$\therefore C = 0$		
$\therefore v^2 = \dfrac{160000}{x}$		
rocket comes to rest when $v=0$		
$0 = \dfrac{160000}{x}$		
$\therefore$ no solution		
$\therefore$ rocket does not come to rest.		

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	**Marks**	**Marker's Comments**
ii) $x = 6400 + 3600$ $= 10000$ $\therefore v^2 = \frac{160000}{10000}$ $= 16$ $v = \pm 4$ km/s Since initial velocity is positive and rocket never comes to rest ie. it will not change direction $\therefore v > 0$ $\therefore v = 4$ km/s	1	$\Rightarrow$ must have explanation to why $v > 0$.

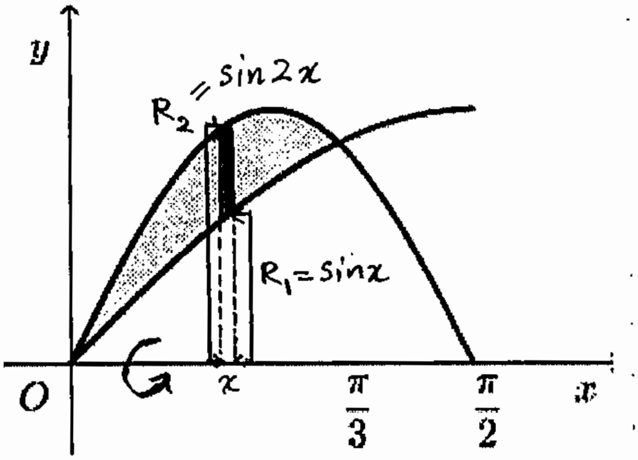
MATHEMATICS Ext 1: Question: 13

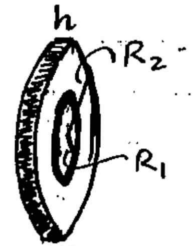
Suggested Solutions	Marks	Marker's Comments
(a)  		Handled well. Minor issues such as failing to take $\sqrt{\ }$ or mistake with signs in cosine rule.
$\angle PBQ = 138° - 90° = 48°$.		
Letting $d = PQ$, then, by cosine rule, $d^2 = BQ^2 + BP^2 - 2(BQ)(BP)\cos 48°$		
Now, $\tan 39° = \frac{150}{BP} \Rightarrow BP = 150 \cot 39°$ and $\tan 22° = \frac{150}{BQ} \Rightarrow BQ = 150 \cot 22°$.	1	
So $d^2 = (150 \cot 39°)^2 + (150 \cot 22°)^2$ $-2(150)^2 \cot 39° \cot 22°$ $= 80114.85 \ldots \ m^2$	1	
So $PQ = \sqrt{d^2} = 283.05 \ldots \ m$ $= 280 \ m$ to nearest 10 m.	1	

MATHEMATICS Ext 1: Question: 13

Suggested Solutions | **Marks** | **Marker's Comments**

(b) vi.





(volume) = (larger cylinder) − (smaller cylinder)

$$= \pi R_2^2 h - \pi R_1^2 h$$

$$= \pi (R_2^2 - R_1^2) h$$

Where, at x, $R_2 = \sin 2x$, $R_1 = \sin x$ and $h = \delta x$.

Also, curves intersect where

$$\sin 2x = \sin x$$

$$2 \sin x \cos x = \sin x$$

So, either $\sin x = 0$ or $\cos x = \frac{1}{2}$.

Over domain: $x = 0$ or $x = \pi/3$.

Primary error in setup: taking $(R_2 - R_1)^2$ instead of $R_2^2 - R_1^2$ - you're subtracting one cylindrical volume from another.

MATHEMATICS Ext 1: Question: 13

Suggested Solutions	Marks	Marker's Comments
So, $V=\pi\int_0^{\pi/3} \sin^2 2x - \sin^2 x \quad dx$	1	Factor of π missing from several responses; HSC will deduct a mark for this, so pay attention to your initial setup.
$=\pi\int_0^{\pi/3} \frac{1-\cos 4x}{2} - \frac{1-\cos 2x}{2} \quad dx$		
$=\frac{\pi}{2}\int_0^{\pi/3} \cos 2x - \cos 4x \quad dx$		
$=\frac{\pi}{2}\left[\frac{1}{2}\sin 2x - \frac{1}{4}\sin 4x \right\|_0^{\pi/3}$	1	
$=\frac{\pi}{2}\left[\left(\frac{1}{2}\sin\frac{2\pi}{3} - \frac{1}{4}\sin\frac{4\pi}{3}\right) - 0\right]$		
$=\frac{\pi}{2}\left[\frac{1}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{4}\left(-\frac{\sqrt{3}}{2}\right)\right]$		Another issue, integrand ordered incorrectly (e.g. $\sin^2 x - \sin^2 2x$, leading to <u>negative</u> volume → shouldn't happen if you're integrating correctly, so check your work. May <u>admit to</u> <u>error</u> and 'drop' sign.
$=\frac{\pi}{2}\cdot\frac{3\sqrt{3}}{8}$		
$=\frac{3\pi\sqrt{3}}{16}$ units^3.	1	

MATHEMATICS Ext 1: Question: 13

Suggested Solutions	Marks	Marker's Comments
(c) Let $S(n)$ be statement		Handled well.
$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\dots+\frac{1}{n(n+1)(n+2)}=\frac{1}{4}-\frac{1}{2(n+1)(n+2)}$		
where $n\in\mathbb{N}$, $n\geq1$.		Only issue with some: need to finish with a proper, concluding statement.
(I) Show $S(1)$ true:		
LHS $=\frac{1}{1\times(1+1)(1+2)}$ RHS $=\frac{1}{4}-\frac{1}{2(1+1)(1+2)}$	①	
$=\frac{1}{6}$ $=\frac{1}{4}-\frac{1}{12}$		
$=\frac{1}{6}$		
∴ LHS = RHS, so $S(1)$ true.		
(II) Assume true for $n=k$; i.e. assume $S(k)$ true where $k\in\mathbb{N}$.		
Then		
$\frac{1}{1\times2\times3}+\dots+\frac{1}{k(k+1)(k+2)}=\frac{1}{4}-\frac{1}{2(k+1)(k+2)}$		— ⊛
is taken to be the case.		
For $S(k+1)$:		
$\left(\frac{1}{1\times2\times3}+\dots+\frac{1}{k(k+1)(k+2)}\right)+\frac{1}{(k+1)((k+1)+1)((k+1)+2)}$		
$=\frac{1}{4}-\frac{1}{2(k+1)(k+2)}+\frac{1}{(k+1)(k+2)(k+3)}$	①	by hypothesis, ⊛

Suggested Solutions	Marks	Marker's Comments

$$= \frac{1}{4} - \left[\frac{1}{2(k+1)(k+2)} \cdot \frac{(k+3)}{(k+3)} - \frac{2}{2(k+1)(k+2)(k+3)} \right]$$

$$= \frac{1}{4} - \left[\frac{(k+1)}{2(k+1)(k+2)(k+3)} \right]$$

$$= \frac{1}{4} - \frac{1}{2((k+1)+1)((k+1)+2)}$$ ①

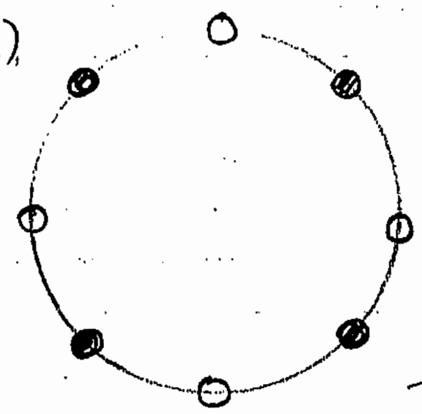
which is of the required form.

Hence we've shown that if $S(k)$ is true, then $S(k+1)$ is true : $S(k) \rightarrow S(k+1)$ —— ⊕

(III) Hence by principle of mathematical induction, since $S(1)$ is true, it follows by ⊕ that $S(2)$ is true; since $S(2)$ true, $S(3)$ is true, etc.

Hence $S(n)$ is true for all integers $n \geqslant 1$ //

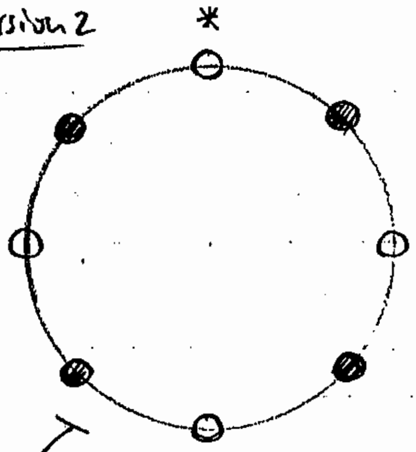
MATHEMATICS Ext 1: Question: 13

Suggested Solutions	Marks	Marker's Comments
(d)  arrange all males, say, giving		Not handled well.
$(4-1)! = 3!$ arrangements.	1	First mark for 'reasonable' start.
Then first F partner can be seated 2 ways. Once this occurs, all other F partners have no choice but to sit in a determined position.		
So, $3! \times 2 = 12$ ways.	1	Final mark for correct answer.

~~ELSE~~ **Suggested Solutions** | **Marks** | **Marker's Comments**

version 2

(d)



Let couples be assigned as

$\{M_i, F_i\}$

where $i = 1, 2, 3, 4$.

wlog, place M_1 at *

$\underset{*}{\underline{M_1}}$ _ _ _ _ _ _ $\underline{F_1}$

or

$\underset{*}{\underline{M_1}}$ $\underline{F_1}$ _ _ _ _ _ _

Then F_1 must be placed at pos^n ② or ⑧ (depending on which side of M_1 she sits.

Choices for male/female couples are then fixed as

$$\frac{1}{M_1} \times \frac{3}{F} \times \frac{1}{M} \times \frac{2}{F} \times \frac{1}{M} \times \frac{1}{F} \times \frac{1}{M} \quad \frac{1}{F_1}$$

or

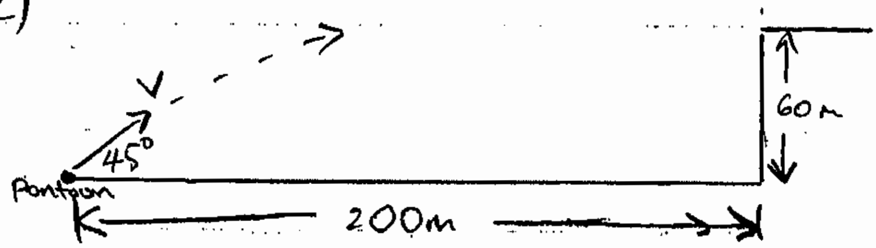
$$\frac{1}{M_1} \times \frac{1}{F_1} \times \frac{3}{M} \times \frac{1}{F} \times \frac{2}{M} \times \frac{1}{F} \times \frac{1}{M} \times \frac{1}{F}$$

So, $3! \times 2 = 12$ ways.

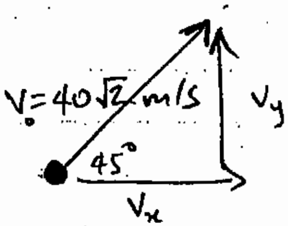
MATHEMATICS Ext 1: Question: 13

Suggested Solutions | **Marks** | **Marker's Comments**

(e)



(i)



Taking $g = 10\,ms^{-2}$:

$V_x = 40\sqrt{2}\sin 45^\circ = 40$ m/s

$V_y = 40\sqrt{2}\cos 45^\circ = 40$ m/s

Horizontal trajectory:

$\ddot{x} = 0$

$\dot{x} = c_1$ for all t.

Hence $\dot{x}(0) = c_1 = 40$ m/s:

so $\dot{x} = 40$ (hold units)

Then $x = 40t + c_2$

Marker's Comments

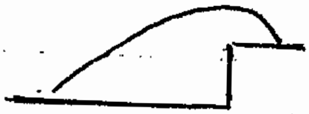
Part (i) handled well.

Only small issues:

* not evaluating sin45°, cos45°, or doing so incorrectly

* not substituting values given for pronumerals

Suggested Solutions	Marks	Marker's Comments
Now, $x(0)=0$, so $c_2=0$,		
so $\boxed{x(t)=40t}$ —①	1	
Vertical trajectory:		
$\ddot{y}=-10$ (hold units)		
so $\dot{y}=-10t+c_3$		
Now, $\dot{y}(0)=40$ m/s, so		
$c_3=40$ m/s.		
$\therefore \dot{y}=-10t+40$		
Hence $y=-\frac{10t^2}{2}+40t+c_4$		
$=-5t^2+40t+c_4$		
Now, $y(0)=0$, so $c_4=0$.		
Hence		
$\boxed{y=-5t^2+40t}$	1	

We look first for x when $y = 60$m (height of cliff), expecting this type of  situation Then, by (i) $60 = -5t^2 + 40t$ $\Rightarrow \quad t^2 - 8t + 12 = 0$ $\Rightarrow \quad (t-2)(t-6) = 0$ when $t = 2$, object reaches 60m in ascent; $t = 6$, reaches 60m in descent; take $t = 6$s.	1	First mark for 'logical' start.
when $x(6) = 40(6)$ $= 240$m. $240 - 200 = 40$ The rocket lands 40m from edge of cliff	1	

①

MATHEMATICS Extension 1: Question 14

Suggested Solutions	Marks Awarded	Marker's Comments
14 a) $y = \frac{1}{\sqrt{1+x^2}}$ $x \le 0$ $x = \frac{1}{\sqrt{1+y^2}}$ $y < 0$ $x^2 = \frac{1}{1+y^2}$ $1+y^2 = \frac{1}{x^2}$ $y^2 = \frac{1}{x^2} - 1$	1	welldone
$y = \pm\sqrt{\frac{1}{x^2} - 1}$ since $y < 0$ $y = -\sqrt{\frac{1}{x}^2 - 1}$	1	for correct answer students need to think about domain given.
b) $x^2 = 8y$ $y = \frac{x^2}{8}$ $\frac{dy}{dx} = \frac{2x}{8}$ $= \frac{x}{4}$ at $x = 4p$ $\frac{dy}{dx} = p$ $\therefore$ $y - 2p^2 = p(x - 4p)$ $y = px - 4p^2 + 2p^2$ $y = px - 2p^2$		

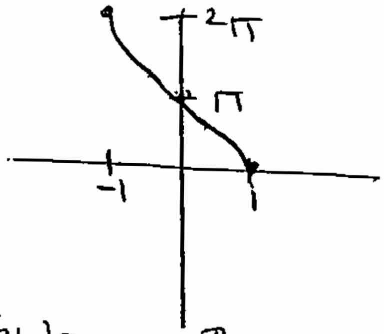
MATHEMATICS Extension 1 : Question 14

Suggested Solutions	Marks Awarded	Marker's Comments
Similarly $y=qx-2q^2$		question said find students needed to derive formula. for tangents
$\therefore \quad y=px-2p^2$		
$- \quad y=qx-2q^2$		
$0=(p-q)x-2(p^2-q^2)$		
$(p-q)x=2(p-q)(p+q)$		
$x=2(p+q)$		
$y=2q(p+q)-2q^2$		
$=2pq+2q^2-2q^2$		
$=\underline{2pq}$		
$M(2(p+q),\ 2pq)$	1	
(ii) $\tan 45^\circ = \left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$		
$m_p=p \quad m_q=q.$		
$1=\frac{\lvert p-q\rvert}{\lvert 1+pq\rvert}$		
$\lvert 1+pq\rvert=\lvert p-q\rvert$	1	
e(iii) $\lvert p-q\rvert=\lvert 1+pq\rvert$.		
$\therefore \quad (p-q)^2=(1+pq)^2$ — ①		
$x=2(p+q) \quad y=2pq$ — ②	1	

③

Suggested Solutions	Marks Awarded	Marker's Comments
MATHEMATICS Extension 1: Question..14..		
$p^2-2pq+q^2=1+2pq+p^2q^2$ Now $p+q=\frac{x}{2}$ $(p+q)^2=p^2+q^2+2pq$ $\frac{x^2}{4}=p^2+q^2+y$ since $y=2pq$ $\therefore\ p^2+q^2-2pq=\frac{x^2}{4}-y-y$ $1+2pq+p^2q^2=1+y+\frac{y^2}{4}$ $\therefore\ \frac{x^2}{4}-2y=1+y+\frac{y^2}{4}$	1	Needed to explain what you were doing
$x^2-8y=4+4y+y^2$ $x^2-y^2-12y-4=0$		
(c) let $u=t^2+2$ $\frac{du}{dt}=2t$. $\int t^2\sqrt{t^2+2}\ t\,dt$. $=\int(u-2)\sqrt{u}\,\frac{du}{2}$	1	students did not put brackets in OR had $\sqrt{u-2}$
$=\frac{1}{2}\int u^{3/2}-2u^{1/2}\,du$.		

④

Suggested Solutions	**Marks Awarded**	**Marker's Comments**
MATHEMATICS Extension 1 14: Question……..		
$= \frac{1}{2}\left(\frac{2u^{5/2}}{5} - \frac{4u^{3/2}}{3}\right)$ $= \frac{1}{5}(t^2+1)^{5/2} - \frac{2(t^2+1)^{3/2}}{3}$	1	Had to integrate substitute back if you lost fractional index did not get ~~final~~ ~~mark~~ CFE.
d) $f(x) = 2\cos^{-1}x \quad -1 \le x \le 1$ $g(x) = \sin^{-1}(2x^2-1) \quad -1 \le x \le 0$ (i) $y = f(x)$. 	1	well done but think about scale π=3.14 some graphs very distorted.
(ii) $f'(x) = \frac{-2}{\sqrt{1-x^2}}$. $g'(x) = \frac{1}{\sqrt{1-(2x-1)^2}} \cdot 4x$ $= \frac{4x}{\sqrt{1-4x^4+4x^2-1}}$ $= \frac{4x}{2\lvert x\rvert\sqrt{1-x^2}}$ Now if $-1 < x < 0$ then $\lvert x\rvert = -x$ so $= \frac{4x}{-2x\sqrt{1-x^2}}$	1	Needn't clear explanation not sufficient just to write $-1 < x < 0$.

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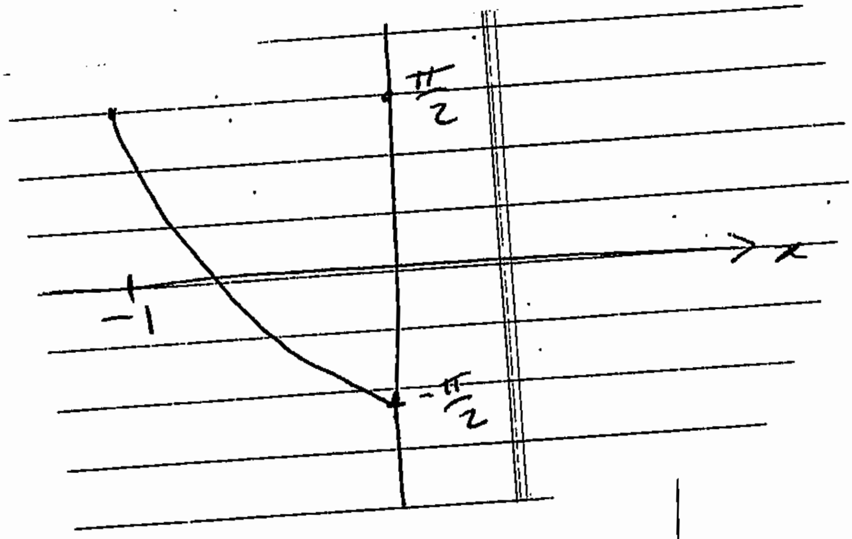
MATHEMATICS Extension 1: Question 14

Suggested Solutions	Marks Awarded	Marker's Comments
$\therefore g'(x) = -\frac{2}{\cancel{2}\sqrt{1-x^2}}$		
$\therefore f'(x) = g'(x)$		
iii) If $f'(x) = g'(x)$ then $f(x) = g(x) + c$	1	
$\sin^{-1}(2x^2-1) = 2\cos^{-1}x + c$ when $x = 0$ $\sin^{-1}\overset{-1}{\cancel{0}} = 2\cos^{-1}0 + c$ $-\frac{\pi}{2} = \pi + c$		value had to be in domain $-1 \le x \le 0$
$c = -\frac{3\pi}{2}$ $\therefore g(x) = f(x) - \frac{3\pi}{2}$	1	some students didn't write $g(x)$ in terms of $f(x)$.
(iv) 		

MATHEMATICS Extension 1: Question 14

Suggested Solutions	Marks Awarded	Marker's Comments
$\therefore g'(x) = \frac{-2}{\cancel{2}\sqrt{1-x^2}}$		
$\therefore f'(x) = g'(x)$		
iii) If $f'(x) = g'(x)$ then $f(x) = g(x) + c$	1	
$\sin^{-1}(2x^2-1) = 2\cos^{-1}x + c$		
when $x = 0$ $\sin^{-1}-1 = 2\cos^{-1}0 + c$ $-\frac{\pi}{2} = \pi + c$		value had to be in domain $-1 \le x \le 0$
$c = -\frac{3\pi}{2}$		
$\therefore g(x) = f(x) - \frac{3\pi}{2}$	1	some students didn't write $g(x)$ in terms of $f(x)$.
(iv)