Student Number: ______________

KAMBALA

AUGUST 2007
YEAR 12
HSC ASSESSMENT TASK#4
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using blue or black pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.

Total marks – 84

- Attempt Questions 1-7.
- All questions are of equal value.

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Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

QUESTION 1 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $\frac{4}{x-1} \geq 1$ for x. **3**

(b) Write down the inverse function of $y = e^{2x+1}$. Express your answer as a function of x. **2**

(c) Use the table of standard integrals to evaluate $\int_0^{\frac{\pi}{12}} \sec 3x \tan 3x \, dx$. **2**

(d) Given $f(x) = \tan^{-1}(\sin x)$ find $f'(\pi)$. **2**

(e) The curves $y = \ln x$ and $y = 1 - x^2$ intersect at the point $P(1, 0)$. **3**

Find the acute angle between the tangents to the curves at P. Give your answer to the nearest degree.

Marks

QUESTION 2 (12 marks) Use a SEPARATE writing booklet.

(a) The point P divides the interval AB in the ratio $k:1$ where A and B are the points $(4,-1)$ and $(0,3)$ respectively.

(i) Write down the co-ordinates of P in terms of k. 1

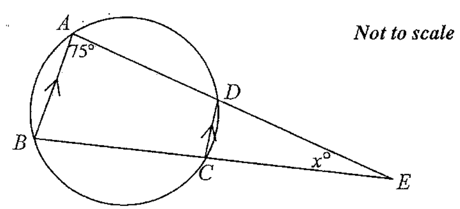
(ii) If P lies on the line $y=2x$, find k and hence find P. 3

(b) The polynomial equation $2x^3-4x^2+5x-3=0$ has roots α, β and γ. 2

Find the exact value of $\frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$.

(c) Evaluate $\int_0^{\log 3} \frac{e^x}{\sqrt{1+e^x}}\,dx$, using the substitution $u=e^x$. 3

(d) In the diagram below, AB is parallel to DC, $\angle BAD=75°$ and $\angle DEC=x°$. 3



Find the value of x, giving reasons for your answer.

Marks

QUESTION 3 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Express $6\cos x+8\sin x$ in the form $R\cos(x-\alpha)$, where $R>0$ and α is an acute angle in radians. 2

(ii) Hence, or otherwise, solve the equation $6\cos x+8\sin x=5$ for $0\le x\le 2\pi$. 2

(b) Find $\frac{d}{dx}\left(\sin^{-1}(x-1)\right)$ in its simplest form, and hence deduce that 4

$$\int_{\frac{1}{2}}^{1} \frac{dx}{\sqrt{x(2-x)}}=\frac{\pi}{6}$$

(c) (i) Sketch the graph of $y=\cos x$, $-\pi\le x\le\pi$, and use this graph to show that $\cos x+x=0$ has only one solution. 2

(ii) Use Newton's method with a first approximation of $x=-1$ to find a better approximation to the root of $\cos x+x=0$. 2

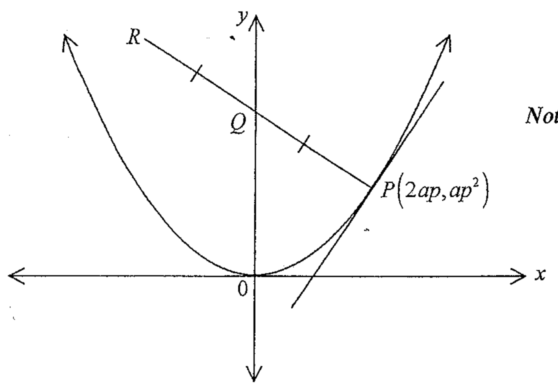
Marks

QUESTION 4 (12 marks) Use a SEPARATE writing booklet.

(a) Using $t = \tan\frac{x}{2}$, prove that $\frac{1+\cos x}{1-\cos x} = \cot^2\frac{x}{2}$ **2**

(b) Use the identity $\sin 2x = 2\sin x\cos x$ to find $\int \sin^2 x\cos^2 x\,dx$. **3**

(c) The normal to the parabola $x^2 = 4ay$ at the point $P(2ap, ap^2)$ meets the y-axis at Q and is produced to R so that $PQ = QR$ as shown.



(i) Show that the equation of the normal to the parabola $x^2 = 4ay$ at the point $P(2ap, ap^2)$ is $x + py = 2ap + ap^3$. **2**

(ii) Find the co-ordinates of Q. **1**

(iii) Show that the co-ordinates of R are $(-2ap, 4a + ap^2)$. **2**

(iv) Find the cartesian equation of the locus of R. **2**

Marks

QUESTION 5 (12 marks) Use a SEPARATE writing booklet.

(a) A vessel used for water is a solid of revolution formed by the rotation of the parabola $9y = 8x^2$ about the y-axis. The depth of the vessel is 8 cm.

(i) Show that when the height of the water is $h\text{ cm}$ from its base the volume is given by $V = \frac{9}{16}\pi h^2\text{ cm}^3$. **2**

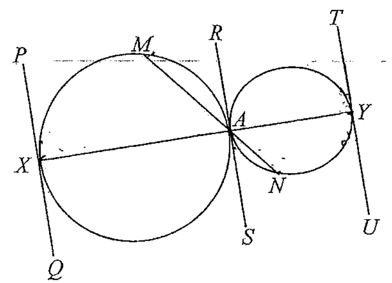
(ii) Show that when the vessel is half full the height of the water is $4\sqrt{2}$ cm. **1**

(iii) If water is poured into the vessel at a rate of $20\text{cm}^3/\text{sec}$, find the rate at which the level of water is rising when the vessel is half full. **2**

(b) (i) Sketch the graph of $y = \tan x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. **1**

(ii) Hence, or otherwise, find values of x in the domain $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ such that the series $1 + \sqrt{3}\tan x + 3\tan^2 x + 3\sqrt{3}\tan^3 x + \ldots$ has a limiting sum. **2**

(c) Two circles touch at the point A. Lines through A meet the circles at X and Y and at M and N respectively, as shown. RS, the tangent at A is shown. **4**



Copy the diagram into your writing booklets.

Prove that PQ and TU, the tangents at X and Y respectively, are parallel.

Marks

QUESTION 6 (12 marks) Use a SEPARATE writing booklet.

(a) The rate of increase of a population P of sandflies at Sand Fly Point on the Milford Track is proportional to the difference between the population, P, and 2000. This rate can be expressed by the differential equation $\frac{dP}{dt} = k(P-2000)$, where k is a constant and t represents time in weeks.

(i) Show that $P = 2000 + Ae^{kt}$, where A is a constant, satisfies the differential equation. **1**

(ii) Initially, the population is 2500 and two weeks later it had increased to 5000. Find the values of A and k. **2**

(iii) After how many weeks will the population of sandflies exceed 10 000? **1**

(b) A particle moves in a straight line so that its velocity after t seconds is v ms^{-1} and its displacement from the origin is x metres.

(i) Given that $\ddot{x} = 10x - 2x^3$ and that $v = 0$ when $x = -1$, find v^2 in terms of x. **2**

(ii) Explain why the motion cannot exist between $x = -1$ and $x = 1$. **1**

(iii) Describe briefly what would have happened if the motion had commenced at $x = 0$ with $v = 0$. **1**

(c) (i) Use Mathematical Induction to prove that for all positive integers n **3**

$$4(1^3 + 2^3 + 3^3 + ... + n^3) = n^2(n+1)^2$$

(ii) Hence find the value of $\lim_{x\to\infty}\left(\frac{1^3+2^3+3^3+...+n^3}{n^4}\right)$ **1**

Marks

QUESTION 7 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch a possible shape of the continuous function $y = f(x)$ given that $f(0) = 3$, $\lim_{x\to\infty} f(x) = 0$ and that it has only one zero, at $x = -1$. **1**

(ii) Explain why $y = f(x)$ has an inflexion point to the right of $x = -1$. **1**

(b) A particle is moving about the origin O according to the rule $x = 4\sin 3t$ where x metres is the displacement from O at time t seconds.

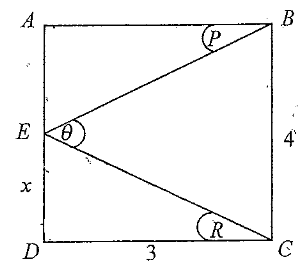
(i) Show that the motion is simple harmonic. **2**

(ii) Determine when the particle is at $x = 2$ metres for the first time. **2**

(iii) Write down the maximum speed of the particle. **1**

(c) A billboard, BC, is 4 metres high and is positioned vertically and parallel to a highway at a height which maximises $\tan\theta$, where θ is the angle subtended at the eyes of the passengers on the top deck of passing double-decker buses. The billboard must be 3 metres from the passengers.

Let $\angle ABE = P$, $\angle DCE = R$ and $ED = x$.



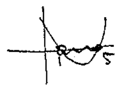
Not to scale

(i) Copy the diagram into your answer booklet and express θ in terms of P and R. **1**

(ii) Show that $\tan\theta = \frac{12}{9-4x+x^2}$. **4**

Hence find the value of x which maximises $\tan\theta$.

End of Assessment

Question	Solutions	Marks	Marking Criteria
1 (a)	$\frac{4}{x-1} \geqslant 1 \qquad x \neq 1$ $4(x-1) \geqslant (x-1)^2$ $(x-1)^2 - 4(x-1) \leqslant 0$ $(x-1)(x-1-4) \leqslant 0$ $(x-1)(x-5) \leqslant 0$  $1 < x \leqslant 5$		
(b)	inverse: $x = e^{2y+1}$ $2y+1 = \ln x$ $2y = \ln x - 1$ $y = \frac{1}{2}(\ln x - 1)$		
(c)	$\int_0^{\frac{\pi}{12}} \sec 3x \tan 3x \, dx$ $= \left[\frac{1}{3} \sec 3x\right]_0^{\frac{\pi}{12}}$ $= \frac{1}{3} \sec \frac{\pi}{4} - \frac{1}{3} \sec 0$ $= \frac{1}{3}(\sqrt{2} - 1)$		
(d)	$f(x) = \tan^{-1}(\sin x)$ $f'(x) = \frac{1}{1+\sin^2 x} \times \cos x$ $f'(\pi) = \frac{\cos \pi}{1+\sin^2 \pi}$ $= -1$		
(e)	$y = \ln x \Rightarrow y' = \frac{1}{x}$ $m_T = 1$ $y = 1 - x^2 \Rightarrow y' = -2x$ $m_T = -2$ $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $= \left\lvert \frac{1 - -2}{1 + 1 \times -2} \right\rvert$ $= 3 \qquad \therefore \theta = 72^\circ$		

Question	Solutions	Marks	Marki
2 (a)(i)	$x_p = \frac{4 \times 1 + 0 \times k}{k+1} \qquad y_p = \frac{-1 \times 1 + 3 \times k}{k+1}$ $P\left(\frac{4}{k+1}, \frac{3k-1}{k+1}\right)$		
(ii)	P lies on $y = 2x$ $\therefore \frac{3k-1}{k+1} = 2 \times \frac{4}{k+1}$ $3k - 1 = 8$ $k = 3$ $\therefore P = (1, 2)$		
(b)	$\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma} = \frac{\gamma + \beta + \alpha}{\alpha\beta\gamma}$ $= \frac{-\frac{(-4)}{2}}{-\frac{(-3)}{2}}$ $= \frac{4}{3}$		
(c)	$\int_0^{\ln 3} \frac{e^x}{\sqrt{1+e^x}} \, dx$ $u = e^x \quad \frac{du}{dx} = e^x$; $x = 0 \quad u = 1$; $x = \ln 3 \quad u = 3$; $\sqrt{1+e^x} = (1+u)^{\frac{1}{2}}$ $= \int_1^3 (1+u)^{-\frac{1}{2}} du$ $= \left[\frac{(1+u)^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^3$ $= 2(\sqrt{4} - \sqrt{2})$ $= 4 - 2\sqrt{2}$		
(d)	$\angle EDC = 75^\circ$ corresp. angles AB||CD $\angle ECD = 75^\circ$ ext. angle of cyclic quad equals int. opposite ∠ $\therefore x = 180^\circ - (2 \times 75^\circ)$ ∠ sum of Δ $= 30^\circ$		

Question	Solutions	Marks	Marking Criteria

3 (a) (i)

$R\cos(x-\alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$

$6\cos x + 8\sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$

$\therefore R\cos\alpha = 6$

$R\sin\alpha = 8$

square + add $\Rightarrow R = 10 \quad (R>0)$

$\tan\alpha = \frac{4}{3} \Rightarrow \alpha = \tan^{-1}\left(\frac{4}{3}\right)$

$= 0.927\ldots$

$6\cos x + 8\sin x = 10\cos(x - 0.927)$

(ii)

$6\cos x + 8\sin x = 5$

ie $10\cos(x-0.927) = 5$

$\cos(x-0.927) = \frac{1}{2}$

$x - 0.927 = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3} + 0.927, \frac{5\pi}{3} + 0.927$

$x = 1.974, 6.163$

(b)

$\frac{d}{dx}\left(\sin^{-1}(x-1)\right) = \frac{1}{\sqrt{1-(1-x)^2}} \times 1$

$= \frac{1}{\sqrt{1-(1-2x+x^2)}}$

$= \frac{1}{\sqrt{2x-x^2}}$

$= \frac{1}{\sqrt{x(2-x)}}$

$\therefore \int_{\frac{1}{2}}^{1} \frac{dx}{\sqrt{x(2-x)}} = \left[\sin^{-1}(x-1)\right]_{\frac{1}{2}}^{1}$

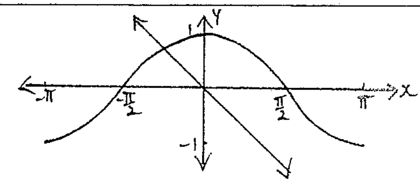
$= \sin^{-1}(0) - \sin^{-1}\left(-\frac{1}{2}\right)$

$= 0 - \left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{6}$

Question	Solutions	Marks	Marking Criteria

3 (c) (i)



$\cos x + x = 0$ ie solve simult.

$y = \cos x$ and $y = -x$

only 1 point of intersection

$\therefore \cos x + x = 0$ has only one solution

(ii)

$f(x) = \cos x + x \qquad f(-1) = -0.45\ldots$

$f'(x) = -\sin x + 1 \qquad f'(-1) = 1.84\ldots$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

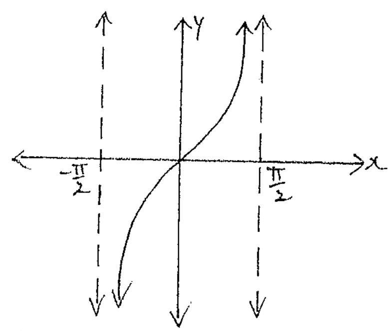
$= -1 - \frac{-0.45\ldots}{1.84\ldots}$

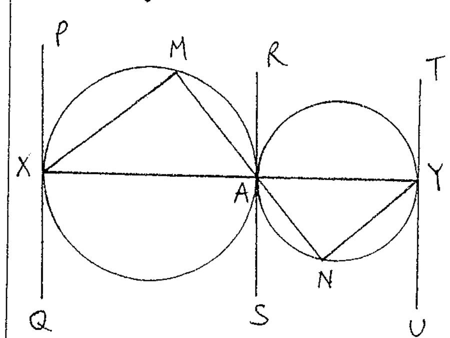
$= -0.75$

$\therefore$ better approximation is -0.75

Question	Solutions	Marks	Marking Criteria
4 (a)	$t = \tan\frac{x}{2} \qquad \cos x = \frac{1-t^2}{1+t^2}$ $\text{LHS} = \frac{1+\cos x}{1-\cos x}$ $= \frac{1+\frac{1-t^2}{1+t^2}}{1-\frac{1-t^2}{1+t^2}}$ $= \frac{\frac{1+t^2+1-t^2}{1+t^2}}{\frac{1+t^2-(1-t^2)}{1+t^2}}$ $= \frac{2}{2t^2}$ $= \frac{1}{t^2}$ $= \frac{1}{\tan^2\frac{x}{2}}$ $= \cot^2\frac{x}{2}$		
(b)	$\sin 2x = 2\sin x\cos x$ $\sin^2 x\cos^2 x = (\sin x\cos x)^2$ $= \left(\frac{1}{2}\sin 2x\right)^2$ $= \frac{1}{4}\sin^2 2x$ $\int \sin^2 x\cos^2 x\,dx = \frac{1}{4}\int \sin^2 2x\,dx$ $\sin^2 2x = \frac{1}{2}(1-\cos 4x)$ $\therefore \int \sin^2 x\cos^2 x\,dx = \frac{1}{8}\int (1-\cos 4x)\,dx$ $= \frac{1}{8}\left(x - \frac{1}{4}\sin 4x\right) + C$		
(c) (i)	$x^2 = 4ay \Rightarrow y = \frac{x^2}{4a}$ $y' = \frac{x}{2a} \quad \therefore m_T = p \quad m_N = -\frac{1}{p}$ eq. normal $y - ap^2 = -\frac{1}{p}(x - 2ap)$ $py - ap^3 = -x + 2ap$ ie $x + py = 2ap + ap^3$		

Question	Solutions	Marks	Marking Criteria
4 (c) (ii)	at Q (on the normal) $x = 0$ $\therefore py = 2ap + ap^3$ $y = 2a + ap^2$ $Q(0,\ 2a + ap^2)$		
(iii)	Q is midpt. of RP $\therefore 0 = \frac{x_R + 2ap}{2} \Rightarrow x_R = -2ap$ $2a + ap^2 = \frac{y_R + ap^2}{2} \Rightarrow y_R = 4a + ap^2$ $\therefore R(-2ap,\ 4a + ap^2)$		
(iv)	$x = -2ap \Rightarrow p = -\frac{x}{2a}$ $y = 4a + ap^2$ $y = 4a + a\left(\frac{-x}{2a}\right)^2$ $y = 4a + \frac{x^2}{4a}$ $4ay = 16a^2 + x^2$ $x^2 = 4ay - 16a^2$ $x^2 = 4a(y - 4a)$		

Question	Solutions	Marks	Marking Criteria
5 (a) (i)	$V=\pi\int_0^h x^2\,dy$ volume about y-axis $=\pi\int_0^h \frac{9y}{8}\,dy$ $=\pi\left[\frac{9y^2}{16}\right]_0^h$ $=\frac{9}{16}\pi h^2 \text{ cm}^3$		
(ii)	vessel is full when $h=8$ $V=\frac{9}{16}\pi\times 8^2=36\pi$ $\therefore \frac{1}{2}V=18\pi$ $18\pi=\frac{9}{16}\pi h^2$ $h^2=32$ $h=\sqrt{32}=4\sqrt{2}$ cm		
(iii)	$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ find $\frac{dh}{dt}$ when $h=4\sqrt{2}$ $\frac{dV}{dt}=20 \text{ cm}^3/\text{s}$ $\frac{dV}{dh}=\frac{9}{8}\pi h$ $20=\frac{9}{8}\pi\times 4\sqrt{2}\times\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{40}{9\pi\sqrt{2}}=\frac{20\sqrt{2}}{9\pi}$ cm/s		
(b) (i)			

Question	Solutions	Marks	Marking Criteria
5 (b) (ii)	$1+\sqrt{3}\tan x+3\tan^2 x+\dots$ For a limiting sum $-1<r<1$ $\therefore -1<\sqrt{3}\tan x<1$ $\frac{-1}{\sqrt{3}}<\tan x<\frac{1}{\sqrt{3}}$ $\frac{-\pi}{6}<x<\frac{\pi}{6}$		
(c)			
	Join XM and YN $\angle PXM=\angle MAX$ $\angle$ between tangent + chord = $\angle$ in the alternate segment $\angle MAX=\angle YAN$ vertically opposite $\angle YAN=\angle UYN$ alt. segment theory $\therefore \angle PXM=\angle UYN$ $\angle SAN=\angle NYA$ alt. segment theory $\angle RAM=\angle SAN$ vert. opposite $\angle RAM=\angle MXA$ alt. segment theory $\therefore \angle MXA=\angle NYA$ $\angle PXA=\angle PXM+\angle MXA$ $\angle UYA=\angle UYN+\angle NYA$ $\therefore \angle PXA=\angle UYA$ alternate angles are equal $\therefore PQ\parallel TU$		When circles touch the line of centres passes through the point of contact. $\therefore AX$ and AY are diameters $\therefore \angle PXA=90°$ and $\angle UYA=90°$ tangent to a circle is perpendicular to the radius drawn to the point of contact. $\angle PXA=\angle UYA$ alternate angles are equal $\therefore PQ\parallel TU$

Question	Solutions	Marks	Marking Criteria
6 (a) (i)	$P = 2000 + Ae^{kt} \Rightarrow Ae^{kt} = P - 2000$ $\frac{dP}{dt} = kAe^{kt}$ $= k(P-2000)$		
(ii)	$t=0 \quad P=2500$ $2500 = 2000 + Ae^0$ $A = 500$ $t = 2 \quad P = 5000$ $5000 = 2000 + 500e^{2k}$ $e^{2k} = 6$ $2k = \ln 6$ $k = \frac{1}{2}\ln 6 = 0.895\ldots$		
(iii)	$P = 10000$ $10000 = 2000 + 500e^{\frac{1}{2}\ln 6 t}$ $e^{\frac{1}{2}\ln 6 t} = 16$ $\frac{1}{2}\ln 6 \, t = \ln 16$ $t = \frac{2\ln 16}{\ln 6}$ $= 3.09$ weeks in the 3rd week		
(b) (i)	$\ddot{x} = 10x - 2x^3$ $\therefore \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 10x - 2x^3$ $\frac{1}{2}v^2 = 5x^2 - \frac{x^4}{2} + c$ $v=0 \quad x=-1 \Rightarrow 0 = 5 - \frac{1}{2} + c$ $c = -4\frac{1}{2}$ $\frac{1}{2}v^2 = 5x^2 - \frac{x^4}{2} - \frac{9}{2}$ $v^2 = 10x^2 - x^4 - 9$		
(ii)	For $-1 < x < 1 \quad v^2 < 0$ which is possible $\therefore$ motion doesn't exist between $x=-1$ and $x=1$		

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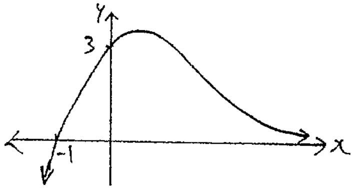
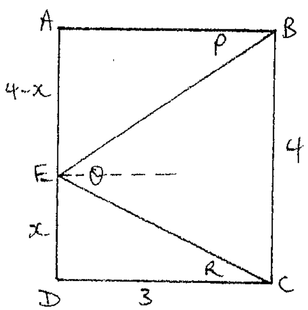
Question	Solutions	Marks	Marking Criteria
6 (b) (ii)	Solve $v^2 < 0$ $10x^2 - x^4 - 9 < 0$ ie $x^4 - 10x^2 + 9 > 0$ $(x^2-9)(x^2-1) > 0$ $x < -3, \ -1 < x < 1, \ x > 3$		
(iii)	$\frac{1}{2}v^2 = 5x^2 - \frac{x^4}{2} + c$ if $x=0$ and $v=0$ then $\ddot{x} = 0$ stationary particle and no acceleration $\therefore$ particle would not move.		
(c) (i)	Show true for $n=1$ $LHS = 4 \quad RHS = 1^2(2)^2 = 4$ $LHS = RHS \ \therefore$ true for $n=1$ Assume true for $n=k$ ie $4(1^3+2^3+\ldots+k^3) = k^2(k+1)^2$ Show true for $n=k+1$ ie $4(1^3+2^3+\ldots+k^3+(k+1)^3) = (k+1)^2(k+2)^2$ $LHS = 4(1^3+2^3+\ldots+k^3) + 4(k+1)^3$ $= k^2(k+1)^2 + 4(k+1)^3$ $= (k+1)^2(k^2+4k+4)$ $= (k+1)^2(k+2)^2$ $= RHS$ $\therefore$ true for $n=k+1$ if true for $n=k$ since true for $n=1$ also true for $n=1+1=2$ & thus true for $n=2+1=3$ & so on for all positive integral n.		
(ii)	$\frac{1^3+2^3+\ldots+n^3}{n^4} = \frac{\frac{1}{4}n^2(n+1)^2}{n^4} = \frac{\frac{1}{4}(n+1)^2}{n^2}$ $\lim_{n\to\infty} \frac{\frac{1}{4}(n+1)^2}{n^2} = \lim_{n\to\infty} \frac{\frac{1}{4}(n^2+2n+1)}{n^2}$ $= \lim_{n\to\infty} \frac{\frac{1}{4}\left(1+\frac{2}{n}+\frac{1}{n^2}\right)}{1}$ $= \frac{1}{4}$ since $\lim_{n\to\infty}\frac{1}{n} = 0$ $\therefore \lim_{n\to\infty} \frac{1^3+2^3+\ldots+n^3}{n^4} = \frac{1}{4}$		

Question	Solutions	Marks	Marking Criteria
7 (a) (i)			
(ii)	concave down around $x=-1$ passes through $(0,3)$ but to return to x-axis ($f(x)\to 0$ from above) must change from concave down to concave up ie pt. of inflexion to the right of $x=-1$		
(b) (i)	$x = 4\sin 3t$ $\dot{x} = 12\cos 3t$ $\ddot{x} = -36\sin 3t$ $= -9(4\sin 3t)$ $= -9x$ which is of the form $\ddot{x} = -n^2x$ $\therefore$ motion is simple harmonic		
(ii)	$2 = 4\sin 3t$ $\sin 3t = \frac{1}{2}$ $3t = \frac{\pi}{6}$ $t = \frac{\pi}{18}$ s		
(iii)	max. speed 12 m/s		
(c) (i)	 $\theta = P + R$		

Year 12 Mathematics Extension 1 Assessment #4 (Trial Examination) August 2007

Question	Solutions	Marks	Marking Criteria
7 (c) (ii)	$\tan\theta = \tan(P+R)$ $= \frac{\tan P + \tan R}{1 - \tan P \tan R}$ $= \frac{\frac{4-x}{3} + \frac{x}{3}}{1 - \frac{4-x}{3}\times\frac{x}{3}}$ $= \frac{\frac{4}{3}}{\frac{9-(4x-x^2)}{9}}$ $= \frac{12}{9-4x+x^2}$ differentiate $\tan\theta$ wrt x $\frac{d(\tan\theta)}{dx} = -12(9-4x+x^2)^{-2}\times(-4+2x)$ $= \frac{48-24x}{(9-4x+x^2)^2}$ max when $\frac{d(\tan\theta)}{dx} = 0$ ie $48-24x = 0$ $x = 2$ test: x: 2^-, 2, 2^+; $\frac{d(\tan\theta)}{dx}$: $+$, 0, $-$ — max $\tan\theta$ is a maximum when $x = 2$		