KAMBALA

ESTO SOL TESTIS

2013
Higher School Certificate
Trial Examination

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in Question 11 – 14
- Write your student number and/or name at the top of every page

Total marks – 70

Section I - Pages 3 – 5

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 6 – 9

60 marks

Attempt Questions 11 – 14

Allow about 1 hour 45 minutes for this section

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ……………………..

Student name / number ....

Marks

Section 1

10 marks
Attempt Questions 1-10
Allow 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 The remainder obtained when $P(x) = x^3 - 3x^2 - 5x + 6$ is divided by $(x-3)$ will be: **1**

(A) -34

(B) -9

(C) 6

(D) 21

2 Which of the following is an expression for $\frac{d}{dx}\left(2^x\right)$? **1**

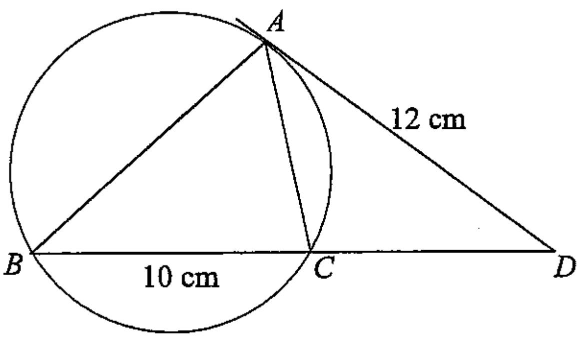
(A) $x2^{x-1}$

(B) 2^{x-1}

(C) 2^x

(D) $2^x \log_e 2$

3



Not to scale

ABC is a triangle inscribed in a circle. The tangent to the circle at A meets BC produced at D where $BC = 10$ cm and $AD = 12$ cm. What is the length of CD ? **1**

(A) 6 cm

(B) 7 cm

(C) 8 cm

(D) 9 cm

Student name / number ....

Marks

4 The equation $2x^3 + x^2 - 13x + 6 = 0$ has roots α, $\frac{1}{\alpha}$ and β. What is the value of β? **1**

(A) 3

(B) 2

(C) -3

(D) -6

5 Which of the following is an expression for $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)$? **1**

(A) $\dfrac{-x^2}{1+x^2}$

(B) $\dfrac{-1}{1+x^2}$

(C) $\dfrac{1}{1+x^2}$

(D) $\dfrac{x^2}{1+x^2}$

6 Which of the following lines is a horizontal asymptote of the curve $y = \dfrac{e^x - 2}{e^x + 2}$? **1**

(A) $y = -2$

(B) $y = -1$

(C) $y = 0$

(D) $y = 2$

7 After t years the number N of individuals in a population is given by $N = 400 + 100e^{-0 \cdot 1t}$. **1**
What is the difference between the initial population size and the limiting population size?

(A) 100

(B) 300

(C) 400

(D) 500

Student name / number ....

Marks

8 What is the derivative of $y = \cos^{-1}(\frac{1}{x})$ with respect to x? 1

(A) $\frac{-1}{\sqrt{x^2-1}}$

(B) $\frac{-1}{x\sqrt{x^2-1}}$

(C) $\frac{1}{\sqrt{x^2-1}}$

(D) $\frac{1}{x\sqrt{x^2-1}}$

9 Which of the following statements is FALSE. 1

(A) $\cos^{-1}(-\theta) = -\cos^{-1}\theta$.

(B) $\sin^{-1}(-\theta) = -\sin^{-1}\theta$

(C) $\tan^{-1}(-\theta) = -\tan^{-1}\theta$

(D) $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

10 Which of the following is the domain of the function $y = \ln\left(x + \sqrt{x^2+1}\right)$? 1

(A) $\{x : x \leq -1, x \geq 1\}$

(B) $\{x : -1 \leq x \leq 1\}$

(C) $\{x : x$ is an element of the set of real numbers$\}$

(D) $\{x : x \geq 1\}$

Student name / number ....

Marks

Section 2

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Attempt each question in a SEPARATE writing booklet. Extra writing booklets are available.
In Questions 11-14 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch the function $y = |x^2 - 4|$. **1**

(ii) At what points is $y = |x^2 - 4|$ not differentiable? **1**

(b) $A(-1, 4)$ and $B(7, -2)$ are two points. Find the coordinates of the point P that divides the interval AB internally in the ratio $3:2$. **2**

(c) Find correct to the nearest degree the acute angle θ between the lines $3x - 2y = 0$ and $x + 3y = 0$. **2**

(d) Find the exact value of $\displaystyle\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\, dx$. **3**

(e) Use the substitution $t = \tan\frac{x}{2}$ to show that $\dfrac{1 + \cos x + \sin x}{1 - \cos x + \sin x} = \cot\frac{x}{2}$. **3**

(f) $P(2at, at^2)$ is a point on the parabola $x^2 = 4ay$ with focus $F(0, a)$.

(i) Use differentiation to show that the tangent to the parabola at P has gradient t and equation $tx - y - at^2 = 0$. **2**

(ii) Show that the shortest distance between the focus and this tangent is $a\sqrt{1 + t^2}$. **1**

Student name / number ....

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) If $x + \frac{1}{x} = 3$, find the value of $x^2 + \frac{1}{x^2}$. **2**

(b) Solve the inequality $\frac{2x-1}{x+2} > 1$. **3**

(c) Use the substitution $u = 6 - x$ to find the exact value of $\int_1^6 x\sqrt{6-x}\,dx$. **3**

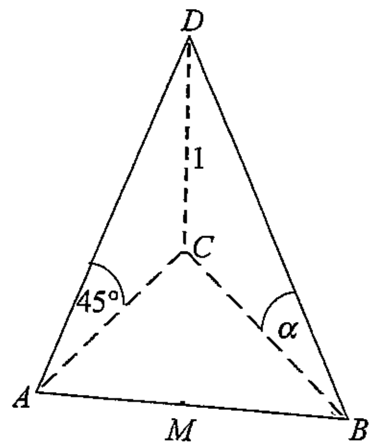
(d) Consider the statement:

$S(n): 2^n - (-1)^n$ *is divisible by 3,* where n is a positive integer.

(i) Show that $S(1)$ and $S(2)$ are true. **1**

(ii) Show that if $S(k)$ is true for all positive integers k then $S(k+2)$ is also true. **2**

(e)



CD is a vertical pole of height 1 metre that stands with its base C on horizontal ground.
A is a point due South of C such that the angle of elevation of D from A is $45°$.
B is a point due East of C such that the angle of elevation of D from B is α.
M is the midpoint of AB.

(i) Show that $AB = \operatorname{cosec}\alpha$. **2**

(ii) Show that $CM = \frac{1}{2}\operatorname{cosec}\alpha$. **2**

Student name / number ....

Marks

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

(i) Show that $f(-x) = -f(x)$ **1**

(ii) Show that $f(x) = 1 - \dfrac{2}{e^{2x}+1}$. **1**

(iii) Explain why $f(x) < 1$ for all values of x. **1**

(b) Consider the function $f(x) = \sin^{-1}(x-1)$.

(i) Find the domain of the function. **1**

(ii) Sketch the graph of the curve $y = f(x)$ showing the endpoints and the x intercept. **2**

(iii) The region in the first quadrant bounded by the curve $y = f(x)$ and the y axis between the lines $y = 0$ and $y = \frac{\pi}{2}$ is rotated through one complete revolution about the y axis. Find in simplest exact form the volume of the solid of revolution. **3**

(c) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds its displacement from a fixed point O on the line is x metres, given by $x = 4\sqrt{2}\cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$, its velocity is v ms^{-1} and its acceleration is $\ddot{x}$ ms^{-2}.

(i) Find the amplitude and period of the motion. **2**

(ii) Find the initial position of the particle and determine if it is initially moving towards or away from O. **2**

(iii) Find the distance travelled by the particle in the first 3 seconds of its motion. **2**

Student name / number ....

Marks

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = 2 - \log_e x$.

(i) Find the equation of the inverse function $f^{-1}(x)$. **1**

(ii) Explain why the x coordinate X of the point of intersection P of the graphs $y = f(x)$ and $y = f^{-1}(x)$ satisfies the equation $e^{2-X} - X = 0$. **2**

(iii) Use two applications of Newton's Method with an initial value of $X = 1 \cdot 5$ to find the value of X correct to two decimal places. **3**

(b) A vertical building of height 60 metres stands on horizontal ground. A particle is projected from a point O at the top of the building with speed $V = 20\sqrt{2}\ \text{ms}^{-1}$ at an angle α above the horizontal. It moves in a vertical plane under gravity where the acceleration due to gravity is $g = 10\ \text{ms}^{-2}$ and hits the ground at a distance 120 metres from the foot of the building. At time t seconds its horizontal and vertical displacements from O are x metres and y metres respectively, given by $x = 20\sqrt{2}\, t\cos\alpha$ and $y = 20\sqrt{2}\, t\sin\alpha - 5t^2$. (Do NOT prove these results.)

(i) Show that $\alpha = \frac{\pi}{4}$ *or* $\alpha = \tan^{-1}\frac{1}{3}$. **2**

(ii) If $\alpha = \tan^{-1}\frac{1}{3}$, find the exact time taken for the particle to hit the ground. **2**

(iii) If $\alpha = \frac{\pi}{4}$, find the exact speed of the particle after 6 seconds. **2**

(c) The acceleration of a creature is given by $\ddot{x} = -\frac{1}{2}u^2 e^{-x}$, where x is the displacement from the origin, and u is the initial velocity at the origin. Given that u = 2m/sec:

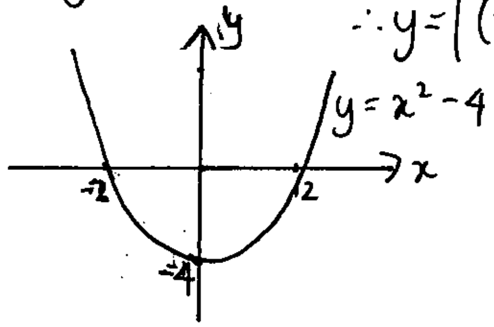
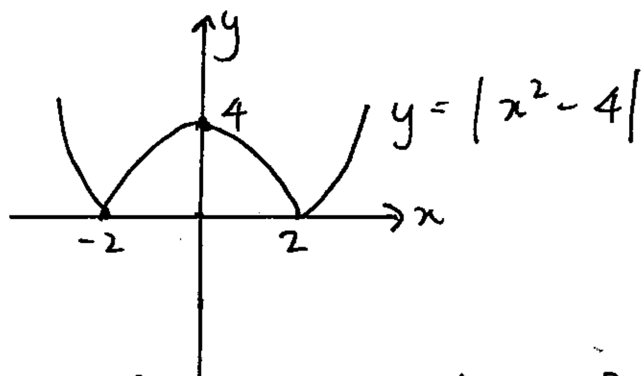
(i) Show that $v^2 = 4e^{-x}$. **2**

(ii) Explain why $v > 0$. **1**

Qn	Solutions	Marks	Comments & Criteria
	Section 1		
1.	$P(x) = x^3 - 3x^2 - 5x + 6$ remainder $= P(3)$ $P(3) = (3)^3 - 3(3)^2 - 5(3) + 6$ $= -9$ B		
2.	$\frac{d}{dx}(2^x)$ $= \frac{d}{dx}(e^{\ln 2^x})$ $= \frac{d}{dx}(e^{x\ln 2})$ $= e^{x\ln 2} . \ln 2$ $= 2^x . \ln 2$ D		
3.	let $CD = x$ $\therefore (x+10)x = (12)^2$ $x^2 + 10x = 144$ $x^2 + 10x - 144 = 0$ $(x+18)(x-8) = 0$ $x = -18, 8$ but x is a length, $\therefore x > 0$ $\therefore x = 8$ cm C		

Qn	Solutions	Marks	Comments & Criteria
4.	$2x^3 + x^2 - 13x + 6 = 0$ roots are $\alpha, \frac{1}{\alpha}, \beta$ $\therefore (\alpha)(\frac{1}{\alpha})(\beta) = -\frac{6}{2}$ $\therefore \beta = -3$ (C)		
5.	$\frac{d}{dx}\tan^{-1}(\frac{1}{x})$ $= \frac{1}{1+(\frac{1}{x})^2}\left(-\frac{1}{x^2}\right)$ $= \frac{-1}{x^2+1}$ (B)		
6.	$y = \frac{e^x - 2}{e^x + 2}$ $\lim_{x \to -\infty} \frac{e^x - 2}{e^x + 2}$ $= \frac{0-2}{0+2}$ $= -1$ $\therefore y = -1$ (B)		

Qn	Solutions	Marks	Comments & Criteria
7.	$N = 400 + 100e^{-0.1t}$ at $t=0$, $N = 400 + 100e^{0}$ $= 400 + 100$ $= 500$ $\therefore$ initial population is 500 $\lim_{t \to \infty} N = 400$ $\therefore$ difference $= 500 - 400$ $= 100$ A		
8.	$\frac{d}{dx} \cos^{-1}\left(\frac{1}{x}\right)$ $= \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot -x^{-2}$ $= \frac{+1}{x^2\sqrt{\frac{x^2-1}{x^2}}}$ $= \frac{1}{x\sqrt{x^2-1}}$ D		
9.	A (since $\cos^{-1}(-\theta) = \cos^{-1}(\theta)$)		
10.	C		

Qn	Solutions	Marks	Comments & Criteria
	Section 2.		
11a)	i) $y = \lvert x^2 - 4 \rvert$ $\therefore y = \lvert (x+2)(x+2) \rvert$		
			
			
(ii)	not differentiable at $x = \pm 2$		
b)	$A(-1, 4)$ $B(7, -2)$		
	$m:n = 3:2$		
	$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$		
	$= \left(\frac{(3)(7) + (2)(-1)}{3+2}, \frac{(3)(-2) + (2)(4)}{3+2} \right)$		
	$= \left(\frac{21 - 2}{5}, \frac{-6 + 8}{5} \right)$		

$$P = \left(\frac{19}{5}, \frac{2}{5} \right)$$

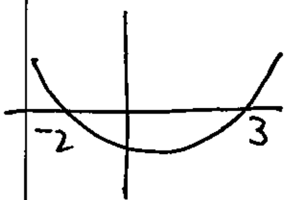
Qn	Solutions	Marks	Comments & Criteria
11c)	$3x-2y=0 \quad x+3y=0$ $m_1=\frac{3}{2} \quad m_2=-\frac{1}{3}$ $\tan\theta=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$ $=\frac{\frac{3}{2}+\frac{1}{3}}{1+\left(\frac{3}{2}\right)\left(-\frac{1}{3}\right)}$ $=\frac{\frac{11}{6}}{1-\frac{1}{2}}$ $=\frac{\frac{11}{6}}{\frac{1}{2}}$ $=\frac{11}{3}$ $\therefore \theta=\tan^{-1}\left(\frac{11}{3}\right)$ $\therefore \theta=75^\circ$ (to nearest degree)		

Qn	Solutions	Marks	Comments & Criteria
11d)	$\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$ $= \left[\sin^{-1}\frac{x}{2}\right]_{\sqrt{2}}^{\sqrt{3}}$ $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{\pi}{3} - \frac{\pi}{4}$ $= \frac{\pi}{12}$		
11e)	$\frac{1+\cos x+\sin x}{1-\cos x+\sin x} = \cot\frac{x}{2}$ let $t = \tan\frac{x}{2}$ $\therefore$ LHS: $\frac{1+\cos x+\sin x}{1-\cos x+\sin x}$ $= \dfrac{1+\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}}$		

Qn	Solutions	Marks	Comments & Criteria
1(e) ctd	$= \dfrac{\dfrac{1+t^2+1-t^2+2t}{1+t^2}}{\dfrac{1+t^2-1+t^2+2t}{1+t^2}}$ $= \dfrac{\dfrac{2t+2}{1+t^2}}{\dfrac{2t^2+2t}{1+t^2}}$ $= \dfrac{2(t+1)}{1+t^2} \cdot \dfrac{1+t^2}{2t(t+1)}$ $= \dfrac{2}{2t}, \quad t^2 \neq -1$ $= \dfrac{1}{t}$ $= \cot \dfrac{x}{2}$ $=$ RHS		

Qn	Solutions	Marks	Comments & Criteria
11f)	$P(2at, at^2) \quad x^2 = 4ay$		
i)	$y = \frac{x^2}{4a}$ $y' = \frac{2x}{4a}$ $= \frac{x}{2a}$ at P, $y' = \frac{2at}{2a}$ $= t$ as required eqn: $y - at^2 = t(x - 2at)$ $y - at^2 = xt - 2at^2$ $\therefore tx - y - at^2 = 0$ as required		
ii)	$F(0, a)$ $d = \left\lvert \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right\rvert$ $= \left\lvert \frac{0 + (-1)(a) + (-at^2)}{\sqrt{t^2 + (-1)^2}} \right\rvert$ $= \left\lvert \frac{-a(1+t^2)}{\sqrt{1+t^2}} \right\rvert$		

$= a\sqrt{1+t^2}$ as required

Qn	Solutions	Marks	Comments & Criteria
12a)	$x+\frac{1}{x}=3$ $\left(x+\frac{1}{x}\right)^2$ $=x^2+2+\frac{1}{x^2}$ $\therefore x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$ $=(3)^2-2$ $=7$		
b)	$\frac{2x-1}{x+2}>1 \quad , x\neq -2$ $(2x-1)(x+2)>(x+2)^2$ $2x^2+3x-2>x^2+4x+4$ $x^2-x-6>0$ $(x-3)(x+2)>0$  $\therefore x<-2, x>3$		

Qn	Solutions	Marks	Comments & Criteria
12c)	$\int_1^6 x\sqrt{6-x}\,dx$ $u=6-x$ When $x=1$, $u=5$ When $x=6$, $u=0$ $\frac{du}{dx}=-1$ $\therefore dx=-du$ $\int_5^0 (6-u).\sqrt{u}.-du$ $=\int_0^5 \left(6u^{\frac{1}{2}}-u^{\frac{3}{2}}\right)du$ $=\left[\frac{2.6u^{\frac{3}{2}}}{3}-\frac{2u^{\frac{5}{2}}}{5}\right]_0^5$ $=\left[4\sqrt{u^3}-\frac{2}{5}\sqrt{u^5}\right]_0^5$ $=\left(4\sqrt{125}-\frac{2}{5}\sqrt{3125}\right)-0$ $=20\sqrt{5}-\frac{2}{5}\times 25\sqrt{5}$ $=20\sqrt{5}-10\sqrt{5}$ $=10\sqrt{5}$		

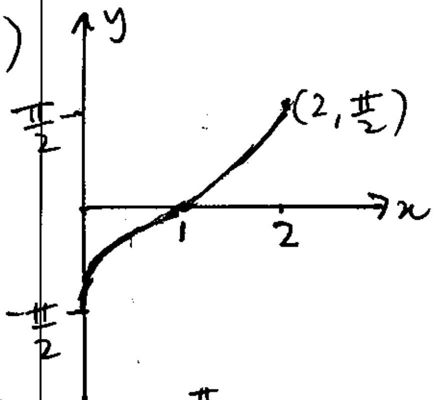
Qn	Solutions	Marks	Comments & Criteria
12d)	$S(n)$: $2^n - (-1)^n$ divisible by 3		
i)	$S(1)$: $2^1 - (-1)^1$ $= 2 + 1$ $= 3$, divisible by 3 $S(2)$: $2^2 - (-1)^2$ $= 4 - 1$ $= 3$, divisible by 3 $\therefore S(1), S(2)$ are true		
ii)	If $S(k)$ is true, then $2^k - (-1)^k = 3M$ for some integer M RTP: $S(k+2)$ is true $S(k+2) = 2^{k+2} - (-1)^{k+2}$ $= 2^2 \cdot 2^k - (-1)^k(-1)^2$ $= (3m + (-1)^k)2^2 - (-1)^k$ $= 12m + 4(-1)^k - (-1)^k$ $= 12M + 3(-1)^k$ $= 3(4M + (-1)^k)$,		

divisible by 3

$\therefore$ true for $S(k+2)$

Qn	Solutions	Marks	Comments & Criteria
12d) iii)	If $S(1)$ and $S(2)$ are true and $S(k+2)$ is true whenever $S(k)$ is true, then, by the Principle of Mathematical Induction, the statement is true for all positive integers n.		
12e) (i)	In ΔABC: $AC = 1$ $BC = \cot\alpha$ $\angle ACB = 90^\circ$ $\therefore (AB)^2 = 1 + \cot^2\alpha$ (by Pythagoras' Theorem) $= \text{cosec}^2\alpha$ $\therefore AB = \text{cosec}\,\alpha$ as required		
ii)	A unique circle can be drawn through A, B and C. $\angle ACB = 90^\circ$, $\therefore AB$ is a diameter of this circle, with centre M. MC and MB are radii. $\therefore MC = MB = \frac{1}{2}\text{cosec}\,\alpha$ as required.		

Qn	Solutions	Marks	Comments & Criteria
13a)	$f(x) = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$		
i)	$f(-x) = \dfrac{e^{-x} - e^{x}}{e^{-x} + e^{x}}$ $-f(x) = \dfrac{-(e^{x} - e^{-x})}{e^{x} + e^{-x}}$ $= \dfrac{e^{-x} - e^{x}}{e^{-x} + e^{x}} = f(-x)$ $\therefore f(-x) = -f(x)$ as required		
ii) RTP	$f(x) = 1 - \dfrac{2}{e^{2x}+1}$ LHS: $\dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$ $= \dfrac{e^{2x} - 1}{e^{2x}+1}$ (multiply by $\dfrac{e^{x}}{e^{x}}$) $= \dfrac{e^{2x}+1-2}{e^{2x}+1}$ $= 1 - \dfrac{2}{e^{2x}+1}$ $=$ RHS		

Qn	Solutions	Marks	Comments & Criteria
13a) iii)	$\frac{2}{e^{2x}+1}$ is positive for all values of x $\therefore f(x) = 1 - \frac{2}{e^{2x}+1}$ will have a maximum value of 1 $\therefore f(x) < 1$		
13b)	$f(x) = \sin^{-1}(x-1)$		
i)	Domain: $\{x : -1 < x-1 < 1\}$ $\therefore \{x : 0 < x < 2\}$		
ii)			
iii)	$V = \pi \int_0^{\frac{\pi}{2}} (1+\sin y)^2 \, dy$ $= \pi \int_0^{\frac{\pi}{2}} \{1 + 2\sin y + \frac{1}{2}(1-\cos 2y)\} \, dy$ $= \pi \left[1\frac{1}{2}y - 2\cos y - \frac{1}{4}\sin 2y\right]_0^{\frac{\pi}{2}}$ $= \pi \{(\frac{3\pi}{4} - 0 - 0) - (0 - 2 - 0)\}$		

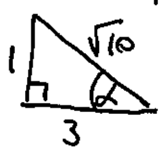
$= \pi \left(\frac{3\pi}{4} + 2\right)$ units^3

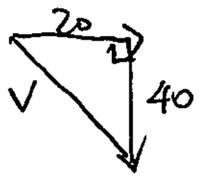
Qn	Solutions	Marks	Comments & Criteria
13c)	$x = 4\sqrt{2}\cos\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$		
i)	amplitude $= 4\sqrt{2}$ metres period $= \dfrac{2\pi}{\frac{\pi}{4}}$ $= 8$ seconds		
ii)	at $t = 0$, $x = 4\sqrt{2}\cos\left(0 - \frac{\pi}{4}\right)$ $x = (4\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)$ $x = 4$ m to the right of 0. $\dot{x} = \frac{dx}{dt} = -4\sqrt{2} \cdot \frac{\pi}{4}\sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$ $= -\pi\sqrt{2}\sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$ at $t = 0$, $\dot{x} = -\pi\sqrt{2}\sin\left(0 - \frac{\pi}{4}\right)$ $\therefore \dot{x} = -\pi\sqrt{2} \cdot -\frac{1}{\sqrt{2}}$ $= \pi > 0$ $\therefore$ particle moving away from 0.		

Qn	Solutions	Marks	Comments & Criteria
13c) iii)	at $t=0$, $x=4$ at $t=1$, $x=4\sqrt{2}\cos\left(\frac{\pi}{4}-\frac{\pi}{4}\right)$ $\therefore x=4\sqrt{2}\cos(0)$ $x=4\sqrt{2}$ at $t=2$, $x=4\sqrt{2}\cos\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$ $\therefore x=4\sqrt{2}\cos\left(\frac{\pi}{4}\right)$ $x=4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$ $\therefore x=4$ at $t=3$, $x=4\sqrt{2}\cos\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)$ $\therefore x=4\sqrt{2}\cos\left(\frac{\pi}{2}\right)$ $x=0$ $\therefore$ Distance travelled $=(4\sqrt{2}-4)+\lvert(4-4\sqrt{2})\rvert+\lvert(0-4)\rvert$ $=4\sqrt{2}-4-4+4\sqrt{2}+4$ $=(8\sqrt{2}-4)$ metres		

Qn	Solutions	Marks	Comments & Criteria
14a)	$f(x) = 2 - \log_e x$		
i)	$\therefore y = 2 - \log_e x$ $\therefore \log_e x = 2 - y$ $x = e^{2-y}$ $\therefore f^{-1}(x) = e^{2-x}$		
ii)	The graphs of $f(x)$ and $f^{-1}(x)$ are reflections in the line $y = x$, so any points of intersection of these graphs must lie on the line $y = x$. If the graphs intersect at a point where $x = X$, then $f(X) = f^{-1}(X) = x$ $\therefore e^{2-x} = X$ $\therefore e^{2-x} - X = 0$ as required		

Qn	Solutions	Marks	Comments & Criteria
14 a) iii)	Let $g(x) = e^{2-x} - x$ $\therefore g'(x) = -e^{2-x} - 1$ $x - \dfrac{g(x)}{g'(x)} = \dfrac{x(e^{2-x}+1) + e^{2-x} - x}{e^{2-x}+1}$ $= \dfrac{e^{2-x}(x+1)}{e^{2-x}+1}$ $= \dfrac{e^{2-x}(x+1)}{e^{2-x}\left(1+\frac{1}{e^{2-x}}\right)}$ $= \dfrac{x+1}{1+(e^{2-x})^{-1}}$ $= \dfrac{x+1}{1+e^{x-2}}$ for $X = 1{\cdot}5$, $X - \dfrac{g(x)}{g'(x)} = \dfrac{2{\cdot}5}{1+e^{-0{\cdot}5}}$ $\doteqdot 1{\cdot}56$ for $X = 1{\cdot}56$, $X - \dfrac{g(x)}{g'(x)} = \dfrac{2{\cdot}56}{1+e^{-0{\cdot}44}}$ $\doteqdot 1{\cdot}56$ $\therefore X = 1{\cdot}56$ (to 2 dec. pl.)		

Qn	Solutions	Marks	Comments & Criteria
14b)	$V = 20\sqrt{2}$ $g = 10$ $x = 20\sqrt{2}\, t \cos\alpha$ $y = 20\sqrt{2}\, t \sin\alpha - 5t^2$		
i)	When $x = 120,\ y = -60$ $\therefore\ t = \dfrac{x}{20\sqrt{2}\cos\alpha}$ $\therefore\ 20\sqrt{2}\left(\dfrac{120}{20\sqrt{2}\cos\alpha}\right)\sin\alpha - 5\left(\dfrac{120}{20\sqrt{2}\cos\alpha}\right)^2 = -60$ $120\tan\alpha - 90\sec^2\alpha = -60$ $4\tan\alpha - 3(1+\tan^2\alpha) = -2$ $(3\tan\alpha - 1)(\tan\alpha - 1) = 0$ $\therefore\ \tan\alpha = \frac{1}{3},\ 1$ $\therefore\ \alpha = \tan^{-1}\left(\frac{1}{3}\right),\ \frac{\pi}{4}$ as required		
ii)	$\tan\alpha = \frac{1}{3}$  $\therefore \cos\alpha = \frac{3}{\sqrt{10}}$ $t = \dfrac{120}{20\sqrt{2}\cos\alpha}$ $\therefore\ t = \dfrac{120\sqrt{10}}{20\sqrt{2}.3}$ $t = 2\sqrt{5}$ seconds		

Qn	Solutions	Marks	Comments & Criteria
14b) iii)	$\alpha = \frac{\pi}{4}$ $\dot{x} = 20 \quad \dot{y} = 20\sqrt{2}\sin\left(\frac{\pi}{4}\right) - 10t$ $\dot{y} = 20\sqrt{2} \cdot \frac{1}{\sqrt{2}} - 10t$ $\dot{y} = 20 - 10t$ at $t = 6$, $\dot{x} = 20$ at $t = 6$, $\dot{y} = 20 - 10(6)$ $\therefore \dot{y} = -40$  $v^2 = 20^2 + 40^2$ $v^2 = 400 + 1600$ $v^2 = 2000$ $v = \sqrt{2000}$ $v = 20\sqrt{5}$ ms^{-1}		

Qn	Solutions	Marks	Comments & Criteria
14c)	$\ddot{x} = -\frac{1}{2}u^2 e^{-x}$		
i)	at $t=0$, $u=2$		
	$\therefore \ddot{x} = -\frac{1}{2}(2)^2 e^{-x}$		
	$\ddot{x} = -2e^{-x}$		
	but $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$		
	$\therefore \frac{1}{2}v^2 = +2e^{-x} + c$		
	$v^2 = 4e^{-x} + 2c$		
	when $t=0$, $x=0$, $v=2$		
	$\therefore 4 = 4e^0 + 2c$		
	$\therefore 2c = 0$		
	$\therefore v^2 = 4e^{-x}$		
ii)	$4e^{-x} > 0$ for all x		
	$\therefore v^2 > 0$ for all x		
$\therefore$	at $t=0$, $V=2$ and V remains positive.		