2017 Trial Higher School Certificate Examination

Killara HIGH SCHOOL

MATHEMATICS EXTENSION 1

21 July 2017

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen.
- NESA approved calculators may be used.
- Commence each new question in a new booklet. Write on both sides of the paper.
- A reference sheet is provided.
- In Question 11-14 show relevant mathematical reasoning and/or calculations
- At the conclusion of the examination, bundle the booklets used in the correct order **including your reference sheet** within this paper and hand to examination supervisor.

Total Marks: 70

Section 1 – 10 marks (pages 2 - 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section 2 – 60 marks (pages 6 - 10)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

NESA NUMBER: .... **# BOOKLETS USED:** .....

Marker's use only.

QUESTION	1-10	11	12	13	14	**Total**
MARKS	$\overline{10}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{70}$

This task constitutes 40 % of the HSC Course Assessment

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided (labelled as page 11)

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1. $\displaystyle\int \sin 2x \, dx =$

I $\quad \sin^2 x + C$ where C is a constant

II $\quad -\dfrac{1}{2}\cos 2x + D$ where D is a constant

III $\quad -\cos^2 x + E$ where E is a constant

Which of above solutions is correct?

(A) II only

(B) I and II only

(C) II and III only

(D) I, II and III

2. In how many ways can a committee of 2 men and 3 women be selected from a group of 6 men and 8 women?

(A) ${}^6P_2 \times {}^8P_3$

(B) ${}^6C_2 \times {}^8C_2$

(C) ${}^6P_3 \times {}^8C_2$

(D) ${}^6C_2 \times {}^8C_3$

3. What are the solutions to the equation $e^{6x} - 7e^{3x} + 6 = 0$?

(A) $x = 1$ and $x = 6$

(B) $x = 0$ and $x = \dfrac{\log_e 6}{2}$

(C) $x = 0$ and $x = \dfrac{\log_e 6}{3}$

(D) 6

4. If P is the point $(-1, 2)$ and K is the point $(3, 5)$, the coordinates of the point that divides the interval PK externally in the ratio $2 : 3$ are:

(A) $(11, 11)$

(B) $\left(\dfrac{3}{5}, \dfrac{16}{5}\right)$

(C) $(-3, -4)$

(D) $(-9, -4)$

5.

$$\lim_{x \to 0} \frac{\sin \dfrac{3x}{2}}{6x} =$$

(A) $\dfrac{1}{4}$

(B) 4

(C) $\dfrac{1}{2}$

(D) 2

6. For the functions $f(x) = \dfrac{1}{x}$ and $g(x) = 2\sin x$ the range of $g(f(x))$ is

(A) $-1 \leq y \leq 1$

(B) $\dfrac{1}{2} \leq y \leq \dfrac{1}{2}$

(C) $-1 \leq y \leq 1$

(D) $-2 \leq y \leq 2$

7. What is the exact value of $\tan\left(\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)\right)$?

(A) $\dfrac{1}{\sqrt{3}}$

(B) $-\dfrac{1}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

8. $\cos^{-1}\left[\cos(\cos^{-1}(-x))\right]$

(A) $\pi - x$

(B) $\pi - \cos^{-1} x$

(C) $\cos^{-1} x$

(D) $-\cos^{-1} x$

9. A parabola has the parametric equation $x = 6t$, $y = 3t^2$. The Cartesian equation of its derivative is:

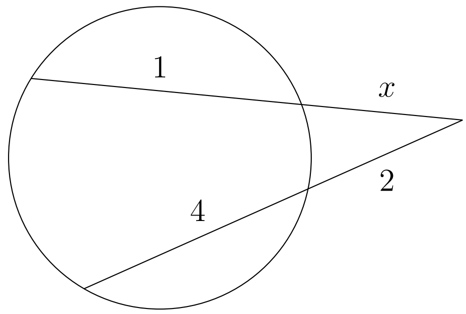
(A) $y' = 6t$

(B) $y' = \dfrac{x^2}{12}$

(C) $y' = \dfrac{x}{6}$

(D) $y' = \dfrac{2x}{9}$

10. Find the value of x.



NOT TO SCALE

(A) $x = 1$

(B) $x = 2$

(C) $x = 3$

(D) $x = 4$

Section II

70 marks
Attempt Questions 11 to 14
Allow approximately 1 hour and 45 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Solve the inequality $\dfrac{5}{x-2} \leq 4$ **3**

(b) Use the substitution $u = x+2$ to evaluate $\displaystyle\int_1^3 \frac{x+1}{(x+2)^2}\,dx$ giving the answer in simplest exact form. **3**

(c) The polynomial $P(x) = 2x^3 + bx^2 - 8x + 5$ has a remainder of 2 when divided by $(x+3)$, calculate the remainder when when divided by $(x+2)$. **3**

(d) i. Show that that $y = e^{-x} - 3x$ has only one real root. **1**

ii. The root lies in the interval $0.2 < x < 0.3$ Taking $x_1 = 0.25$ as the first approximation, use one application of Newton's method to determine a better approximation. Give your answer correct to 3 significant figures. **2**

(e) Using the word COWABUNGA

i. How many arrangements of the 9 letters are possible? **1**

ii. How many arrangements of the 9 letters are possible if the 3 letters COW have to be placed together? Note the letters COW can be placed in any order but they must be together. **2**

Question 12 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) The polynomial $P(x) = x^3 - 3x^2 + kx + 48$ has roots α, β and γ **3**

Two of the roots are equal in magnitude but opposite in sign. Find the third root and hence find the value of k.

(b) A particle whose displacement is x moves in simple harmonic motion. Given that the displacement equation is of the form $x = a\cos(nt + \alpha)$ and $\ddot{x} = -4x$.

Initially the particle has displacement of $x = \sqrt{3}$ and velocity of $\dot{x} = 6$.

i. Find the value of n. **1**

ii. Find the displacement equation. **3**

(c) Determine the domain and range of $y = \dfrac{1}{2}\sin^{-1}(1 - 3x)$ and hence sketch the graph. **3**

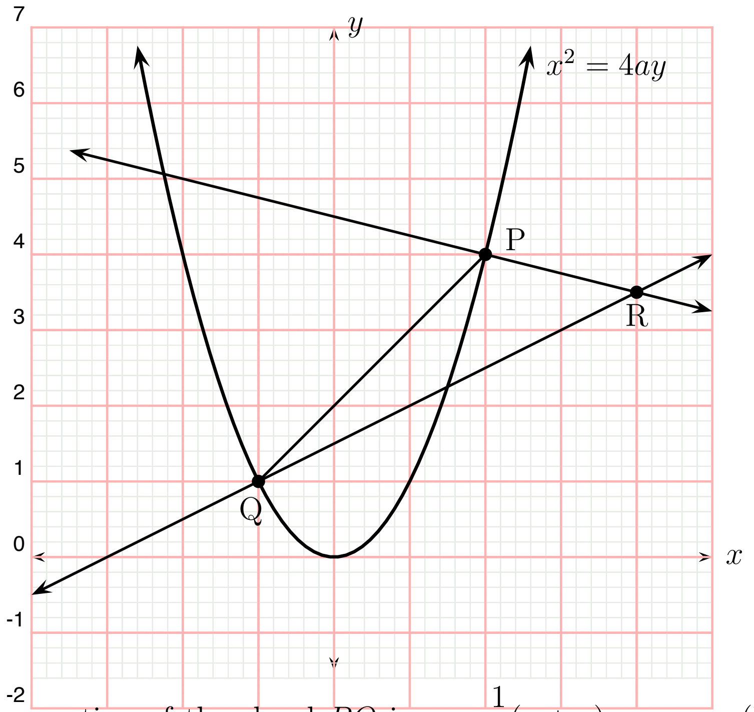
Question 12 continues on page 8

Question 12 (continued)

(d) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ as shown below.

The equation of the normal to the parabola P is $x + py = 2ap + ap^3$.

The equation of the normal to the parabola at Q is $x + qy = 2aq + aq^3$.



i. The equation of the chord PQ is $y = \frac{1}{2}(p+q)x - apq$ (DO NOT show this). If the chord passes through $(0, 5a)$ show that $pq = -5$. **1**

ii. Show that the point of intersection, R, of the normals at P and Q has the coordinates $[-apq(p+q),\ a(p^2 + pq + q^2 + 2)]$. **2**

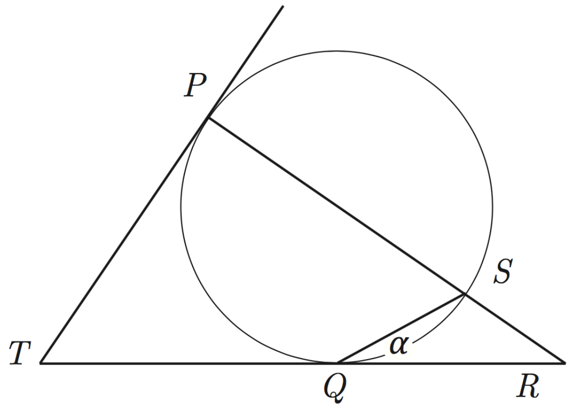
iii. Find the equation of the locus of R if the chord PQ passes through $(0, 5a)$. **2**

Question 13 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) i. Show that $y = \dfrac{x^2 + x + 1}{x+1} = x + \dfrac{1}{x+1}$ **1**

ii. Sketch the graph of the function, clearly showing all key features **3**

(b) PS is a diameter of the circle PSQ. The tangents at P and Q meet at T. The tangent TQ and the diameter PS are produced to meet at R. Let $\angle SQR = \alpha$.



i. Prove that $\angle TPQ = \dfrac{\pi}{2} - \alpha$. **2**

ii. Prove that $\angle PTQ = 2\alpha$. **2**

(c) The acceleration of a body A is given by $a = 18x(x^2 + 1)$ where x cm is the displacement after time t seconds. Initially, A starts from the origin with velocity $3\,cms^{-1}$.

i. Show that $v = 3(x^2 + 1)$. **2**

ii. Find x in terms of t. **2**

(d) By writing $\sqrt{3}\sin 3x - \cos 3x$ in the form $R\sin(3x - \alpha)$, where α is acute and $R > 0$. or otherwise, find the general solutions to the equation: **3**

$$\sqrt{3}\sin 3x - \cos 3x = 0$$

Question 14 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) A boys throws a ball with velocity $v\,ms^{-1}$ at an angle of $45°$ to the horizontal.

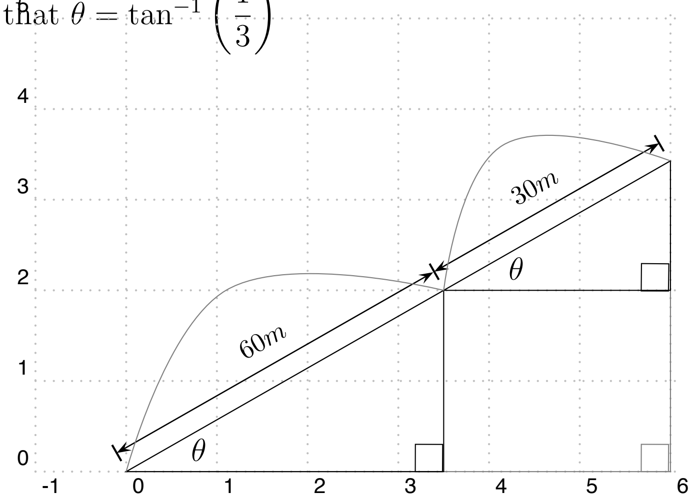
i. Derive expressions for the horizontal and vertical displacement of the ball from the point of projection. **2**

ii. Show that the Cartesian equation of the path of the ball is **2**

$$y = x - \frac{gx^2}{v^2}$$

where g is the acceleration due to gravity.

iii. A boy is standing on a hill inclined at θ to the horizontal as shown in the diagram below. He throws the ball at the same angle of elevation of $45°$ and at the same speed of $v\,ms^{-1}$. If he can throw the ball 60 m down the hill but only 30 m up the hill, use the results in part ii. to show that $\theta = \tan^{-1}\left(\frac{1}{3}\right)$ **3**



(b) The surface area of a spherical bubble is increasing at a constant rate of $1.9mm^2$ per second. Find the rate of increase of its volume when the radius is $0.6mm$. The volume of a sphere is given by $V = \frac{4}{3}\pi R^3$, and the surface area is given by $A = 4\pi R^2$. **3**

(c) i. Using the expansions of $\cos(A+B)$ and $\cos(A-B)$, show that: **2**

$$\cos x - \cos(x + 2\theta) = 2\sin(x+\theta)\sin\theta$$

ii. Hence prove by mathematical induction that **3**

$$\sum_{r=1}^{n} \sin(2r-1)\theta = \frac{1-\cos(2n\theta)}{2\sin\theta}$$

End of Examination ☺

2017 | Trial Higher School Certificate Examination

Killara HIGH SCHOOL

MATHEMATICS EXTENSION 1

NESA NUMBER: ....

Section 1 –Multiple Choice Answer Sheet

Use this multiple choice answer sheet for Questions 1 - 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

Multiple Choice Answer Sheet

Student Number Ext 1 - Answers - 2017 Trial.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ☒ B ☒ C ○ D ○

1. A ○ B ○ C ○ D ●
2. A ○ B ○ C ○ D ●
3. A ○ B ○ C ● D ○
4. A ○ B ○ C ○ D ●
5. A ● B ○ C ○ D ○
6. A ○ B ○ C ○ D ●
7. A ● B ○ C ○ D ○
8. A ○ B ● C ○ D ○
9. A ○ B ○ C ● D ○
10. A ○ B ○ C ● D ○

Ext 1 - Multiple Choice Answers - 2017 Trial.

1 $y=(\sin x)^2$

$y' = 2\sin x\cos x = \sin 2x$

$y = -\frac{1}{2}\cos 2x$

$y' = -\frac{1}{2} \times 2 \times -\sin 2x = \sin 2x$

$y = -\cos^2 x = -(\cos x)^2$

$y' = 2 \times -x - \sin x \times \cos x$

$= 2\sin x\cos x = \sin 2x$

$\therefore$ (D)

2 need 2 men from 6, 3 women from 8

$\therefore \; {}^{6}C_2 \times {}^{8}C_3$ it is C not P as a committee

$\therefore$ (D) (order is unimportant)

3 → you can sub. in values + test

OR $e^{6x} - 7e^{3x} + 6 = 0$

let $u = e^{3x}$ $\therefore u^2 - 7u + 6 = 0$

$(u-6)(u-1) = 0$

$\therefore u = 6, 1$

when $u = 1$

$e^{3x} = 1$

$\ln(e^{3x}) = \ln(1)$

$\therefore 3x = 0$

$\therefore x = 0$

when $u = 6$

$e^{3x} = 6$

$\ln(e^{3x}) = \ln(6)$

$\therefore 3x = \ln(6)$

$\therefore x = \frac{\ln(6)}{3}$

$\therefore$ (C)

4 $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$(-1, 2)$ $(3, 5)$ $(-2, 3)$

(x_1, y_1) (x_2, y_2) (m, n)

$= \left(\frac{-2\times 3 + 3\times -1}{1}, \frac{-2\times 5 + 3\times 2}{1}\right)$

$= (-9, 4)$ $\therefore$ (D)

5 $\lim_{x\to 0} \frac{\sin\frac{3x}{2}}{6x}$

$= \lim_{x\to 0} \frac{1}{4}\left(\frac{\sin\frac{3x}{2}}{\frac{6x}{4}}\right)$

$= \lim_{x\to 0} \frac{1}{4}\left(\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)$ $\left(\text{as } \lim_{x\to 0} \frac{\sin x}{x} = 1\right)$

$= \frac{1}{4}$ $\therefore$ (A)

6 $g[f(x)] = 2\sin\left(\frac{1}{x}\right)$

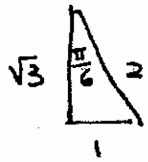
as $\frac{1}{x}$ has domain all real x except $x \neq 0$

$\therefore$ this is just like $2\sin x$ in terms of range

$\sin x$ $-1 \leq y \leq 1$

$2\sin x$ $\underline{-2 \leq y \leq 2}$ $\therefore$ (D)

7 $\tan\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$



$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\therefore \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \quad \therefore (A)$

8 $\cos^{-1}\left[\cos\left(\cos^{-1}(-x)\right)\right]$

$= \cos^{-1}\left[\cos\left(\pi - \cos^{-1}(x)\right)\right]$

$= \cos^{-1}\left[\cos\pi - x\right]$

$= \pi - \cos^{-1}(x) \quad \therefore (B)$

9 $x = 6t \ , \ y = 3t^2$

$t = \frac{x}{6}$

$y = 3\times\left(\frac{x}{6}\right)^2 = \frac{3x^2}{36}$

$y = \frac{x^2}{12}$

$y' = \frac{2x}{12} = \frac{x}{6} \quad \therefore (C)$

10 $x(x+1) = 2(2+4)$

$x^2 + x = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$\therefore x = -4, 3 \quad$ as $\quad x \nless 0$

$\therefore x = 3 \quad \therefore (C)$

Question 11 Solutions

(a) Solve the inequality $\frac{5}{x-2} \le 4$ **3**

3 marks answer and excluding x=2
2 marks correct answer not excluding x=2
1 mark correct expression with (x-2) factor

$5(x-2) \le 4(x-2)^2$

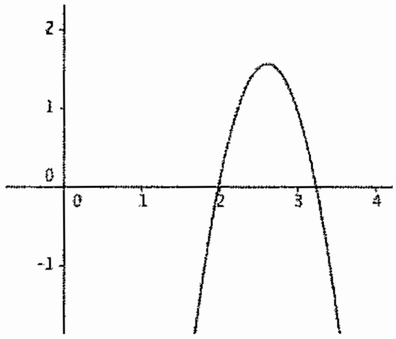
$5(x-2) - 4(x-2)^2 \le 0$

$(x-2)[5-4x+8] \le 0$ (1)

$(x-2)(13-4x) \le 0$

$x \le 2 \quad x \ge \frac{13}{4}$ (2)

$x < 2$ as $x \ne 2 \quad x \ge \frac{13}{4}$ (3)



Question was done well. There was no penalty if the solution was not graphed or tested

(b) Use the substitution $u = x+2$ to evaluate $\int_1^3 \frac{x+1}{(x+2)^2}\,dx$ giving the answer in simplest exact form. **3**

$u = x+2$

$\frac{du}{dx} = 1$

$du = dx$

$x+1 = u-1$

terminals when $x=1 \quad u=3$

$x=3 \quad u=5$

$$\int_3^5 \frac{(u-1)}{u^2}\,du$$

$$= \int_3^5 \frac{1}{u} - u^{-2}\,du$$

$$= \left[\ln u - \frac{u^{-1}}{-1}\right]_3^5$$

$$= \left[\ln u + \frac{1}{u}\right]_3^5$$

$$= (\ln 5 + \frac{1}{5}) - (\ln 3 + \frac{1}{3})$$

$$= \ln(\frac{5}{3}) - \frac{2}{15}$$

Common error was forgetting that $\int \frac{1}{u}du = \ln u$, *and not changing the terminals*

Many students lost a mark for not simplifying the last line

(c) The polynomial $P(x) = 2x^3 + bx^2 - 8x + 5$ has a remainder of 2 when divided by $(x+3)$, calculate the remainder when when divided by $(x+2)$. **3**

3 marks correctly value of remainder with correct working 2 marks correct value of b 1 mark correct substitution(-3)

$P(-3) = 2(-27) + 9b + 24 + 5 = 2$ (1)

$-25 + 9b = 2$

$9b = 27$

$b = 3$ (2)

$P(x) = 2x^3 + 3x^2 - 8x + 5$

$P(-2) = 2(-8) + 3(4) + 16 + 5$

$= 17$ (3)

Question was done well

(d) i. Show that that $y = e^{-x} - 3x$ has only one real root. 1

ii. The root lies in the interval $0.2 < x < 0.3$ Taking $x_1 = 0.25$ as the first approximation , use one application of Newton's method to determine a better approximation. Give your answer correct to 3 significant figures. 2

(i)

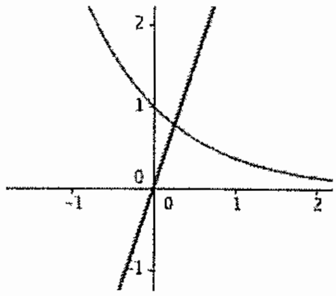
1 mark correct working showing graphs or equivalent working

$e^{-x} - 3x = 0$

$e^{-x} - 3x = 0$

$e^{-x} = 3x$

sketching graphs they intersect at only 1 point $\therefore$ 1 solution only



This question was done poorly. Students who graphed the function were successful.
Many students showed that f'(x)<0 for all x and then stated that there were no turning points. This was not sufficient to show there is only 1 root, as there could be no roots. You must show that f(x) changes sign as well.

Some students found the descriminant and showed that $\Delta < 0$. *This is incorrect as the descriminant is only valid for quadratics.*

(ii)

2 marks correct value of x_2 with correct working
1 mark correct derivative and substitution into the correct formula, or 1 error

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x) = e^{-x} - 3x$$

$$f'(x) = -e^{-x} - 3 \qquad (1)$$

$$x_2 = 0.25 - \frac{\left[e^{-0.25} - 3(0.25)\right]}{\left[-e^{-0.25} - 3\right]}$$

$$= 0.2576....$$

$$= 0.258 \qquad (2)$$

Done well. There was no penalty for incorrect rounding this time, but students need to take care.

(e) Using the word COWABUNGA

i. How many arrangements of the 9 letters are possible? 1

ii. How many arrangements of the 9 letters are possible if the 3 letters COW have to be place together? Note the letters COW can be placed in any order but they must be together. 2

(i)

1 mark correct answer

$\frac{9!}{2!}$ or $\frac{2262880}{2} = 181440$

(ii)

2 marks correct answer no working required

1 mark correct answer with 1 error

$\frac{7! \times 3!}{2!} = 15120$

$COW------$

$-COW-----$

$--COW----$

$---COW---$

$----COW--$

$-----COW-$

$------COW$

$\frac{6! \times 3! \times 7}{2} = \frac{30240}{2} = 15120$

Done well, common error was not allowing for Double letter A

Yr12 Ext 1 Marking Scheme

Q12

a. $x^3 - 3x^2 + kx + 48$ roots α, β, γ

$\alpha + \beta + \gamma = \frac{-b}{a} = 3$ ①

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = k$ ②

$\alpha\beta\gamma = \frac{-d}{a} = -48$ ③

(1 mark)

Let $\beta = -\alpha$ $\therefore$ ① $\rightarrow$ $\alpha + -\alpha + \gamma = 3$

$\gamma = 3$ (1 mark)

③ $\alpha \cdot -\alpha\gamma = -48$

$-\alpha^2 \times 3 = -48$

$3\alpha^2 = 48$

$\alpha^2 = 16$

$\alpha = 4, \quad -\alpha = -4$

$\therefore$ ② Becomes

$\alpha \cdot -\alpha + \cancel{\alpha\gamma} - \cancel{\alpha\gamma} = k$

$4 \times -4 = k$

$\therefore k = -16$ (1 mark)

(lose 1 mark if α is incorrect)

12b $\quad x = a\cos(nt+\epsilon)$

$$\ddot{x} = -4x \qquad t=0,\ x=\sqrt{3},\ \dot{x}=6$$

i

$$\ddot{x} = -n^2 x$$
$$-4x = -n^2 x$$
$$\therefore n^2 = 4$$
$$\therefore n = 2$$

ii $\quad$ when $t=0$

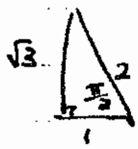
$$x = a\cos\epsilon$$
$$\therefore \sqrt{3} = a\cos\epsilon \quad (1)$$

as $\quad x = a\cos(nt+\epsilon)$
$$\dot{x} = -na\sin(nt+\epsilon)$$
when $t=0$ $\quad 6 = -2a\sin\epsilon$
$$a\sin\epsilon = -3 \quad (2)$$

1 mark

$(2) \div (1) \quad \dfrac{a\sin\epsilon}{a\cos\epsilon} = \dfrac{-3}{\sqrt{3}} = -\sqrt{3}$

$$\therefore \tan\epsilon = -\sqrt{3}$$
$$\therefore \epsilon = \frac{-\pi}{3}$$



(1 mark,
(ECF
awarded)

now get a

$$\sqrt{3} = a\cos\epsilon \quad (1)$$
$$\sqrt{3} = a.\cos\left(-\frac{\pi}{3}\right)$$
$$\sqrt{3} = a \times \frac{1}{2}$$
$$\therefore a = 2\sqrt{3}$$

$$\therefore \quad x = 2\sqrt{3}\cos\left(2t - \frac{\pi}{3}\right)$$

(1 mark)
(ECF awarded)

12b
OR

$\frac{1}{2}v^2 = \int -4x\,dx$

$\frac{1}{2}v^2 = -2x^2 + c$

$v^2 = -4x^2 + c$ when $x = \sqrt{3}$, $v = 6$

$6^2 = -4 \times (\sqrt{3})^2 + c$

$6^2 = -12 + c$

$36 = -12 + c$

$\therefore c = 48$

$\therefore v^2 = -4x^2 + 48$

$v^2 = n^2(a^2 - x^2)$

$v^2 = 4(12 - x^2)$

$\therefore a^2 = 12$

$a = \sqrt{12} = 2\sqrt{3}$

$x = 2\sqrt{3}\cos(2t + \alpha)$

when $t = 0$, $x = \sqrt{3}$

$\sqrt{3} = 2\sqrt{3}\cos\alpha$

$\frac{1}{2} = \cos\alpha \quad \therefore \alpha = \pm\frac{\pi}{3}$ $\rightarrow$ note $\dot{x} = -4\sqrt{3}\sin\left(2t \pm \frac{\pi}{3}\right)$

<u>check</u> $\dot{x} = 6, t = 0 \quad 6 = -4\sqrt{3}\left(\sin -\frac{\pi}{3}\right)$

$\therefore x = 2\sqrt{3}\cos\left(2t - \frac{\pi}{3}\right)$

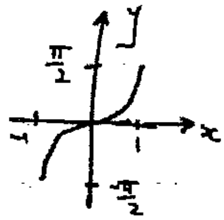
due to
S	A ✓
T	C ✓

(f only 1 error & process good) 2/...

12c $\quad y = \frac{1}{2}\sin^{-1}(1-3x)$

$y = \sin^{-1}x \qquad$ domain $-1 \le x \le 1$

range $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$



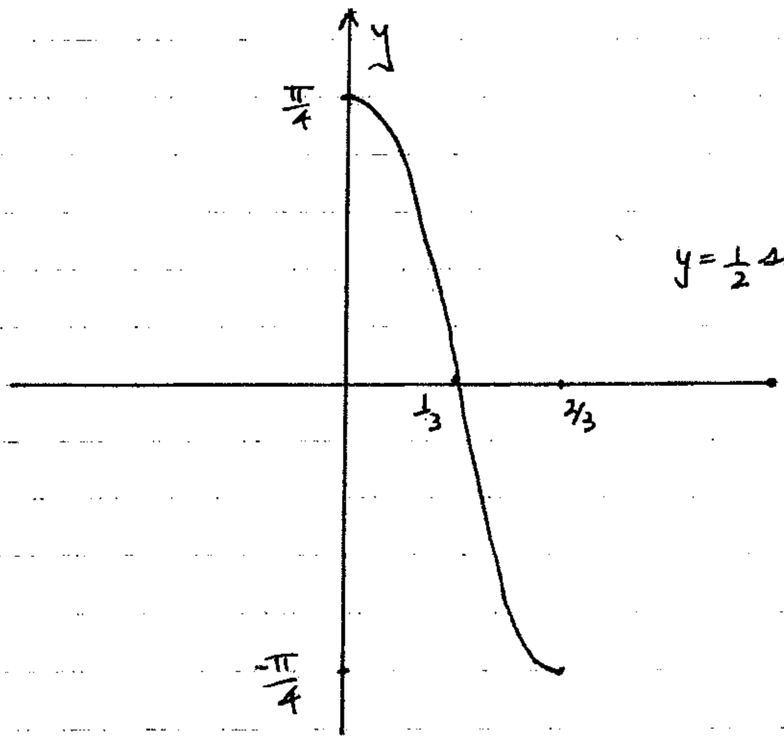
domain $\quad -1 \le 1-3x \le 1$

$-2 \le -3x \le 0$

$\frac{2}{3} \ge x \ge 0 \qquad \therefore \qquad 0 \le x \le \frac{2}{3}$ $\Big\}$ 1 mark

range is half normal range

$\therefore \quad \frac{-\pi}{4} \le y \le \frac{\pi}{4}$ $\Big\}$ 1 mark



$y = \frac{1}{2}\sin^{-1}(1-3x)$

(1 mark)
graph.

12di $y = \frac{1}{2}(p+q)x - apq$

sub $(0, 5a)$

$5a = \frac{1}{2}(p+q) \times 0 - apq$

$(5a = -apq) \div -a$

$pq = -5$

12d ii

$x + py = 2ap + ap^3 \longrightarrow x = 2ap + ap^3 - qy$

$x + qy = 2aq + aq^3$

$\downarrow$

$-py + 2ap + ap^3 + qy = 2aq + aq^3$

$y(q-p) = 2aq - 2ap + aq^3 - ap^3$

$y(q-p) = 2a(q-p) + a(q^3 - p^3)$

$y\cancel{(q-p)} = 2a\cancel{(q-p)} + a\cancel{(q-p)}(q^2 + pq + p^2)$

$y = 2a + a(q^2 + pq + q^2)$

$y = a(p^2 + pq + q^2 + 2)$ (1 mark)

sub. into $x = 2ap + ap^3 - py$

$x = 2ap + ap^3 - pa(p^2 + pq + q^2 + 2)$

$x = \cancel{2ap} + \cancel{ap^3} - \cancel{ap^3} - ap^2q - apq^2 - \cancel{2ap}$

$\therefore x = -apq(p+q)$ (1 mark)

(1 mark if valid attempt but mess up y and x)

12d iii use $pq = -5$

$x = -apq(p+q)$

$\therefore p+q = \frac{-x}{apq} = \frac{-x}{-5a} = \frac{x}{5a}$

$y = a(p^2 + pq + q^2 + 2)$
$y = a(p^2 + q^2 - 5 + 2)$

$y = a\left[(p+q)^2 - 2pq - 3\right]$ (1 mark)

$y = a\left[\left(\frac{x}{5a}\right)^2 - 2 \times -5 - 3\right]$

$y = a\left(\frac{x^2}{25a^2} + 7\right)$

$y = \frac{x^2}{25a} + 7a$ (1 mark)

put into form $(x-h)^2 = 4a(y-k)$

$y - 7a = \frac{x^2}{25a}$

$25a(y - 7a) = x^2$

(1 mark if worked forwards)

Markers Feedback X1 Trial Q12

Polynomials,

a was very well done by most students a few made minor errors, but most got full marks.

SHM

b, many students had trouble with part ii

→ when you need to get the angle 'α' it is easier to get 2 equations ① from x and ② from $\dot{x}$ to get $\tan\alpha$, then get a.

→ many did $\frac{1}{2}v^2 = \int -4x\,dx$

this could enable them to get 'a' easily but was more challenging to get α, as have to use $v=6, t=0$.

→ some integrated inverse trig functions doing way too much work for the marks.

Inverse Trig

c most calculated the domain & range correctly However, about 30% of students showed room for improvement showing insufficient skills.

→ Many drew the graph incorrectly – this is a shame as substituting $x=0$ would quickly confirm where the graph lay.

Parametrics

d i done very well

ii most did this question well, about 20% of students need to Revise their Parametrics work on finding intersection of 2 points. students should show $q^3 - p^3 = (q-p)(q^2+qp+p^2)$ when breaking down difference of 2 cubes, to ensure they get all marks.

iii about 40% of students struggled.
they need to remember the purpose is to eliminate p and q → using $pq=-5$, $p+q=\frac{x}{5a}$ and $p^2+q^2=(p+q)^2-2pq$

Q13

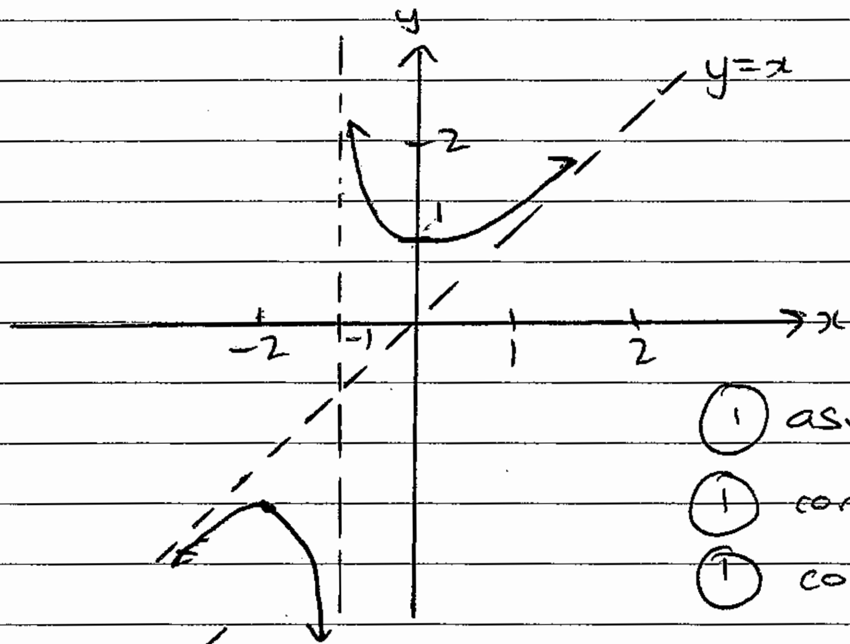
a) i) $\dfrac{x^2+x+1}{x+1} = \dfrac{x(x+1)+1}{x+1} \qquad x \neq -1$

$= x + \dfrac{1}{x+1}$

Many students still work RHS → LHS not what was asked

1 mark correct algebraic method working from LHS → RHS

a) ii)



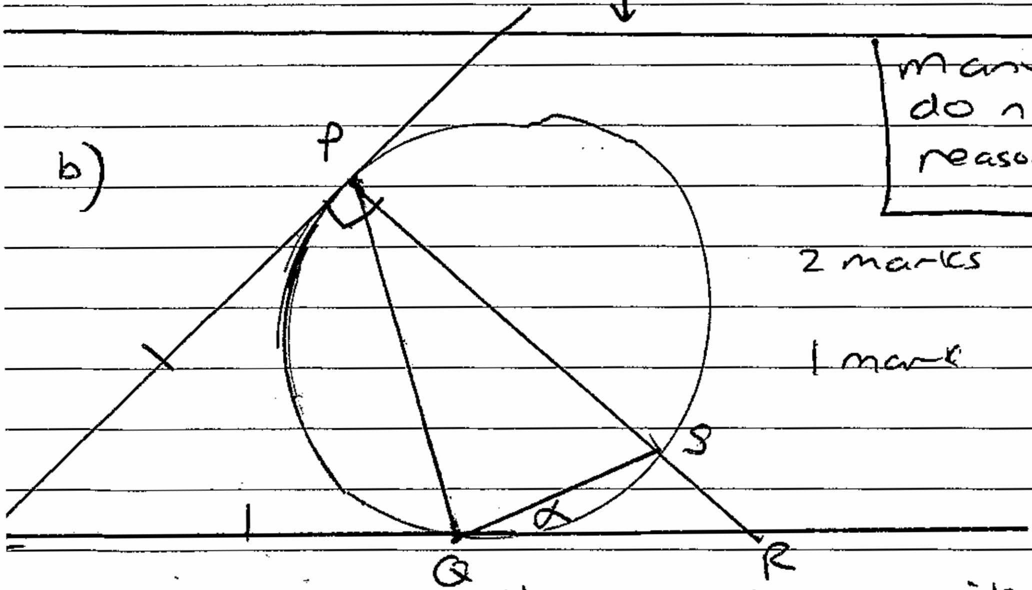
Many students not accurate enough with graph near asymptotes

1 asymptotes

1 correct y intercept

1 correct shape

b)



Many students still do not use correct reasons → see syllabus!

2 marks 2 correct reasons that lead to proof

1 mark one correct reason

$\angle TPS = \pi/2$ (tangent to a circle is perpendicular to the radius drawn to point of contact) ✓1

$\angle SQR = \angle SPQ = \alpha$ (angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment) ✓1

$\therefore \angle TPQ = \pi/2 - \alpha$

b) iii) In ΔTPQ

$TP = TQ$ (tangents to a circle ~~from an external point~~ are equal)

$\therefore \angle TPQ = \angle TQP$ (equal angles opposite equal sides in isosceles ΔTPQ)

$\therefore \angle PTQ = \pi - \left(\frac{\pi}{2} - \alpha + \pi/2 - \alpha\right)$ (angle sum of $\Delta TPQ = \pi$)

$\therefore \angle PTQ = 2\alpha$

* 2 marks all correct reasons
✓ 1 mark one incorrect reason

Generally well done but many still using incorrect reasoning or very inefficient methods

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c) i) $\ddot{x} = 18x(x^2+1)$

$\therefore \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 18x^3 + 18x$

$\therefore \int \frac{d}{dx}\left(\frac{1}{2}v^2\right).dx = \int 18x^3 + 18x \, .dx$ ✓ 1 must have dx!

many don't indicate what they are integrating wrt.

$\therefore \frac{1}{2}v^2 = \frac{18x^4}{4} + \frac{18x^2}{2} + \frac{c}{2}$

$\therefore v^2 = 9x^4 + 18x^2 + c$

when $x=0, v=3 \rightarrow 9 = c$

$\therefore v^2 = 9x^4 + 18x^2 + 9$

$v^2 = 9(x^4+2x^2+1)$

$= 9(x^2+1)^2$

but when $x=0, v=3 \rightarrow v>0$

$\therefore v = 3(x^2+1)$ ✓ 1 must have reason for excluding negative

* Many students did not see the factorisation
* MANY gave no reason for ignoring the negative and lost a mark

(13)

c) ii) $v = \frac{dx}{dt} = 3(x^2+1)$

$\therefore \frac{dt}{dx} = \frac{1}{3(x^2+1)}$

$\therefore \int \frac{dt}{dx} \cdot dx = \frac{1}{3}\int \frac{1}{x^2+1} \cdot dx$ ✓ 1 setting up to apply standard integral

$\therefore t = \frac{1}{3}\tan^{-1}(x) + c$

when $t=0,\ x=0 \rightarrow c=0$

$\therefore t = \frac{1}{3}\tan^{-1}(x)$

$\therefore x = \tan(3t)$ ✓ 1 successfully rearranging to achieve solution

* Many failed to recognise standard integral
* some left answer as $t = \frac{1}{3}\tan^{-1}(x)$
→ look at question

d) $\sqrt{3}\sin 3x - \cos 3x = R\sin(3x - \alpha)$

$\therefore R = \sqrt{(\sqrt{3})^2 + 1} = 2$

$\tan\alpha = \frac{1}{\sqrt{3}} \rightarrow \alpha = \pi/6$ ✓ 1 for correct α

✓ 1 for correct R

$\therefore \sqrt{3}\sin 3x - \cos 3x = 2\sin(3x - \pi/6)$

$\therefore \sqrt{3}\sin 3x - \cos 3x = 2\sin(3x - \pi/6) = 0$

$\therefore 2\sin(3x - \pi/6) = 0$

$\therefore \sin(3x - \pi/6) = 0$

$\therefore 3x - \pi/6 = n\pi$

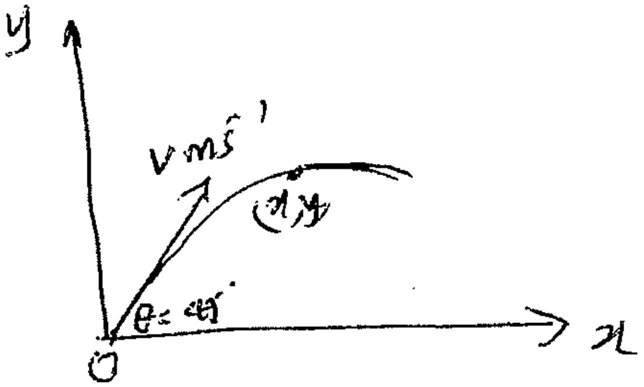
$3x = n\pi + \pi/6$

$x = \frac{n\pi}{3} + \frac{\pi}{18}$ ✓ 1

* Students who used general sol^n formula $\sin\theta = a \rightarrow \theta = n\pi + (-1)^n \sin^{-1} a$ failed to recognise that $a = 0$

Q14

a) (i)



$\rightarrow \quad \ddot{x} = 0$

$\int \frac{d\dot{x}}{dt} \cdot dt = \int 0\, dt$

$\dot{x} = C$

$t = 0,\ \dot{x} = V\cos\theta$

$\dot{x} = V\cos\theta$ (1 mark)

$\int \frac{dx}{dt} \cdot dt = \int V\cos\theta\, dt$

$x = V\cos\theta \cdot t + D$

$t = 0,\ x = 0$

$\therefore D = 0$

$\therefore x = V\cos\theta \cdot t$

as $\theta = 45°$

$\boxed{x = \frac{Vt}{\sqrt{2}}}$

$\uparrow \quad \ddot{y} = -g$ (1 mark)

$\int \frac{d\dot{y}}{dt} \cdot dt = \int -g\, dt$

$\dot{y} = -gt + E$

$t = 0,\ \dot{y} = V\sin\theta$

$\therefore E = V\sin\theta$

$\dot{y} = -gt + V\sin\theta$

$\int \frac{dy}{dt} \cdot dt = \int -gt + V\sin\theta\, dt$

$y = -\frac{1}{2}gt^2 + V\sin\theta \cdot t + F, \quad \theta = 45°$

$t = 0,\ y = 0 \Rightarrow F = 0 \quad \boxed{\therefore y = -\frac{1}{2}gt^2 + \frac{V}{\sqrt{2}}t}$

Many students did not clearly show:

(i) integrating with respect to what?

(ii) evaluating the constants

(iii) the direction of the $\ddot{x}$ and $\ddot{y}$ as they are vector quantities

Q14 a(ii)

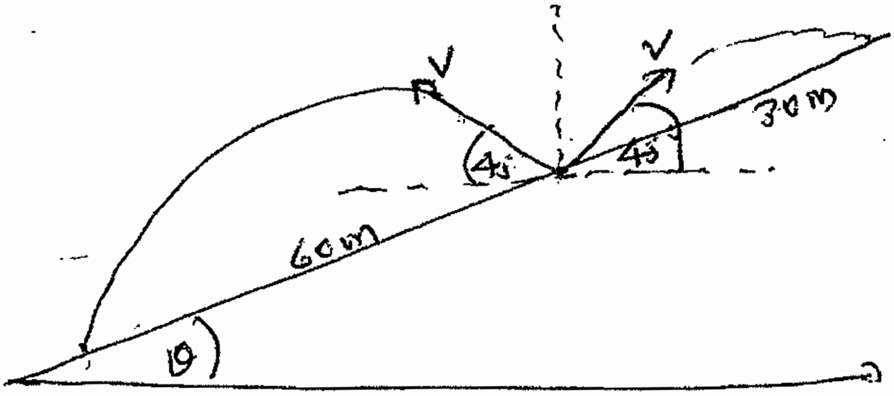
$$x = \frac{vt}{\sqrt{2}} \Rightarrow t = \frac{\sqrt{2}x}{v} \longrightarrow \text{1 mark}$$

$$\left.\begin{aligned} y &= \frac{vt}{\sqrt{2}} - \frac{1}{2}gt^2 \\ &= \frac{v}{\sqrt{2}}\cdot\frac{\sqrt{2}x}{v} - \frac{1}{2}g\cdot\frac{2x^2}{v^2} \end{aligned}\right\} \text{— (1 mark)}$$

$$= x - \frac{gx^2}{v^2}$$

Most got this correctly

(iii)



Applying the result of $y = x - \frac{gx^2}{v^2}$ from a(ii),

For the downhill motion $\longrightarrow$ (1 mark)

$$-60\sin\theta = 60\cos\theta - \frac{g}{v^2}(60\cos\theta)^2$$

(i.e) $$60\cos\theta + 60\sin\theta = \frac{g}{v^2}(60\cos\theta)^2 \quad \text{— (1)}$$

For the uphill motion $\longrightarrow$ (1 mark)

$$30\sin\theta = 30\cos\theta - \frac{g}{v^2}(30\cos\theta)^2$$

(i.e) $$30\cos\theta - 30\sin\theta = \frac{g}{v^2}(30\cos\theta)^2 \quad \text{— (2)}$$

$$\left.\begin{aligned} (1) \div (2) \quad \frac{2(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)} &= 4 \\ \Rightarrow \quad 3\sin\theta &= \cos\theta \end{aligned}\right\} \text{(1 mark)}$$

$$\Rightarrow \tan\theta = \frac{1}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

Many students ~~[illegible]~~ ~~[illegible]~~ experienced difficulty with this part

Could not get the equations (1) and (2) with correct signs, again because of not paying attention to the detail

Q14(b)

S = surface Area of the bubble

V = volume of bubble

$V = \frac{4}{3}\pi r^3$ $\qquad$ $S = 4\pi r^2$

$\frac{dV}{dr} = 4\pi r^2$ $\qquad$ $\frac{dS}{dr} = 8\pi$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dS} \cdot \frac{dS}{dt}$$

$$= 4\pi r^2 \cdot \frac{1}{8\pi r} \cdot (1.9)$$

$$= \frac{1.9r}{2}$$

when r = 0.6mm,

$$\frac{dV}{dt} = \frac{1.9 \times 0.6}{2}$$

$$= \underline{0.57 \text{ mm}^3/\text{s}}$$

Many got the correct but some did not show clear working

Q14 (c) $\cos x - \cos(x+2\theta)$

$= \cos[(x+\theta)-\theta] - \cos[(x+\theta)+\theta]$ — (1 mark)

$= \cos(x+\theta)\cancel{\cos\theta} + \sin(x+\theta)\sin\theta - \cancel{\cos(x+\theta)\cos\theta}$
$+ \sin(x+\theta)\sin\theta$

$= 2\sin(x+\theta)\sin\theta$ (1 mark)

Many got this correct. Any relevant method o.k.

$$\sum_{r=1}^{n} \sin(2r-1)\theta = \frac{1-\cos 2n\theta}{2\sin\theta}$$

For $n=1$ LHS $= \sin(2-1)\theta = \sin\theta$

RHS $= \dfrac{1-\cos 2\theta}{2\sin\theta} = \dfrac{2\sin^2\theta}{2\sin\theta} = \sin\theta$

$\Rightarrow$ LHS = RHS

i.e. The proposition is true for $n=1$ $\longrightarrow$ (1 mark)

Assume true for $n=k$

i.e. $\displaystyle\sum_{r=1}^{k} \sin(2r-1)\theta = \frac{1-\cos 2k\theta}{2\sin\theta}$

For $n=k+1$ to prove that

$\displaystyle\sum_{r=1}^{k+1} \sin(2r-1)\theta = \frac{1-\cos 2(k+1)\theta}{2\sin\theta}$ } 2 mark

LHS $\displaystyle\sum_{r=1}^{k+1} \sin(2r-1)\theta = \sum_{r=1}^{k} \sin(2r-1)\theta + \sin[(2(k+1)-1)\theta]$

using the assumption

$= \dfrac{1-\cos 2k\theta}{2\sin\theta}$
$+ \sin[2(k+1)-1]\theta$

$= \dfrac{1-\cos 2k\theta + 2\sin\theta\sin(2k+1)\theta}{2\sin\theta}$

~~let $x = 2k$~~

~~$= 1 - \cos x -$~~

Let $x = 2k\theta$

Some students struggled as they could not use the result from previous part.

$$= \frac{1 - \cos x + 2\sin\theta\,(x+\theta)}{2\sin\theta}$$

$$= \frac{1 - \cancel{\cos x} + \cancel{\cos x} - \cos(x+2\theta)}{2\sin\theta}$$

$$= \frac{1 - \cos(x+2\theta)}{2\sin\theta}$$ (1 mark)

$$= \frac{1 - \cos\left[2(k+1) - 1\right]\theta}{2\sin\theta}$$

$= R(t)$

$\therefore$ The proposition is true for $n = k+1$.

The proposition is true for $n=1$ and if it is true for $n=k$ then it is true for $n=k+1$.

Therefore by the method of mathematical induction, it is true for all positive integers