Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2019 Mathematics Extension 1 Trial Examination

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks:

Section I – 10 marks. (Pages 3 – 6)

- Allow about 15 minutes for this section

Section II – 60 marks. (Pages 7 – 14)

- Allow about 1 hour and 45 minutes for this section

Section I (10 marks)	Multiple Choice	/10
Section II (60 marks)	Question 11	/15
	Question 12	/15
	Question 13	/15
	Question 14	/15
	Total	**/70**

This question paper must not be removed from the examination room.
This assessment task constitutes 30% of the course.

Section I

10 marks
Allow about 15 minutes for this section

Use the multiple-choice sheet for Question 1–10

---

1 The polynomial $f(x)=2x^3+5x^2+qx-12=0$ has a remainder of -9 when divided by $(x-1)$.

The value of q is:

(A) 5

(B) 4

(C) 2

(D) −4

2 The size of the acute angle between the lines $y=3x$ and $2x+y-3=0$ to the nearest degree is:

(A) 8°

(B) 32°

(C) 45°

(D) 82°

3 The integration of $\int x^2\sqrt{1+3x^3}\,dx$ using the substitution $u=1+3x^3$ is equal to:

(A) $\frac{2}{9}(1+3x^3)^{\frac{3}{2}}+C$

(B) $\frac{2}{27}(1+3x^3)^{\frac{3}{2}}+C$

(C) $\frac{1}{18}(1+3x^3)^{-\frac{1}{2}}+C$

(D) $\frac{2}{9}(1+3x^3)^{-\frac{1}{2}}+C$

4 Which expression is equivalent to $3\cos\theta - \sqrt{3}\sin\theta$?

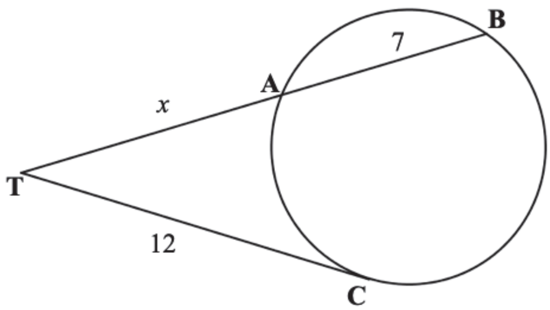
(A) $\sqrt{3}\sin\left(\theta + \frac{\pi}{6}\right)$

(B) $2\sqrt{3}\sin\left(\theta + \frac{\pi}{6}\right)$

(C) $2\sqrt{3}\cos\left(\theta - \frac{\pi}{6}\right)$

(D) $2\sqrt{3}\cos\left(\theta + \frac{\pi}{6}\right)$

5 In the diagram below, the line TB is a secant intersecting the circle ABC at the points A and B. TC is a tangent at the point C. The length *x* is equal to:



(A) 5

(B) 9

(C) 14

(D) 16

Section I continues on next page

6 How many ways can 9 people sit at a round table, if 2 particular people must sit together?

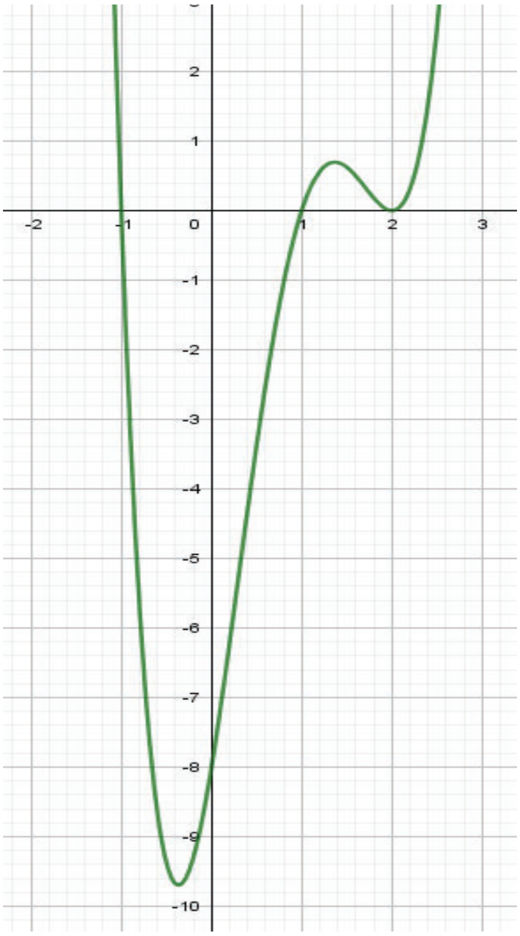
(A) $7!$

(B) $8!$

(C) $7! \times 2$

(D) $8! \times 2$

7



The diagram above shows the graph of $y = a(x+b)(x+c)(x+d)^2$
What are possible values of a, b, c and d?

(A) $a = -8, b = -1, c = 1, d = -2$

(B) $a = -8, b = 1, c = -1, d = 2$

(C) $a = 2, b = 1, c = -1, d = 2$

(D) $a = 2, b = 1, c = -1, d = -2$

8 If $f(x) = \frac{x(x-1)}{x-2}$,which of the following lines will be an asymptote of $y = f(x)$?

(A) $y = x$

(B) $y = x - 1$

(C) $y = x + 1$

(D) $y = x - 2$

9 Which of the following is the integral of

$$\int \frac{1}{e^x + 1} dx \,?$$

(A) $\frac{1}{2}\log_e(e^{2x} + e^x) + C$

(B) $\log_e(e^x + 1) + C$

(C) $x - \log_e(e^x + 1) + C$

(D) $-\log_e\left(\frac{e^x}{e^x+1}\right) + C$

10 The acceleration, a m/s^2, of a particle moving in a straight line is given by

$$a = \frac{v}{\log_e v}$$

where v is the velocity of the particle in ms^{-1} at time t seconds and the initial velocity of the particle was 5 ms^{-1}.

The expression that represents velocity is:

(A) $v = e^{2t}$

(B) $v = e^{2t} + 4$

(C) $v = e^{\sqrt{2t} + \log_e 5}$

(D) $v = e^{\sqrt{2t + (\log_e 5)^2}}$

End of Section I

Section II

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Solve the inequality **3**

$$\frac{3x+1}{x^2-1} \leq 0$$

(b) Find the point P which divides the line AB joining $A(4,6)$ and $B(13,5)$ externally in the ratio $4:1$ **2**

(c) Evaluate: **2**

$$\lim_{x \to 0} \frac{sin\frac{x}{3}}{x}$$

(d) Find the domain of **2**

$$f(x) = \sqrt{x+3} + \sqrt{x-2}$$

(e) Prove that **2**

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

(f) Differentiate **2**

$$f(x) = \left(\tan^{-1}\frac{x}{3}\right)^2$$

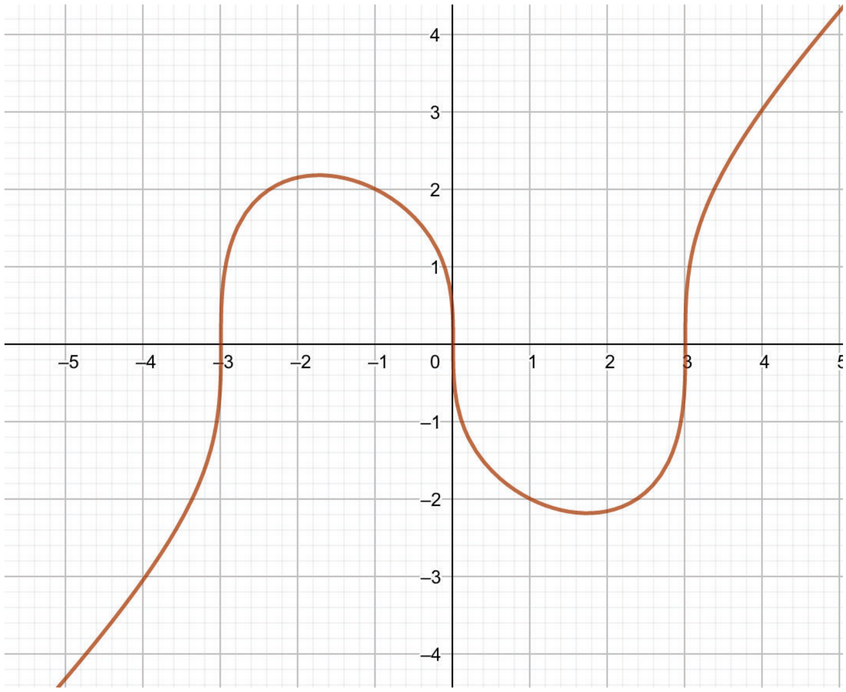
(g) Prove using the factorial formula for ${}^{n}P_{r}$ that **2**

$${}^{n+1}P_r = {}^{n}P_r + r.\ {}^{n}P_{r-1}$$

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) The curve $f(x) = (x^3 - 9x)^{\frac{1}{3}}$ is shown below:



(i) Taking an initial approximation of $x_1 = -2.5$, use one application of Newton's Method to obtain another approximation to the root of $f(x) = 0$ (Give your answer correct to 4 decimal places) **3**

(ii) Explain why using $x_1 = -2.5$ and Newtons Method does not produce a better approximation to the root than the original estimate. **1**

(b) A hot pan is cooling in a room of constant temperature 22°C. At time t minutes its temperature decreases according to the equation,

$$\frac{dT}{dt} = -k(T - 22)$$

where k is a positive constant.

(i) Verify that $T = 22 + Ae^{-kt}$ is a solution to the equation, where A is a constant. **1**

(ii) If a pan initially at 180 °C cools to 100 °C in 6 minutes, how long will it take for the pan to become cool enough to touch (40 °C)? Give your answer correct to the nearest minute. **3**

(c) A particle is moving in a straight line with Simple Harmonic motion. At time t seconds it has a displacement x metres from the fixed point O, on the line where $x = (\cos t + \sin t)^2$, velocity v m/s and acceleration a m/s^2.

(i) Show that $a = -4(x - 1)$ **2**

(ii) Find the extreme positions of the particle during its motion **1**

(d) Find $\int \sin^2 2x \, dx$ **2**

(e) A spherical balloon is expanding so that its surface area is increasing at a constant rate of 0.027 cm^2 per second.

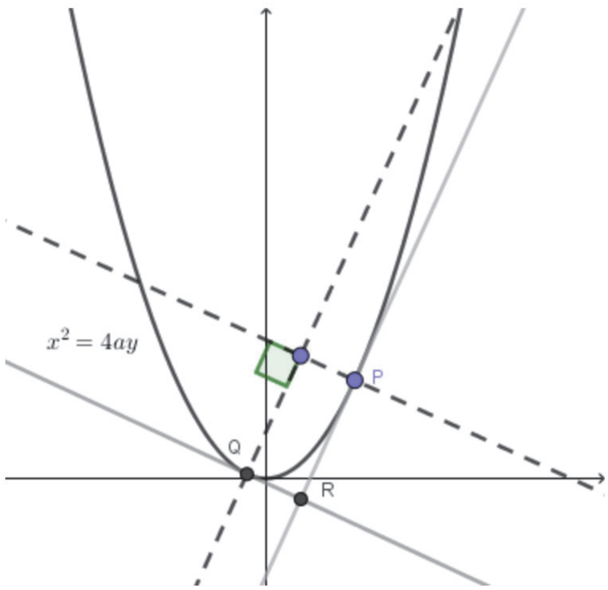
Find the rate of change of the volume when the radius is 7cm. **2**
(Give your answer correct to 4 decimal places)

(The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, and the Surface Area is $A = 4\pi r^2$)

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a)



In the diagram above, the normal at $P(2at, at^2)$ and $Q(2aq, aq^2)$ on the parabola $x^2 = 4ay$ are perpendicular.

(i) The equations of the normal at P and Q are $x + ty = at^3 + 2at$ and $x + qy = aq^3 + 2aq$. DO NOT PROVE THIS. **1**

Show that the relationship between q and t is $q = -\frac{1}{t}$

(ii) The tangents at P and Q are $y = tx - at^2$ and $y = qx - aq^2$ respectively. DO NOT PROVE THIS. **2**

Hence show that the coordinates of R, the point of intersection of the tangents at P and Q are

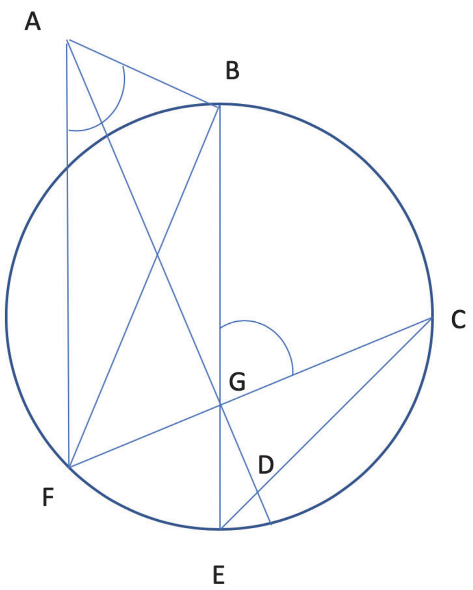
$$\left[a\left(t - \frac{1}{t}\right), -a\right]$$

(iii) Describe the locus of R and its relationship to the parabola $x^2 = 4ay$ **1**

Question 13 continues on page 11

Question 13 (continued)

(b)



In the diagram above, the chords BE and CF intersect at the point G. The point A is chosen such that $\angle BAF = \angle BGC$. AG is produced to intersect CE at D.

Copy the diagram into your writing booklet.

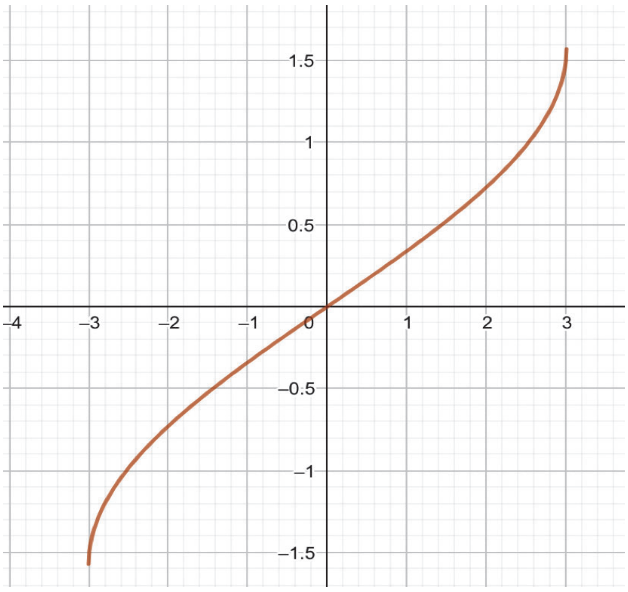
(i) Explain why ABGF is a cyclic quadrilateral **1**

(ii) Hence prove that $\angle FCE = \angle FAG$ **2**

Question 13 continues on page 12

Question 13 (continued)

(c) The sketch below shows the graph of $f(x) = \sin^{-1}\left(\frac{x}{3}\right)$



(i) Determine the inverse function of $f(x) = \sin^{-1}\left(\frac{x}{3}\right)$ **1**

(ii) Hence or otherwise calculate the area between the curve $y = f(x)$ and the x-axis, between $x = 0$ and $x = 3$ as shown in the diagram above. **3**

(d) A particle is moving along the x-axis in simple harmonic motion centred at the origin.

When $x = 3$cm, the velocity of the particle is 8 cm/s and when $x = 5$ cm, the velocity of the particle is 4 cm/s.

(i) Determine the period of the motion **3**

(ii) Hence, write an equation to represent v^2 in terms of x **1**

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Prove by mathematical induction that **4**

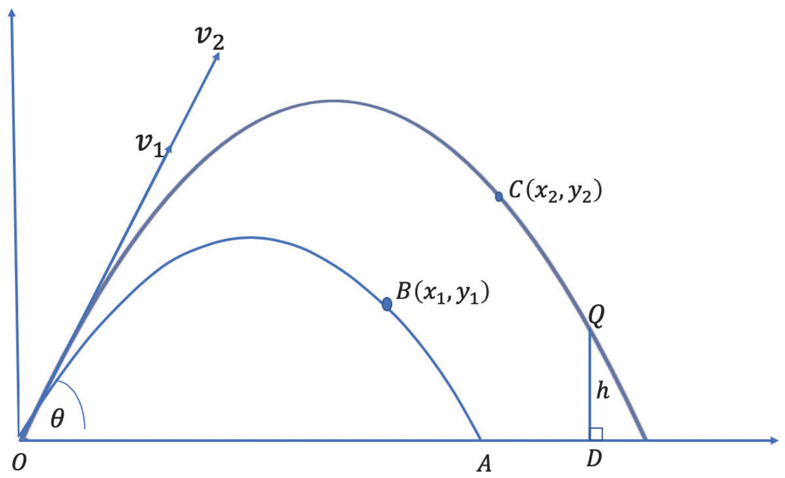
$$\sin(x + n\pi) = (-1)^n \sin x \qquad \text{for all integer } n \geq 1$$

(b) Find the inverse function of: **2**

$$f(x) = \frac{e^x - e^{-x}}{2}$$

(c) Two stones are projected simultaneously from the point O with initial velocities v_1 and v_2 (but $v_2 > v_1$) in a vertical plane at an angle θ to the horizontal.
The slower stone hits the ground at a point A.
At time t, let $B(x_1,\ y_1)$ and C$(x_2,\ y_2)$ be two points on the respective paths of flights.

The equations of motion are:	
$\ddot{x} = 0$	$\ddot{y} = -g$
$\dot{x} = v\ cos\theta$	$\dot{y} = -gt + v\ sin\theta$
$x = v\ cos\theta\ t$	$y = -\frac{gt^2}{2} + \ v\ sin\theta\ t$
DO NOT PROVE THIS	



(i) Find the gradient of BC and explain why it is independent of time t **3**

Let QD be a wall of height h on level ground. When the second stone just clears a wall, the first stone hits the ground at A.

(ii) Prove that $AD = h\ cot\theta$ **2**

Question 14 continued on page 15

Let T be the time of flight of the slower stone. At the same instant, let ϕ be the angle made by the downward flight of the faster stone with the horizontal.

(iii) Show that **2**

$$\tan(-\phi) = tan\theta - \frac{gT}{v_2 cos\theta}$$

(iv) Hence, show that **2**

$$v_2(tan\theta + tan\phi) = 2v_1\, tan\theta$$

End of Paper

Year 12 Mathematics Extension 1 Trial Marking scheme

Multiple choice

1	Solution	Marking Criteria	Marker's feedback
1-7	Multiple choice answers are attached		
8	C	$x^2 - x = (x+1)(x-2) + 2$ Oblique asymptote is $y = x + 1$	
9	C	$\int \frac{1}{e^x+1} dx$ $= \int \frac{e^x + 1 - e^x}{e^x+1} dx$ $= \int 1 - \frac{e^x}{e^x+1} dx$ $= x - \ln(e^x+1) + C$	
10	D	$a = \frac{v}{lnv}$ $\frac{dv}{dt} = \frac{v}{lnv}$ $\frac{lnv}{v} dv = dt$ $\int_5^v \frac{lnv}{v} dv = \int_0^t dt$ $\left[\frac{(lnv)^2}{2}\right]_5^v = [t]_0^t$ $(lnv)^2 - (ln5)^2 = 2t$ $(lnv)^2 = 2t + (ln5)^2$ $t = 0, v = 5.$ Hence, $lnv = \sqrt{2t + (ln5)^2}$ $v = e^{\sqrt{2t+(ln5)^2}}$	

Ext 1 trial   M/choice Q1~10

1 $f(x) = 2x^3 + 5x^2 + qx - 12$

$f(1) = -9$

$-9 = 2 + 5 + q - 12$

$-9 = -5 + q$

$\therefore q = -4$ (D) ✓

---

2 $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 . m_2}\right|$ $m_1 = 3$, $m_2 = -2$

$\tan\theta = \left|\dfrac{3 - -2}{1 + 3.-2}\right| = \left|\dfrac{5}{-5}\right| = 1$

$\therefore \theta = \tan^{-1}(1) = 45°$ (C) ✓

---

3 $\int x^2 \sqrt{1+3x^3}\, dx$ $u = 1 + 3x^3$

$\dfrac{du}{dx} = 9x^2$

$= \dfrac{1}{9}\int \sqrt{1+3x^3} . 9x^2 . dx$ $du = 9x^2 . dx$

$= \dfrac{1}{9}\int \sqrt{u}\, du$

$= \dfrac{1}{9}\int u^{1/2}$

$= \dfrac{1}{9}\left[\dfrac{2u^{3/2}}{3}\right] + c$

$= \dfrac{2}{27}\left(1+3x^3\right)^{3/2} + c$ (B)

4 $3\cos\theta - \sqrt{3}\sin\theta$

$$\cos(\phi+\theta) = \cos\phi\cos\theta - \sin\phi\sin\theta$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \quad \therefore \phi = \frac{\pi}{6}$$

$$R = \sqrt{3^2+(\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$$

$$\therefore 2\sqrt{3}\cos\left(\theta + \frac{\pi}{6}\right) \quad (D)$$

---

5 $CT^2 = TA.TB$

$12^2 = x(x+7)$

$x^2+7x-144=0$

$(x+16)(x-9)=0$

$x=-16, 9$

$\therefore x = 9$ (B) ✓

---

9 people.

6 ⇒ treat as 8 people, as 1 group of 2 but 2 people can switch

$(8-1)! = 7!$

and account for 2 switching $= 7! \times 2$ (C)

7 $(x+d)^2$

$d = -2$

$\therefore$ A or D

$b=1, c=-1$ good for both

y-int: $1 \times -1 \times (-2)^2 = -4$ for $(x+b)(x+c)(x+d)^2$

$\therefore a = 2$

$\therefore$ (D) ✓

---

8 $y = \frac{x(x-1)}{x-2} \Rightarrow \frac{x^2-x}{x-2}$

$$\begin{array}{r} x+1 \\ x-2 \overline{) x^2 - x} \\ -\underline{x^2 - 2x} \\ x \\ -\underline{x-2} \\ 2 \end{array}$$

$\therefore y = x+1$ (C) ✓

---

9 $\int \frac{1}{e^x+1}$

~~try~~ $\Rightarrow y = \ln(e^x+1) + c$

$y' = \frac{e^x}{e^x+1}$

$y = x - \ln(e^x+1)$

$y' = 1 - \frac{e^x}{e^x+1}$

$y' = \frac{e^x+1-e^x}{e^x+1}$ = original

$\therefore$ (C) ✓

10 $a = \frac{v}{\ln v}$ $t=0, v=5$

try B as when $t=0, v=5$

$e^0 + 4 = 5$

$v = e^{2t} + 4$

$\frac{dv}{dt} = 2.e^{2t}$

$\therefore$ A & B out

try C $t=0$

$v = e^{0+\ln 5} = 5$

$V = e^{\sqrt{2t} + \ln 5}$

$\frac{dv}{dt} =$

$y = (2t)^{1/2} + \ln 5$

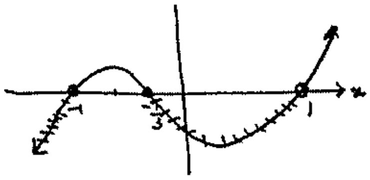
$y' = \frac{1}{2} \cdot (2t)^{-1/2} = \frac{1}{\sqrt{t}\,2} = \frac{1}{\sqrt{2t}}$.

$\therefore \ln\left(e^{\sqrt{2t} + \ln 5}\right)$

= not the answer

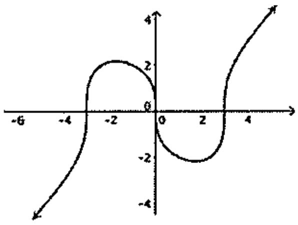
$\therefore$ must be (D)

Question 11

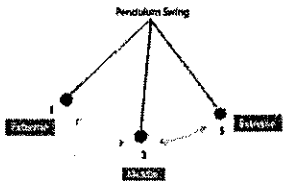
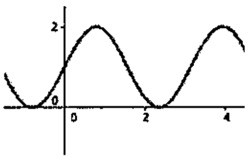
Question Number	Solution	Marking Criteria	Marker's feedback
a)	$\frac{3x+1}{x^2-1} \leq 0$ $\frac{(3x+1)(x^2-1)^2}{x^2-1} \leq 0 \times (x^2-1)^2$ $(3x+1)(x+1)(x-1) \leq 0$  $x \neq \pm 1$ *Hence,* $x < -1, \quad -\frac{1}{3} \leq x < 1$	1 mark: Multiplies both sides by $(x^2-1)^2$ Solves the inequality correctly 2 Mark: combines with the restriction $x \neq \pm 1$ to give the correct final solution 1 mark only: if both restrictions are not applied	*The most common error was not applying the restriction $x \neq \pm 1$.* *Many students failed to draw a diagram of the cubic to aid the solution and thus making unnecessary errors.* *Those students who resorted to testing different regions must understand that this is a lengthy process and where time is precious is not a recommended method.*
b)	$A(4,6)$ and $B(13,5)$ $4:1$ $P\left(\frac{13\times 4-1\times 4}{4-1}, \frac{5\times 4-1\times 6}{4-1}\right)$ $P\left(16, \frac{14}{3}\right)$	2 Marks: substitutes into the correct formula and correct evaluation 1 mark: substitutes into the correct formula, but minor error in evaluation	*Generally well done, however, there are still students who do not know how to transform the given internal division formula to external division contexts.*
d)	$= \lim_{x\to 0} \frac{\sin\frac{x}{3}}{x}$ $= \frac{1}{3} \times \lim_{x\to 0} \frac{\sin\frac{x}{3}}{\frac{x}{3}}$ (must have this line) $= \frac{1}{3} \times 1$ $= \frac{1}{3}$	1 mark: Manipulates the expression correctly (All lines of working must be shown) 1 mark: Applies the small angle formula to give the correct solution	*Well done. But some students seem to think that you can take the $\frac{1}{3}$ from the angle.* *The following solution is erroneous.* $\lim_{x\to 0} \frac{\sin\frac{x}{3}}{x} = \frac{1}{3} \lim_{x\to 0} \frac{\sin x}{x} = \frac{1}{3}$

d)	$$f(x)=\sqrt{x+3}+\sqrt{x-2}$$ Domain of $\sqrt{x+3}+\sqrt{x-2}$ $$x+3\geq 0 \ \ and\ x-2\geq 0$$ $$\therefore \quad x\geq -3 \ \ and\ x\geq 2$$ Hence, $x\geq 2$.	1 mark: gives the domains both components correctly 1 Mark: combines the two results to give the final answer	*Some students presented bald answer for this question and were not awarded any marks. Students.* *Also, please note that the region where both inequalities hold good is the solution to this problem. A quick sketch will be very helpful.*
e)	Let $$\cos^{-1}(-x)=y$$ Hence, $-x=\cos y$ Or $x=-\cos y$ 1 mark $=\cos(\pi-y)$ $$\therefore \ \pi-y=\cos^{-1}x$$ $$y=\pi-\cos^{-1}x$$	2 mark: correct proof showing all lines of working 1 mark: expresses $x=-\cos y$	*Apart from this students have used methods such as differentiation and graphical methods. However, solutions that just amounted to verifying the relation was not sufficient.*
f)	$$f(x)=\left(\tan^{-1}\frac{x}{3}\right)^2$$ $$f'(x)=2\left(\tan^{-1}\frac{x}{3}\right)\times\frac{3}{9+x^2}$$ $$=\frac{6\tan^{-1}\frac{x}{3}}{9+x^2}$$	1 mark: correctly applies chain rule 1 mark: correctly differentiates $\tan^{-1}\frac{x}{3}$	*Well done.*
g)	$${}^{n+1}P_r={}^{n}P_r+r.\,{}^{n}P_{r-1}$$ RHS $$=\frac{n!}{(n-r)!}+r.\frac{n!}{(n-r+1)!}$$ $$=\frac{n!}{(n-r)!}\left[1+\frac{r}{(n-r+1)}\right]$$ $$=\frac{n!}{(n-r)!}.\frac{n-r+1-r}{n-r+1}$$ $$=\frac{n!}{(n-r)!}\frac{n+1}{n-r+1}$$ $$=\frac{(n+1)!}{(n-r+1)!}$$ $$={}^{n+1}P_r$$	2 marks : correct proof with all working lines of working shown 1 mark: correct definitions for n_{P_r} and $n_{P_{r-1}}$ and attempts to take common factor out.	*Students must give the correct definition of* ${}^{n}P_r$

Question 12 (15 marks) Use the Question 1 Writing Booklet.

(a)(i)	$f(x)=(x^3-9x)^{\frac{1}{3}}$  initial approximation $x_1=-2.5$, use one appln Newton's Method $f'(x)=\frac{1}{3}(x^3-9x)^{-\frac{2}{3}}(3x^2-9)$ $=(x^3-9x)^{-\frac{2}{3}}(x^2-3)$ then $x_2=-x_1-\frac{f(x_1)}{f'(x_1)}$ $=-x_1-\frac{(x^3-9x)^{\frac{1}{3}}}{(x^3-9x)^{-\frac{2}{3}}(x^2-3)}$ $=-x_1-\frac{(x^3-9x)}{(x^2-3)}$ $=$ -4.6154 *OR* $f'(-2.5)=0.8988.....$ $f(-2.5)=(-2.5^3+9\times2.5)^{\frac{1}{3}}$ $=1.9014....$ then $x_2=-2.5-\frac{f(x_1)}{f'(x_1)}$ $=-2.5-\frac{1.9014.....}{0.8988....}$ $=-4.6154$	3 marks Correct answer from correct working 2 marks Correct expression of $f'(x)$ And correct value of $f(2.5)$ 1 mark Correct expression of $f'(x)$	A nice algebraic solution given Most students did well. -1 mark deducted if solution not given to 4 dp Several students had incorrect answer from insufficient working and gained 1 mark only
(ii)	Explain why using $x_1=-2.5$ and Newtons Method does not produce a better approximation the root than the original estimate. As the function is close to vertical near the root, the tangent drawn at $x=-2.5$ intersects the x-axis further away from the root.	1 mark correct explanation	Mostly done well, but a concerning number of students have a lack of understanding of this

(b)(i)	A hot pan is cooling in a room of constant temperature 22°C. At time t minutes its temperature decreases according to the equation. $\frac{dT}{dt} = -k(T-22)$ where k is a positive constant $T = 22 + Ae^{-kt}$ $\therefore Ae^{-kt} = T - 22$ $\frac{dT}{dt} = -kAe^{-kt}$ $= -k(T-22)$	**1 mark correct calculation**	Mostly done well. Students should note the requirement to VERIFY, meaning establish the correctness of. A solution by differentiating is valid and a short answer for 1 mark
(ii)	If the pan cools from 180 °C to 100 °C in 6 minutes, how long will it take for the pan to become cool enough to touch (40 °C)? $T = 22 + Ae^{-kt}$ $t = 0 \; T = 180^oC$ $180 = 22 + Ae^{-k\times 0}$ $180 - 22 = A$ $A = 158$ $T = 22 + 158e^{-kt}$ $t = 6 \; T = 100^oC$ $100 = 22 + 158e^{-6k}$ $78 = 158e^{-6k}$ $T = 40^oC$ $40 = 22 + 158e^{-kt}$ $-kt = \ln(\frac{18}{158})$ $t = ...$	3 marks correct value of t from correct working 2 marks Correct Value of A and Correct value of k 1 mark Correct value of A	Mostly done well. There was no penalty for rounding, but the time should be rounded to 19 mins to be cool enough to touch

(c) (i)	$x = (\cos t + \sin t)^2$ $= \cos^2 t + 2\sin t\cos t + \sin^2 t$ $= 1 + 2\sin t\cos t$ $x - 1 = 2\sin t\cos t$ $\dot{x} = 2\sin t(-\sin t) + 2\cos t(\cos t)$ $= 2\cos^2 t - 2\sin^2 t$ $\ddot{x} = 4\cos t(-\sin t) - 4\sin t(\cos t)$ $= -8\sin t\cos t$ $\ddot{x} = -4(2\sin t\cos t)$ $= -4(x-1)$	2 marks correct expression for $\ddot{x}$ from correct working 1 mark correct expression for $\dot{x}$ from correct working	Mostly done well. This is a SHOW THAT question Students that skipped steps lost a mark $\ddot{x} = -4(2\sin t\cos t)$ $= -4(x-1)$
(ii)	Find the extreme positions of the particle during its motion  Extreme position when $x = 1 + 2\sin t$ *then* $0 \le 1 + 2\sin t \le 2$  OR $\dot{x} = 0$ $2(\cos^2 t - \sin^2 t) = 0$ $\cos^2 t = \sin^2 t$ $\cos t = \pm\sin t$ $\cos t = \sin t$ when $t = \frac{\pi}{4}$ $x = 1 + 2\times\frac{1}{2} = 2$ $\cos t = -\sin t$ when $t = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $x = 1 - 2\times\frac{1}{2} = 0$	1 mark correct answer from correct working	Many of the students gave long solutions when considering the max and min positions of the function $y = 1 + 2\sin t$ Was sufficient Many students ignored the second solution

(d)	$\int \sin^2 2x\,dx$ $\cos 2A = 1-2\sin^2 2x$ $\sin^2 A = \frac{1-2\cos 2A}{2}$ $\sin^2 2A = \frac{1-2\cos 4A}{2}$ $\int \sin^2 2x\,dx = \int \frac{1-2\cos 4x}{2}dx$ $= \frac{1}{2}\int 1-\cos 4x\,dx$ $= \frac{1}{2}(x-\frac{\sin 4x}{4})+c$ $= \frac{x}{2}-\frac{\sin 4x}{8}+c$	2 marks correct integral 1 mark correct integral expression using double angle	Mostly Done well Some students tried to use substitution unsuccessfully
(e)	A spherical balloon is expanding so that its surface area is increasing at a constant rate of 0.027 cm^2 per second. Find the rate of change of the volume when the radius is 7cm. $V = \frac{4}{3}\pi r^3$ $A = 4\pi r^3$ $\quad \frac{dV}{dt} = \frac{dV}{dr}\times\frac{dr}{dt}$ $\frac{dA}{dt} = 8\pi r \times \frac{dr}{dt}$ $\quad = 4\pi r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{0.027}{8\pi r}$ $\quad = 4\pi\times 7^2 \times \frac{0.027}{8\pi\times 7}$ $= \frac{0.027\times 7}{2} = \ldots$ OR $\frac{dV}{dA} = \frac{dV}{dr}\times\frac{dr}{dA} = \frac{r}{2}$ $\frac{dV}{dt} = \frac{dA}{dt}\times\frac{dV}{dA}$	2 marks correct answer from correct working 1 mark correct expression for $\frac{dr}{dt}$	Mostly done well Students that were unsuccessful tried to find an expression for $\frac{dV}{dA}$ Without using the chain rule

End of Question 12

Q13 X1 Markers Feedback

a i) most did well, some didn't use $m_{NP} \cdot m_{NQ} = -1$
or showed insufficient working

ii) very well done :)

iii) poorly done ⇒ $y = -a$ is the directrix - 2U knowledge...

b i) well done, many did way too much working
for 1 mark wasting valuable time

ii) well done - students need to conclude and be
careful with reasons.

c i) mostly well done - some showed no working
so a mark couldn't be given

ii) About half got full marks. Many lost a mark
by forgetting $y = \sin^{-1} x$ has $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

→ Some totally ignored the hint in i) and tried to
integrate $y = \sin^{-1}\left(\frac{x}{3}\right)$ by differentiating it !!
→ a couple of X2 students used "integration by parts"
instead which is fine :)

d i) those who formed 2 equations using $v^2 = n^2(a^2 - x^2)$
or $\ddot{x} = -n^2 x$ and integrating were successful
in gaining most or all marks.
⇒ students should realise that using Trig is a poor
choice and the above 2 methods are superior in this case.

ii) most who chose the correct method(s) in i)
easily got this mark :)

Q13

a **i**

$x + ty = at^3 + 2at$

$y = \frac{-1}{t}x + \frac{at^{3}}{t} + \frac{2at}{t}$

$\therefore m_{NP} = \frac{-1}{t}$

$x + qy = aq^3 + 2aq$

$y = -\frac{1}{q}x + aq^2 + 2a$

$\therefore m_{NQ} = -\frac{1}{q}$

as $m_{NP} \cdot m_{NQ} = -1$

$\frac{-1}{t} \cdot \frac{-1}{q} = -1$

$\left(\frac{1}{qt} = -1\right) \times q$

$\therefore q = \frac{-1}{t}$ [1]

ii

$y = tx - at^2$

$y = qx - aq^2$

$\therefore tx - at^2 = qx - aq^2$

$tx - qx = at^2 - aq^2$

$x(t-q) = a(t^2 - q^2)$

$x(t-q) = a(t-q)(t+q)$

$x = a(t+q)$

from i, $q = \frac{-1}{t}$

$\therefore x = a\left(t - \frac{1}{t}\right)$ [1]

$y = tx - at^2$

sub $x = a\left(t - \frac{1}{t}\right)$

$y = t\cdot a\left(t - \frac{1}{t}\right) - at^2$

$y = at^2 - a - at^2$

$\therefore \underline{y = -a}$ [1]

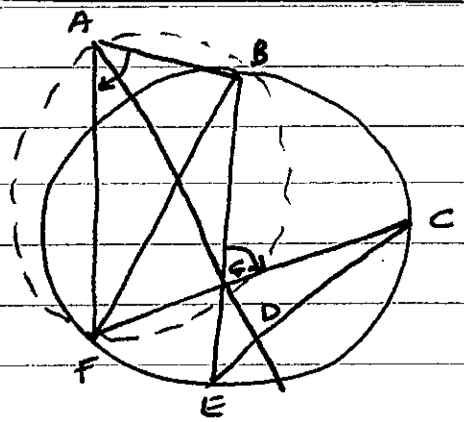
$\therefore R$ is $\left\{ a\left(t - \frac{1}{t}\right), -a \right\}$

iii locus of R is $\underline{y = -a}$ [1]

this means R moves along

<u>the directrix</u> of the parabola $x^2 = 4ay$

b



i $\angle BGC = \angle BAF$ given

$\therefore$ ABGF is a cyclic quadrilateral

as the exterior angle of a cyclic Quad ($\angle BGC$)

equals the interior opposite angle. [1]

ii $\angle FCE = \angle FBE$ (angles in the same segment of circle BCFE are equal) [1]

$\angle FBE = \angle FAG$ (angles in the same segment of circle BAFG are equal)

$\therefore \angle FCE = \angle FBE = \angle FAG \Rightarrow \angle FCE = \angle FAG$ [1]

c i $f(x) = \sin^{-1}\left(\frac{x}{3}\right)$

$y = \sin^{-1}\left(\frac{x}{3}\right)$

reflect across $y = x$

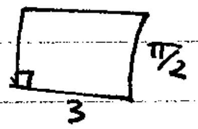
$x = \sin^{-1}\left(\frac{y}{3}\right)$

$\sin(x) = \sin\left\{\sin^{-1}\left(\frac{y}{3}\right)\right\}$

$\sin x = \frac{y}{3}$

$\therefore y = 3\sin x$ [1]

ii we know at $x = 3$, $y = \frac{\pi}{2}$

$\therefore$ 

$- \int_0^{\pi/2} 3\sin x = \int_0^3 \sin^{-1}\frac{x}{3}\,dx$ [1]

$3\int_0^{\frac{\pi}{2}} \sin x\,dx = 3\left[-\cos x\right]_0^{\pi/2}$

$= 3\left[-\cos\frac{\pi}{2} - -\cos(0)\right]$

$= 3 \times 1 = 3$ units^2 [1]

$\therefore$ Area $= \frac{3\pi}{2} - 3$ units^2 [1]

c) – For X2 students, who did integration by parts...

$$\int_0^3 \sin^{-1}\frac{x}{3}\,dx$$

$$= \int_0^3 1.\left[\sin^{-1}\frac{x}{3}\right]dx$$

$$\int f_1 . f_2 = F_1 . f_2 - \int F_1 . f_2'$$

let $1 = f_1 \Rightarrow x = F_1$

$f_2 = \sin^{-1}\frac{x}{3}$

$$f_2' = \frac{1}{3} \times \frac{1}{\sqrt{1-\frac{x^2}{9}}}$$

$$= \frac{1}{\sqrt{9}\sqrt{1-\frac{x^2}{9}}}$$

$$= \frac{1}{\sqrt{9-x^2}}$$

$$\therefore = \left[x . \sin^{-1}\frac{x}{3}\right]_0^3 - \int_0^3 x . \frac{1}{\sqrt{9-x^2}}$$

$$= \left[x . \sin^{-1}\frac{x}{3}\right]_0^3 + \int_0^3 \frac{-x}{\sqrt{9-x^2}}$$

Note

$y = (9-x^2)^{1/2}$

$y' = \frac{1}{2}(9-x^2)^{-1/2} . -2x$

$y' = \frac{-x}{\sqrt{9-x^2}}$

$$= \left(3 . \sin^{-1} 1 - 3 . \sin^{-1} 0\right) + \left[\sqrt{9-x^2}\right]_0^3$$

$$= \left(\frac{3\pi}{2} - 0\right) + \left(\sqrt{9-9} - \sqrt{9-0}\right)$$

$$= \frac{3\pi}{2} - 3 \text{ units}^2$$

d $v^2 = n^2(a^2 - x^2)$ $x = 3, v = 8$

$x = 5, v = 4$

i $v^2 = n^2(a^2 - x^2)$

$8^2 = n^2(a^2 - 3^2)$

$64 = n^2(a^2 - 9)$ (1)

$v^2 = n^2(a^2 - x^2)$

$4^2 = n^2(a^2 - 5^2)$

$16 = n^2(a^2 - 25)$ (2)

(1) ÷ (2)

$$\frac{64}{16} = \frac{n^2(a^2-9)}{n^2(a^2-25)}$$

$$4 = \frac{a^2-9}{a^2-25}$$

$4a^2 - 100 = a^2 - 9$

$3a^2 = 91$

$\therefore a^2 = \frac{91}{3}$ [1]

sub $x = 3$, $v = 8$

$v^2 = n^2(a^2 - x^2)$

$64 = n^2\left(\frac{91}{3} - 9\right)$

$\left(64 = n^2 \times \frac{64}{3}\right) \times \frac{3}{64}$

$\therefore n^2 = 3$ [1]

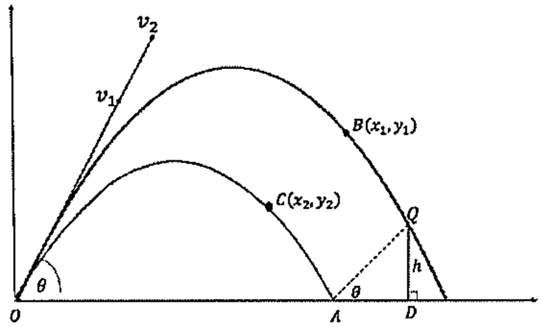
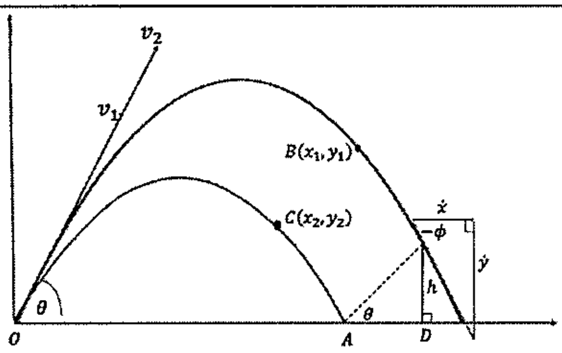
$n = \pm\sqrt{3}$ → as 'n', $n = \sqrt{3}$

period $= \frac{2\pi}{n} = \frac{2\pi}{\sqrt{3}}$ [1]

ii $v^2 = n^2(a^2 - x^2) \Rightarrow v^2 = 3\left(\frac{91}{3} - x^2\right) = 91 - 3x^2$ [1]

Question 14

Question Number	Solution	Marking Criteria	Marker's feedback
a)	$\sin(x+n\pi) = (-1)^n sinx$ for $n \geq 1$ Let $n = 1$ LHS $\sin(x+\pi) = -sinx$ RHS $(-1)^1 sinx = -sinx$ LHS = RHS, hence the result is true. Assume the result is true for $n = k$ Hence, $\sin(x+k\pi) = (-1)^k sinx$ (****) We need to prove, if the result is true for n = k, then it is true for n = k+1. $\text{Sin}(x+(k+1)\pi)$ $= \sin(x+k\pi+\pi)$ $= \sin(x+k\pi)\, cos\pi + \cos(x+k\pi)\, sin\pi$ $= (-1)^k \sin x \times (-1)$ using (****) $= (-1)^{k+1} \sin x$	1 mark: proves the result for n = 1 1 mark: writes the statements for n= k and n = k+1 2 mark: proves the result is true for n = k+1 and gives a conclusion Only 1 mark: a minor error in the proof	*Most students received the first two marks.* *Students who used* $\text{Sin}(x+(k+1)\pi)$ $= \sin(x+k\pi+\pi)$ $= \sin(x+\pi+k\pi$ *found the expansion difficult and less successful.* *Note: Always, keep the results from previous questions in mind. In many cases, they may prove to be useful in solving the later problems. In this case, step 1 and 2 were useful.*
b)	$f(x) = \frac{e^x - e^{-x}}{2}$ Let $y = \frac{e^x - e^{-x}}{2}$ $2y = e^x - e^{-x}$ Multiply by e^x $2ye^x = e^{2x} - 1$ $e^{2x} - 2ye^x - 1 = 0$ $e^x = \frac{2y \pm \sqrt{4y^2+4}}{2}$ $e^x = y \pm \sqrt{y^2+1}$ $\therefore x = \ln(y \pm \sqrt{y^2+1})$ $y < \sqrt{y^2+1}$, hence $x = \ln(y - \sqrt{y^2+1})$ is undefined, and rejected. Hence, $x = \ln(y + \sqrt{y^2+1})$ The inverse is y= $\ln(x + \sqrt{x^2+1})$	1 mark: forms a quadratic in e^x and solves for *x*. 1 mark: gives reasoning for rejecting $\ln(y - \sqrt{y^2+1})$ as a root and gives the inverse function	*Very poorly done. Those students who identified this as a quadratic in* e^x *were more successful.*

b)(i)	$B(x_1,\ y_1)$ and $C(x_2,\ y_2)$ Gradient of BC $=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{-\frac{gt^2}{2}+v_2\ sin\theta\ t+\frac{gt^2}{2}-v_1\ sin\theta\ t}{v_2cos\theta t-v_1cos\theta t}$ $=\frac{(v_2-v_1)sin\theta\ t}{(v_2-v_1)cos\theta t}$ $=\ tan\theta$ which is independent of t	2 marks: correct answer from correct working 1 mark: minor error in working	well done.
b)(ii)	 From (i), gradient of BC = $tan\theta$ Hence, gradient of AQ = $tan\theta$ Hence, in ΔADQ , $$tan\theta=\frac{h}{AD}$$ Hence, $AD\ =\ hcot\theta$	1 mark: identifies gradient of AQ = gradient of BC 1 mark: proves AD = $hcot\theta$ with working	students must refer to the result from (a) to get the full mark for this question. Many students used lengthy processes wasting significant amount of time. Again, understanding the meaning of the results you prove is very important.
b)(iii)	 At the point Q at time T, $$\dot{y}=\ -gT+v_2sin\theta$$ $$\dot{x}=\ v_2cos\theta$$ Hence, tan $(-\phi)=\frac{\dot{y}}{\dot{x}}$ $=\frac{-gT+v_2sin\theta}{v_2cos\theta}$ $=\frac{-gT}{v_2cos\theta}+tan\theta$	1 mark: writes expressions for $\dot{y}$ and $\dot{x}$ 1 mark: gives the expression for tan $(-\phi)=\frac{\dot{y}}{\dot{x}}$	A Please note: $$\tan(-\phi)\neq\frac{y}{x}$$ Gradient is difference in y over difference in x. Basic principles remain the same. Please do not waste your time trying to fudge the results.

b)(iv)	At A, y = 0 $\therefore y = -\frac{gt^2}{2} + v_1 sin\theta t = 0$ $\therefore t = 0,$ $-\frac{gt}{2} + v_1 sin\theta = 0$ Then $T = t = \frac{2v_1 sin\theta}{g}$ Substitute T in $\tan(-\phi) = \frac{-gT}{v_2 cos\theta} + tan\theta$ $\tan(-\phi) = \frac{-g \times 2v_1 sin\theta}{gv_2 cos\theta} + tan\theta$ $-\tan\phi = \frac{-2v_1}{v_2} tan\theta + tan\theta$ $\frac{2v_1}{v_2} tan\theta = tan\phi + tan\theta$ $2v_1 tan\theta = v_2(tan\phi + tan\theta)$	1 mark: proves the expression for T 1 mark: substitutes into $\tan(-\phi) = \frac{-gT}{v_2 cos\theta} + tan\theta$ result and proves the result.	*Generally, well done. students must use the result* $T = \frac{2v_1 sin\theta}{g}$ *~~to~~ prove this result.*