KINCOPPAL-ROSE BAY
Cor Unum

KINCOPPAL-ROSE BAY
SCHOOL OF THE SACRED HEART

TRIAL EXAMINATION

YEAR 12

2000
MATHEMATICS

3 UNIT (ADDITIONAL)
AND
3/4 UNIT (COMMON)

Time allowed – Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on the last page
- Board approved calculators may be used
- Answer each question in a SEPARATE Writing Booklet
- You may ask for extra Writing Booklets if you need them.

QUESTION 1 **(Start a new Booklet)** **Marks**

(a) Solve $\dfrac{x^2 - 4}{x} \geq 3$ 3

(b) (i) Sketch the graph of $y = |2 - 3x|$

(ii) Hence, or otherwise, solve $|2 - 3x| < x$ 3

(c) Find the acute angle between the lines 2
$3x - 2y - 5 = 0$ and $x - 5y - 3 = 0$.

(d) On the number plane, A and B are points with co-ordinates $(5, 3)$ and $(1, -3)$ respectively. P is the point on AB which divides the interval AB externally in the ratio $3:2$.

(i) Show that the co-ordinates of P are $(-7, -15)$.

(ii) C is the point such that the line through P which is parallel to BC meets the line AC at Q $(8, -12)$. Find the co-ordinates of C. 4

QUESTION 2 **(Start a new Booklet)** **Marks**

(a) Evaluate the following: **8**

(i) $\displaystyle\int_0^2 \frac{dx}{1+(x-1)^2}$

(ii) $\displaystyle\int \frac{e^x\,dx}{\sqrt{1-e^{2x}}}$

(iii) $\displaystyle\int_0^{\pi/2} \frac{\cos x\,dx}{1+\sin^2 x}$ using $u = \sin x$

(b) Show that $\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

Hence or otherwise, evaluate $\displaystyle\int_0^{\pi/6} 2\cos 3x\cos x\,dx$ **4**

QUESTION 3 **(Start a new Booklet)** **Marks**

(a) (i) Assuming $\cos x \neq 0$, make $\tan x$ the subject of $\sin (x + \theta) = a \cos x$.

(ii) Use the result from (i) to find the exact value of $\tan x$ when $\sin (x + \frac{\pi}{3}) = 2 \cos x$ and the value(s) of x, $0 \leq x \leq 2\pi$ correct to 4 decimal places. 4

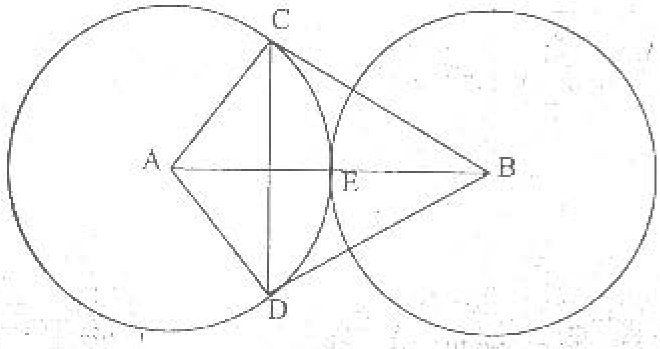
(b) Differentiate $\ln\left\{\dfrac{1 - \sin x}{1 + \sin x}\right\}$ with respect to x.
Express your answer in simplest form. 3

(c) Show that $\displaystyle\lim_{h \to 0} \frac{1 - \cos 2h}{h^2} = 2$ 2

(d) Prove by Mathematical Induction that $2^{3n} - 1$ is divisible by 7. 3

QUESTION 4 **(Start a new Booklet)** **Marks**

(a) 8



Two circles of equal radius and with centres at A and B respectively touch each other externally at E. BC and BD are tangents from B to the circle with centre A.

(i) Copy the diagram.

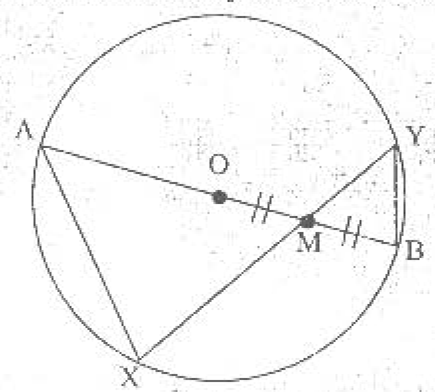
(ii) Show that BCAD is a cyclic quadrilateral.

(iii) Show that E is the centre of the circle which passes through B, C, A and D.

(iv) Show that $\angle CBA = \angle DBA = 30^0$.

(v) Show that triangle BCD is equilateral.

(b) In the diagram below, AB is a diameter of a circle, whose centre is the point O. The chord XY passes through M, the mid point of OB. AX and BY are joined.



(i) Prove the two triangles formed (triangles AXM and MYB) are similar. If XM = 8 cm and YM = 6 cm, find the length of the radius of the circle. 4

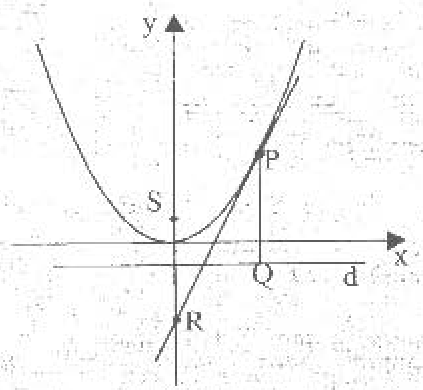
QUESTION 5 **(Start a new Booklet)** **Marks**

(a) (i) Show that the normal to the parabola $x^2 = 4ay$ at the point $T(2at, at^2)$ has equation

$$x + ty = 2at + at^3.$$

(ii) Hence show that there is only one normal to the parabola which passes through its focus. 4

(b)



$P(2at, at^2)$ is a point on the parabola $x^2 = 4ay$. S is the focus of the parabola.
P Q is the perpendicular from P to the directrix d of the parabola.
The tangent at P to the parabola cuts the axis of the parabola at the point R.

(i) Show that the tangent at P to the parabola has equation

$$tx - y - at^2 = 0.$$

(ii) Show that PR and QS bisect each other.

(iii) Show that PR and QS are perpendicular to each other.
State with reason what type of quadrilateral PQRS is. 8

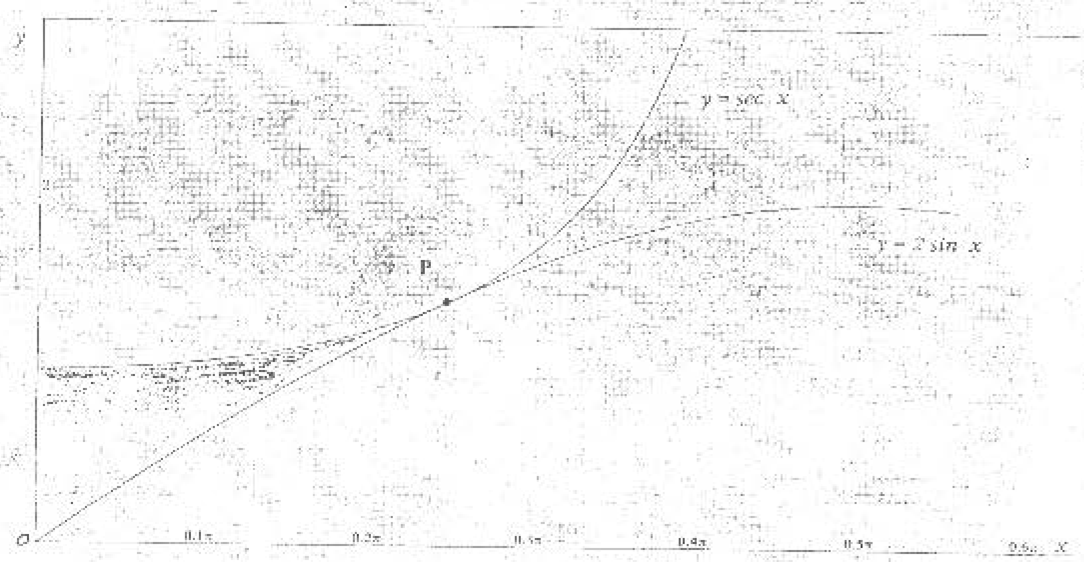
QUESTION 6 **(Start a new Booklet)** **Marks**

(a) You are given that 0.8 is an approximate root of the equation $e^{-x} - 0.5x = 0$. Using one application of Newton's Method, find a better approximation, correct to 3 decimal places. 3

(b) (i) Differentiate $\cos^4 x$. 3

(ii) Hence, find the exact value of $\int_0^{\frac{\pi}{4}} \sin x \cos^3 x \, dx$.

(c)



P is the point of intersection of the graphs $y = \sec x$ and $y = 2 \sin x$ 6

in the domain $0 < x < \dfrac{\pi}{2}$.

(i) Verify that P is the point $\left(\dfrac{\pi}{4}, \sqrt{2}\right)$.

(ii) The shaded region is rotated about the x - *axis*. Find the volume of the solid of revolution formed.

QUESTION 7 **(Start a new Booklet)** **Marks**

(a) If α, β, γ are the roots of $5x^3 - 2x^2 - 4x + 7 = 0$, find the value of $\alpha^2 + \beta^2 + \gamma^2$. 2

(b) (i) Express $\sqrt{3} \cos x - \sin x$ in the form $R \cos (x + \alpha)$, $R > 0$. 4

(ii) Hence, find the general solutions for $\sqrt{3} \cos x - \sin x = 1$.

(c) (i) Sketch the curve $y = \cos^{-1} 2x$. 6

(ii) State the domain and range of the function.

(iii) Find the equation of the tangent to the curve $y = \cos^{-1} 2x$ at the point where the curve cuts the y - *axis*.

The End

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Question 1

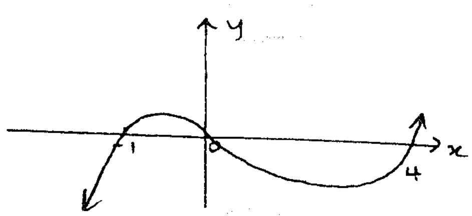
(a) $\dfrac{x^2-4}{x} \geq 3$ $\quad \times\, x^2$

$x(x^2-4) \geq 3x^2$

$x(x^2-3x-4) \geq 0$

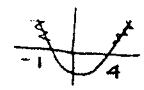
$x(x-4)(x+1) \geq 0$ (1)

(3)



(1)

$x \neq 0$

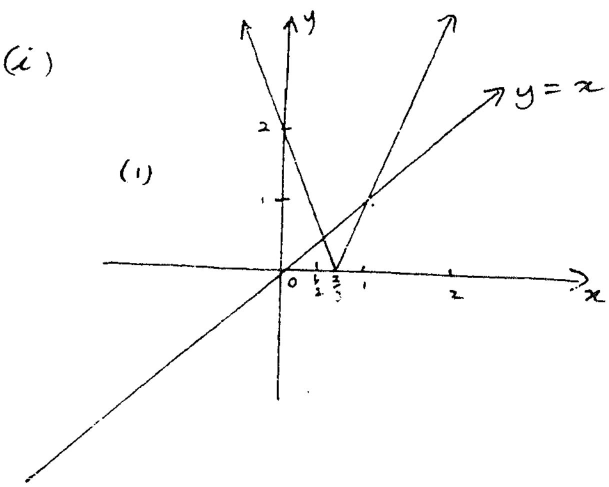
$x > 0$	$x < 0$
$x^2-4 \geq 3x$	$x^2-4 \leq 3x$
$(x^2-3x-4) \geq 0$	
$(x-4)(x+1) \geq 0$	
	$-1 \leq x \leq 4$
$x \leq -1$ or $x \geq 4$	
$\therefore x \geq 4$	$-1 \leq x < 0$

Solution:

(1) $x \geq 4$, $-1 \leq x < \cancel{4}\, 0$

3

(b) (i)



(1)

$2-3x = 0$

$2 = 3x$

$x = \frac{2}{3}$

(ii) $|2-3x| < x$

(2)

from graph

$\frac{1}{2} < x < 1$

2

Critical pts:

$2-3x = x$

$2 = 4x$

$\frac{2}{4} = x$

$\therefore x = \frac{1}{2}$

$-(2-3x) = x$

$-2+3x = x$

$-2 = -2x$

$x = 1$

(c) $3x - 2y - 5 = 0$

$3x - 5 = 2y$

$y = \frac{3}{2}x - \frac{5}{2}$

$\therefore m_1 = \frac{3}{2}$

$x - 5y - 3 = 0$

$x - 3 = 5y$

$y = \frac{1}{5}x - \frac{3}{5}$

$m_2 = \frac{1}{5}$

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

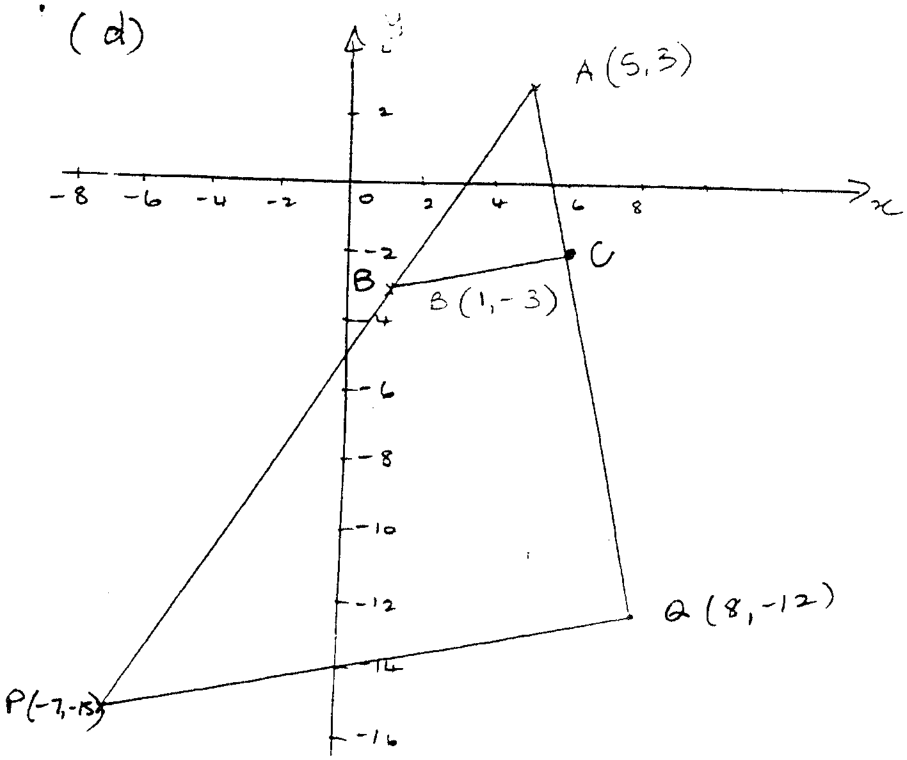
$$= \left| \frac{\frac{3}{2} - \frac{1}{5}}{1 + \frac{3}{2} \cdot \frac{1}{5}} \right|$$

$$= 1$$

$\therefore \underline{\theta = 45^\circ}$

2 marks

(d)



i) $\begin{matrix} -3 \\ 2 \end{matrix} \times \begin{matrix} 5, & 3 \\ 1, & -3 \end{matrix}$

$$\left(\frac{-3 \times 1 + 2 \times 5}{-3+2} , \frac{-3 \times -3 + 2 \times 3}{-3+2} \right)$$

$$= \left(\frac{-3+10}{-1} , \frac{9+6}{-1} \right)$$

$$= (-7, -15)$$

ii) Since P divides AB externally in ratio of 3 : 2, then B divides AP internally in ratio 1 : 2.

Since PQ || BC, C divides AQ internally in ratio 1 : 2. A (5, 3), Q (8, −12)

$\begin{matrix} 1 \\ 2 \end{matrix} \times \begin{matrix} 5 & 3 \\ 8 & -12 \end{matrix}$ $\left(\frac{1 \times 8 + 2 \times 5}{3} , \frac{1 \times -12 + 2 \times 3}{3} \right)$

$= \left(\frac{18}{3} \right.$ [illegible]

QUESTION 2

(a) (i) $\displaystyle\int_0^2 \frac{dx}{1+(x-1)^2} = \left[\tan^{-1}(x-1)\right]_0^2$

$= \tan^{-1} 1 - \tan^{-1}(-1)$

$= -\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)$

$= \frac{\pi}{2}$

(ii) $\displaystyle\int \frac{e^x\,dx}{\sqrt{1-e^{2x}}} = \sin^{-1}(e^x) + C$

(iii) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

using

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,d$

when $x = \frac{\pi}{2}$, $u =$

$x = 0$, $u =$

$= \displaystyle\int_0^1 \frac{du}{1+u^2}$

$= \left[\tan^{-1} u\right]_0^1$

$= \tan^{-1} 1 - \tan^{-1} 0$

$= \frac{\pi}{4} - 0$

$= \frac{\pi}{4}$

2 (b) $LHS = \cos A \cos B - \sin A \sin B + \cos A \cos B + \sin A \sin B$

$= 2 \cos A \cos B$

$$\therefore \int_0^{\pi/6} 2 \cos 3x \cos x \, dx$$

$$= \int_0^{\pi/6} \cos 4x + \cos 2x \, dx$$

$$= \left[\frac{1}{4} \sin 4x + \frac{1}{2} \sin 2x \right]_0^{\pi/6}$$

$$= \frac{3\sqrt{3}}{8}$$

(a)(i) $\sin(x+\theta) = a\cos x$

$$\sin x\cos\theta + \sin\theta\cos x = a\cos x$$

$\div \cos x$, $\cos x \neq 0$

$$\tan x\cos\theta + \sin\theta = a$$

$$\tan x = \frac{a-\sin\theta}{\cos\theta}$$ (2)

(ii)

$$\tan x = \frac{2-\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}$$

$$= \frac{2-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$ (1)

$$= 4-\sqrt{3}$$

Hence $x = \tan^{-1}(4-\sqrt{3})$, $0 \leq x \leq 2\pi$

$= 1.1555$, (1) 4.2971 (2)

(b) $y = \ln\left(\frac{1-\sin x}{1+\sin x}\right)$ $= \ln(1-\sin x) - \ln(1+\sin$

$= \frac{-\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x}$

let $f(x) = \frac{1-\sin x}{1+\sin x}$

$= \frac{-\cos x - \sin x\cos x - \cos x + \cos}{(1-\sin x)(1+\sin x)}$

$$\therefore f'(x) = \frac{(1+\sin x)(-\cos x) - (1-\sin x)(\cos x)}{(1+\sin x)^2}$$

$= \frac{\ \ \cos x}{\ } = -2\sec$

$$f'(x) = \frac{-\cos x \quad -\cos x}{(1+\sin x)^2}$$

$$= \frac{-2\cos x}{(1+\sin x)^2}$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)} = \frac{-2\cos x}{(1+\sin x)^2} \times \frac{1+\sin x}{1-\sin x}$$

$$= \frac{-2\cos x}{1-\sin^2 x}$$

$$= -2\sec x$$

(c) $$\lim_{h\to 0} \frac{1-(1-2\sin^2 h)}{h^2}$$

$$= \lim_{h\to 0} \frac{2\sin^2 h}{h^2}$$

$$= 2 \lim_{h\to 0} \frac{\sin h}{h} \times \frac{\sin h}{h}$$

$$= 2 \times 1 \times 1$$

$$= 2.$$

(d) Step 1: When $n=1$, $2^{3n}-1 = 2^3-1 = 7$

which is divisible by 7.

(1) Thus the statement is true for $n=1$

Step 2: Assume true for $n=k$

ie: $2^{3k}-1 = 7m$ where m is any integer

Step 3: Prove true for $n=k+1$

(1) ie: $2^{3k+3}-1$ is divisible by 7.

Now $2^{3k+3}-1 = 2^{3k}.2^3-1$

$= (7m+1)2^3-1$

Since $2^{3k} = 7m+1$ from Step 2

$= (7m+1)8-1$

$= 56m+8-1$

$= 56m+7$

(1) $= 7(8m+1)$

(2) $= 7p$ which is divisible by 7

where $p = 8m+1$ and $8m+1$ is an integer.

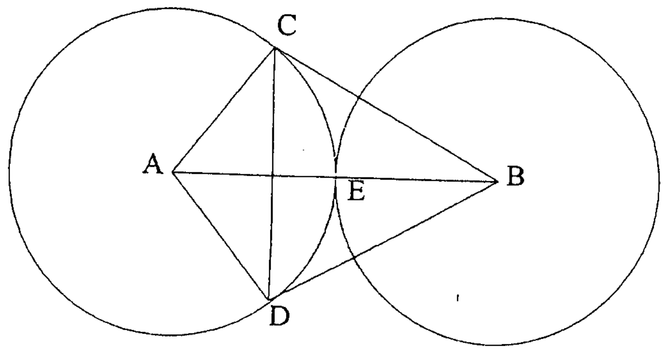
Thus $2^{3(k+1)}-1$ is divisible by 7.

Thus if it is true for $n=k$, it is true for $n=k+1$. It is true for $n=1$, then it is true for $n=2$, and so it is true for all positive n.

Solutions and Marking Scheme

Question 4

(a) i.



(1 mark)

ii. ∠BCA = ∠BDA = 90□ (BC and BD are tangents to circle with centre A)
∴∠BCA + ∠BDA = 180□

(2) ∠BCA + ∠BDA + ∠CAD + ∠CBD = 360□ (Angle sum of a quadrilateral)
∴ 180□ + ∠CAD + ∠CBD = 360□
∴ ∠CAD + ∠CBD = 360□ - 180□
= 180□
∴ BCAD is a cyclical quadrilateral (Opposite angles add to 180□) (2 marks)

(iii) ∠BCA = ∠BDA = 90□ (Above)
∴AB is the diameter of the circle which passes through B, C, A and D
(angle in a semicircle is 90□)
Now AE = BE (circle A and circle B have the equal radii, given)
∴ E is the MP of AB (the diameter)
∴ E is the centre of the circle.
(2 marks)

(iv) AE = BE (above)
AC = AE (equal radii)
AB = 2AC
∠BCA = 90□ (above)

In Δ ABC,
sin (∠CBA) = Opp/ Adj
= AC / AB
= AC / 2AC
= ½
∴∠CBA = 30□

Similarly in Δ ABD,
∠DBA = 30□.

(2 marks)

or AC = AE (radii)
AE = EC (radii circle centre E)
∴ Δ ACE is equilateral
∴ ∠CAE = 60°
now ∠ACB = 90° from before
∴ ∠CBA = 30° (angle sum Δ)
∠CBA = ∠ABD since AB is line of symmetry

v. ∠CBD = ∠CBA + ∠DBA
= 30□ + 30□
= 60□
∠BCD + ∠CDB + ∠CBD = 180□ (angle sum of a triangle)
∠BCD + ∠CDB + 60□ = 180□
∴∠BCD + ∠CDB = 120□
But ∠BCD = ∠CDB (base angles of isosceles triangle, BC = BD, tangents to a circle from an external point are equal in length)
∴∠BCD = ∠CDB = 60□
∴Δ CBD is equilateral (all angles are equal) (2 marks)

(b) i. ∠BAX = ∠XYB (Angles subtended by common arc XB)
∠AXY = ∠YBA (Angles subtended by common arc AY)
∠AMX = ∠BMY (Vertically opposite angles)
∴Δ AXM ||| ∴Δ MYB (AAA) (2 marks)

We know that
AB = 2OB (diameter = 2 x radius)
OB = 2BM (M is MP of OB)
∴AB = 4BM and AM = 3BM
Corresponding sides of similar triangles are in a common ratio.
∴XM/BM =AM/YM
ie 8/BM = AM/6
or 6 x 8 = BM x AM
or 48 = BM x (3BM)
∴ 48/3 = BM x BM
BM = 4 cm and the radius = 2BM = 8 cm (2 marks)

Δ AXM ||| Δ MYB

$\cancel{\frac{XM}{YB}} = \cancel{\frac{AX}{MY}}$ $\frac{AM}{YM} = \frac{XM}{MB}$

$\frac{8}{}$ $\frac{3x}{6} = \frac{8}{x}$

$3x^2 = 48$

$x^2 = 16$

$x = 4$

radius = 8 cm

Question 5

(a) i. $x^2 = 4ay$

$\therefore y = x^2/4a$

$dy/dx = 2x/4a = x/2a$

$\therefore m = x/2a = t$ (when $x = 2at$)

$\therefore$ m of normal $= -1/t$

$(x1,y1) = (2at, at^2)$

now $y - y1 =$ (m of normal) $(x - x1)$

or $y - at^2 = -1/t\,(x - 2at)$

$ty - at^3 = -x + 2at$

$\therefore$ $x + ty = 2at + at^3$. (2 marks)

For the normal to pass through the focus, (0,a) must satisfy the equation of the normal.
ie sub (0,a) into $x + ty = 2at + at^3$

$0 + ta = 2at + at^3$

$0 = at + at^3$

$at + at^3 = 0$

$at(1 + t^2) = 0$

there is only one real solution: $t = o$.

$\therefore$ there is only one normal to the parabola which passes through its focus. (2 marks)

(b) i. At $P(2at, at^2)$, $y = x^2/4a$, $\therefore dy/dx = 2x/4a = x/2a$ and $m = 2at/2a = t$

Equation of tangent at P:

$y - y_P = m(x - x_P)$

$y - at^2 = t(x - 2at)$

$y - at^2 = tx - 2at^2$

$\therefore tx - y - at^2 = 0$ (1) (2 marks) ii **P(2at, at²)**

Directrix has equation $y = -a$ and thus **Q(2at, -a)**
R intersects the y-axis and so $x = 0$ at R and substituting this value into (1) yields:

$$-y - at^2 = 0$$
$$\therefore y = -at^2 \text{ and } \mathbf{R(0, -at^2)}$$

S is the focus ie **S(0,a)**.

Gradient of PR = m_{PR} = t (from above)
Equation of PR: $tx - y - at^2 = 0$
$\therefore y = tx - at^2$ (2)

Gradient of QS = $m_{QS} = (y_Q - y_S)/(x_Q - x_S)$
$= (-a - a)/(2at-0)$
$= -2a / 2at$
$= -1/t$

Equation of QS: $y - y_S = m_{QS}(x - x_S)$
$y - a = -1/t(x - 0)$
$\therefore y = -x/t + a$ (3)

Let Intersection of QS and PR be $M(x_M, y_M)$

For M, set (2) = (3)

$$tx - at^2 = -x/t + a$$
$$t^2x - at^3 = -x + at$$
$$t^2x + x = at^3 + at$$
$$x(t^2 + 1) = at(t^2 + 1)$$
$$\therefore x_M = at \quad (4)$$

Sub (4) into (3): $\therefore y_M = -at/t + a$
$= -a + a$
$= 0$

$\therefore M(x_M, y_M) = \mathbf{M(at, 0)}$

now the midpoint of QS $= MP_{QS}(\frac{1}{2}(x_Q + x_S), \frac{1}{2}(y_Q + y_S))$
$= MP_{QS}(\frac{1}{2}(2at + 0), \frac{1}{2}(-a + a))$
$= MP_{QS}(at, 0)$
$= M(at, 0)$

Similarly the midpoint of PR $= MP_{PR}(\frac{1}{2}(x_Q + x_S), \frac{1}{2}(y_Q + y_S))$
$= MP_{PR}(\frac{1}{2}(2at + 0), \frac{1}{2}(at^2 - at^2))$
$= MP_{PR}(at, 0)$
$= M(at, 0)$

PQ and RS bisect each other at M since M is the midpoint of both PQ and RS.
(4 marks)

iii Gradient of PR = m_{PR} = t (from above)

Gradient of QS = m_{QS} = -1/t (from above)

So $m_{QS} = -1/t$
$= -1/m_{PR}$

$\therefore$ PR is perpendicular to QS. (1 mark)

iv. PQRS is a rhombus since the diagonals QS and PR bisect each other at right angles.
(1 mark)

Question 6 solutions

(a) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ Now $y = e^{-x} - 0.5x$

$y' = -e^{-x} - 0.5$

$$x_2 = 0.8 - \frac{(e^{-0.8} - 0.5(0.8))}{(-e^{-0.8} - 0.5)}$$

$$= 0.8 - \frac{(e^{-0.8} - 0.4)}{(-e^{-0.8} - 0.5)}$$

$\dfrac{0.04932\ldots}{-0.9493\ldots}$

$$= 0.8 - (-0.0519619\ldots)$$

$$= 0.8519619\ldots$$

$\therefore x_2 = 0.852 \quad (3\ \text{D.P})$

Question 6 Solutions

(a) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

Now $y = e^{-x} - 0.5x$

$y' = -e^{-x} - 0.5$

$$x_2 = 0.8 - \frac{(e^{-0.8} - 0.5(0.8))}{(-e^{-0.8} - 0.5)}$$

$$= 0.8 - \frac{(e^{-0.8} - 0.4)}{(-e^{-0.8} - 0.5)}$$

$$\frac{0.04932\ldots}{-0.9493\ldots}$$

$$= 0.8 - (-0.0519619\ldots)$$

$$= 0.8519619\ldots$$

$\therefore x_2 = 0.852$ (3 D.P)

(c) (i) When $x = \frac{\pi}{4}$, $y = \sec \frac{\pi}{4}$

$= \sqrt{2}$

When $x = \frac{\pi}{4}$, $y = 2 \sin \frac{\pi}{4}$

$= 2 \times \frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$

$\therefore$ P $\left(\frac{\pi}{4}, \sqrt{2}\right)$ is a pt of int.

(ii) $V = \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx - \pi \int_0^{\frac{\pi}{4}} (2 \sin x)^2 \, dx$

$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x - 4\pi \sin^2 x \, dx$

$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx - 4\pi \left(\frac{1}{2} \int_0^{\frac{\pi}{4}} 1 - \cos 2x \, dx \right)$

$= \pi \Big[\tan x \Big]_0^{\frac{\pi}{4}} - 2\pi \left[\left(x - \frac{1}{2} \sin 2x \right) \right]_0^{\frac{\pi}{4}}$

$= \pi \left[\tan \frac{\pi}{4} - \tan 0 \right] - 2\pi \left[\frac{\pi}{4} - \frac{1}{2} \sin \frac{\pi}{2} \right]$

$= \pi [1] - 2\pi \left[\frac{\pi}{4} - \frac{1}{2} \right]$

$= \pi - \frac{2\pi^2}{4} + \pi$

$= 2\pi - \frac{\pi^2}{2}$ (cubic units)

Question 7

(a) $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{2}{5}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = -\frac{4}{5}$

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$$
$$= \left(\frac{2}{5}\right)^2 - 2\left(-\frac{4}{5}\right)$$
$$= \frac{44}{25}$$
$$= 1\frac{19}{25}.$$

(b)(i) $\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$

$$= R(\cos x\cos\alpha - \sin x\sin\alpha)$$

Equating coeff's $\sqrt{3} = R\cos\alpha$

$1 = R$

$\therefore \tan\alpha = \frac{1}{\sqrt{3}}$ & $R = 2$

$\therefore \alpha = \frac{\pi}{6}$ $\therefore \sqrt{3}\cos x - \sin x = 2\cos\left(x + \frac{\pi}{6}\right)$

(ii) $2\cos\left(x + \frac{\pi}{6}\right) = 1$

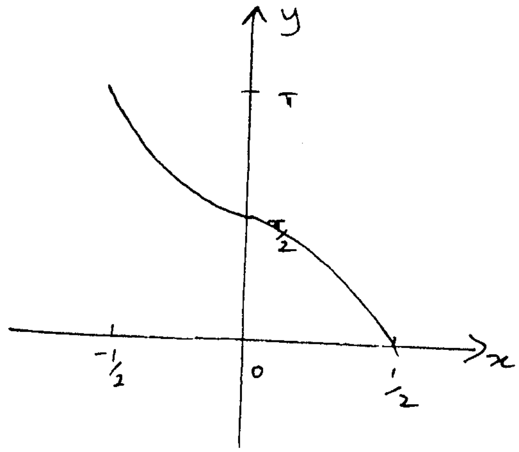
$\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$

$x + \frac{\pi}{6} = 2\pi n \pm \frac{\pi}{3}$

$x = 2\pi n + \frac{\pi}{6}$ 7

Q7 (c)

(i)



$y = \cos^{-1} x$

(ii) Domain: $-1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$

Range: $0 \leq y \leq \pi$

(iii) $y = \cos^{-1} 2x$

$y' = \frac{-1}{\sqrt{1-(2x)^2}} \cdot 2$

$= \frac{-2}{\sqrt{1-4x^2}}$

Cuts y axis when $x = 0$

$\therefore$ At $x = 0$, $y = \frac{\pi}{2}$, $y' = -\frac{2}{1} = -2$

$\therefore$ Equation of tangent is

$y - \frac{\pi}{2} = -2(x - 0)$

$y - \frac{\pi}{2} = -2x$

or $y = \frac{\pi}{2} - $

(a) (i) $y = \cos^4 x$

let $u = \cos x$ $\therefore y = u^4$

$\frac{du}{dx} = -\sin x$ $\qquad \frac{dy}{du} = 4u^3$

$$\frac{dy}{dx} = 4\cos^3 x \cdot -\sin x$$

$$= -4\sin x\cos^3 x$$

(ii)

$$\int_0^{\frac{\pi}{4}} \sin x \cos^3 x \, dx$$

$$= -\frac{1}{4}\int_0^{\frac{\pi}{4}} -4\sin x\cos^3 x \, dx$$

$$= -\frac{1}{4}\left[\cos^4 x\right]_0^{\frac{\pi}{4}}$$

$$= -\frac{1}{4}\left[\left(\frac{1}{\sqrt{2}}\right)^4 - 1\right]$$

$$= -\frac{1}{4}\left[\frac{1}{4} - 1\right]$$

$$= -\frac{1}{4} \times -\frac{3}{4}$$

$$= \frac{3}{16}$$