2008
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start a new booklet for each question

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value

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Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve the inequality $\dfrac{4-2x}{x+5} \leq 2$ **3**

(b) Find $\lim\limits_{x \to 0} \dfrac{\sin\left(\dfrac{\pi}{3}x\right)}{2x}$ **2**

(c) Given that $0 < \alpha < \dfrac{\pi}{2}$, show that $S_\infty = 2\cot\alpha$ for the geometric series **2**

$$\sin 2\alpha + \sin 2\alpha \cos^2\alpha + \sin 2\alpha \cos^4\alpha + \sin 2\alpha \cos^6\alpha + \ldots$$

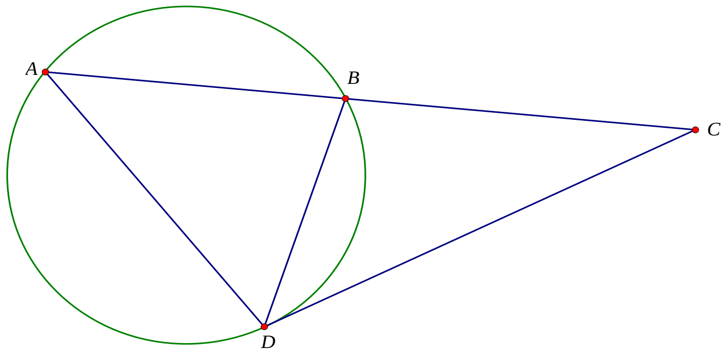
(d) A function is given by the rule $y = \dfrac{x+1}{x+2}$. Find the rule for the inverse function. **2**

(e) Find $\displaystyle\int \frac{-2}{\sqrt{9-4x^2}}\,dx$ **3**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

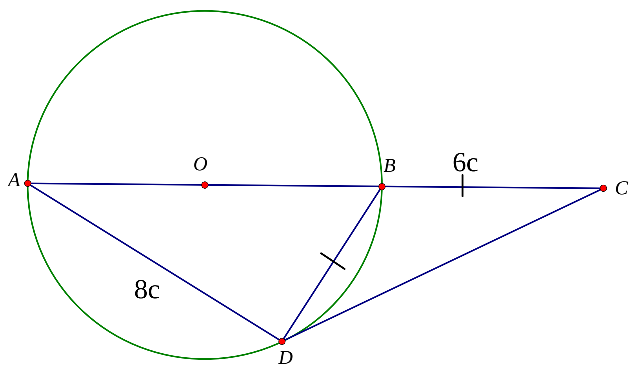
(a) *CD* is a tangent to the circle and *AC* is a secant to the circle intersecting the circle at *B*.



(i) Prove that $\Delta CDB \,|||\, \Delta CAD$. **2**

(ii) Hence show that $(CD)^2 = CA \times CB$ **1**

(b)



AC is a secant to the circle, centre *O*, intersecting the circle at *B*.
CD is a tangent to the circle.
BC is 6cm and *AD* is 8cm.

Find the exact length of *DC*, giving reasons. **2**

Question 2 continued on page 5

Question 2 (continued) **Marks**

(c) On one expressway driving to the Blue Mountains there are five toll gates.
Three of these gates are automatically operated and the other two are manually operated.
Drivers with an official “etag” are able to use any of the gates but those without must use one of the manually operated gates.

Si Ting, Yuki and Ikuko drive through the toll gates everyday with their “etag”.

(i) One day Si Ting has left her “etag” at home. **2**
Find the number of ways in which the three drivers can go through the toll gates so that they all use different gates.

(ii) On another day all drivers have their “etags”. **2**
Find the number of ways in which the three drivers can go through the toll gates so that exactly one goes through a manually operated gate.

(d) Find the constant term in the expansion of $\left(x - \frac{1}{2x^3}\right)^{20}$ **3**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$. The focus S is the point $(0, a)$.
The tangent at P meets the y axis at Q.

(i) Given that the equation of the tangent at P is $y = px - ap^2$, find the coordinates of Q **1**

(ii) Prove that $SP = SQ$ **2**

(b) (i) Express $\cos\theta - \sqrt{3}\sin\theta$ in the form $R\cos(\theta + \alpha)$ where α is in radians. **2**

(ii) Hence, or otherwise, graph $\cos\theta - \sqrt{3}\sin\theta$ for $0 \le \theta \le 2\pi$, indicating all **intercepts**. **3**

(c) Use the process of Mathematical Induction to prove that for all positive integers n, where $n \ge 1$: **4**

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + ... + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The function $f(x) = x^3 - \ln(x+1)$ has one root between 0.5 and 1.

(i) Show that the root lies between 0.8 and 0.9. **1**

(ii) Hence, use the Halving of the Interval method once, to find the root correct to one decimal place. You must justify your choice. **2**

(b) Let $f(x) = \dfrac{x}{x^2 - 1}$

(i) For what values of x is $f(x)$ undefined. **1**

(ii) Show that $y = f(x)$ is odd. **1**

(iii) Show that $y = f(x)$ is decreasing for all values of x for which the function is defined. **2**

(iv) Hence sketch $y = f(x)$. **2**

(c) (i) Write down the expression for $\tan 2A$ in terms of $\tan A$. **1**

(ii) Given $f(a) = a\cot a$ (i.e. $f(b) = b\cot b$ etc), show that $f(2a) = (1 - \tan^2 a)\, f(a)$ **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate $\sin^{-1}(e^{-x})$ **2**

(b) Two of the roots $x^3 + ax^2 + b = 0$ are reciprocals of each other, a, b are real numbers.

(i) Show that the third root is equal to $-b$. **1**

(ii) Show that $a = b - \dfrac{1}{b}$. **3**

(c) The two curves $y = e^x - 1$ and $y = 2e^{-x}$ intersect at the point P.

(i) Solve the equations simultaneously in order to show that the coordinates of the point P are $(\ln 2, 1)$ **3**

(ii) Find the acute angle between the tangents to the curves $y = e^x - 1$ and $y = 2e^{-x}$ at the point P. Give your answer correct to the nearest degree. **3**

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use the substitution $x = u^2 \quad (u > 0)$ to show **4**

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{x}\sqrt{1-x}}\, dx = \frac{\pi}{6}$$

(b) From the top of a cliff an observer spots two ships out at sea. One is on a true bearing of $042°$ with an angle of depression of 6° while the other is on a true bearing of $312°$ with an angle of depression of 4°. **3**
If the two ships are 200 metres apart, find the height of the cliff, to the nearest metre.

(c) Consider the expansion $(7+3x)^{25}$

(i) Show that $\dfrac{t_{k+1}}{t_k} = \dfrac{3(25-k)}{7(k+1)}$, **3**

where t_k is defined to be the coefficient of the term in x^k, $0 \le k \le 25$.

(ii) Hence, or otherwise, find the greatest coefficient t_k. **2**

You may leave your answer in the form $\dbinom{25}{k} 7^c 3^d$.

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the function $f(x) = \cos^{-1}\sqrt{x}$

(i) Explain why the domain of $f(x)$ is $0 \leq x \leq 1$. **1**

(ii) Find the range of the function and sketch the graph of $y = f(x)$ **2**

(iii) Use Simpson's Rule with three function values to find an approximation to the area bounded by the curve $y = f(x)$ and the coordinate axes. **2**

(iv) Use integration to find the exact area bounded by the curve $y = f(x)$ and the coordinate axes. **4**

(b) (i) Write down the expansion of $x(1+x)^n$. **1**

(ii) By first differentiating with respect to *x*, then substituting $x = 1$, show that ${}^nC_0 + 2\,{}^nC_1 + 3\,{}^nC_2 + ... + (n+1)\,{}^nC_n = (n+2)2^{n-1}$ **2**

End of Question 7

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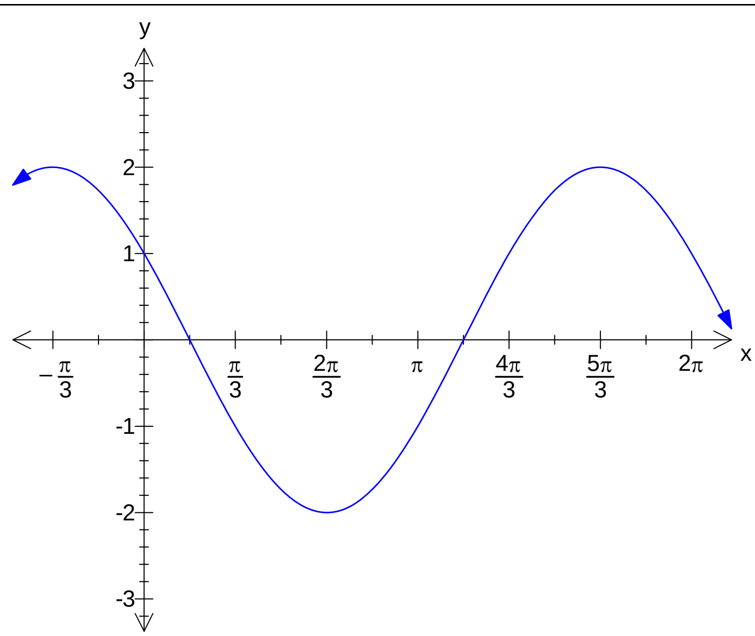
Mathematics Extension 1 HSC Trial Exam 2008 Solutions

Question	Criteria	Marks	Bands
1(a)	$\frac{4-2x}{x+5} \le 2 \quad where\ x \ne -5$ $4-2x \le 2x+10$ $-4x \le 6$ $x \ge -1.5$ $By\, checking:\ \ x < -5\ and\ \ x \ge -1.5$	1 1 1	
1(b)	$\lim_{x\to 0}\frac{\sin\left(\frac{\pi}{3}x\right)}{2x} = \lim_{x\to 0}\frac{\sin\left(\frac{\pi}{3}x\right)}{\left(\frac{\pi}{3}x\right)} \times \frac{\left(\frac{\pi}{3}x\right)}{2x}$ $= 1 \times \frac{\left(\frac{\pi}{3}x\right)}{2x}$ $= \frac{\pi}{6}$	1 1	
1(c)	$a = \sin 2\alpha, \quad r = \cos^2\alpha$ $S_\infty = \frac{\sin 2\alpha}{1-\cos^2\alpha}$ $= \frac{2\sin\alpha\cos\alpha}{\sin^2\alpha} \quad substitution\, and\ cancelling$ $= \frac{2\cos\alpha}{\sin\alpha}$ $= 2\cot\alpha$	1 1	
1(d)	$f: y = \frac{x+1}{x+2}$ $f^{-1}: x = \frac{y+1}{y+2}$ $xy + 2x = y + 1$ $xy - y = 1 = 2x$ $y(x-1) = 1-2x$ $y = \frac{1-2x}{x-1}$	1 1	
1(e)	$(e) \quad \int \frac{-2}{\sqrt{9-4x^2}}\,dx$ $method\ 1:$ $\quad or \quad$ $method\ 2:$ $-2\int \frac{1}{\sqrt{4(\frac{9}{4}-x^2)}}\,dx \qquad 2\int \frac{-1}{\sqrt{4(\frac{9}{4}-x^2)}}\,dx$ $-2\int \frac{1}{2\sqrt{(\frac{9}{4}-x^2)}}\,dx \qquad 2\int \frac{-1}{2\sqrt{(\frac{9}{4}-x^2)}}\,dx$ $-\int \frac{1}{\sqrt{(\frac{9}{4}-x^2)}}\,dx \qquad \int \frac{-1}{\sqrt{(\frac{9}{4}-x^2)}}\,dx$ $= -\sin^{-1}\left(\frac{x}{\frac{3}{2}}\right) \qquad = \cos^{-1}\left(\frac{x}{\frac{3}{2}}\right)$ $= -\sin^{-1}\frac{2x}{3} + C \qquad = \cos^{-1}\frac{2x}{3} + C$	1 1 1	

Question	Criteria	Marks	Bands
2(a)(i)	$\angle CDB = \angle DAB$ angle between the tangent and the chord equal to the angle in the alternate segment $\angle ACD = \angle BCD$ common $\therefore \Delta CDB \mid\mid\mid \Delta CAD$	**1** **1**	
2(a)(ii)	$\frac{CD}{CA} = \frac{CB}{CD}$ matching sides of simialr triangles $\therefore CD^2 = CB.CA$	**1**	
2(b)	$\angle ADB = 90^\circ$ angle in a semi-circle is a right angle $AB = 10cm$ Pythagoras $CD^2 = CA \times CB$ $CD^2 = 16 \times 6$ $\therefore CD = \sqrt{96}$ or $4\sqrt{6}$ Since ΔBDC is isosceles (2 sides equal) $\angle BDC = \angle BCD$ (base angles equal) Or $\angle BCD = \angle BAD$ (both equal to $\angle BDC$) $\therefore \Delta ADC$ also isosceles $\therefore AD = DC = 8$	**1** **1**	
2(c)(i)	Si Ting 2 choices Yuki 4 choices Ikuko 3 choices $2 \times 4 \times 3 = 24$	**1** **1**	
2(c)(ii)	Counting how many ways that one person goes through manual gate 1: 3 choices for person through manual gate 1 3 choices for auto gate for person 2 3 choices for auto gate for person 3 $3 \times 3 \times 3$ Counting how many ways that one person goes through manual gate 2: 3 choices for person through manual gate 2 3 choices for auto gate for person 2 3 choices for auto gate for person 3 $3 \times 3 \times 3$ Total = 54 ways	**1** **1**	

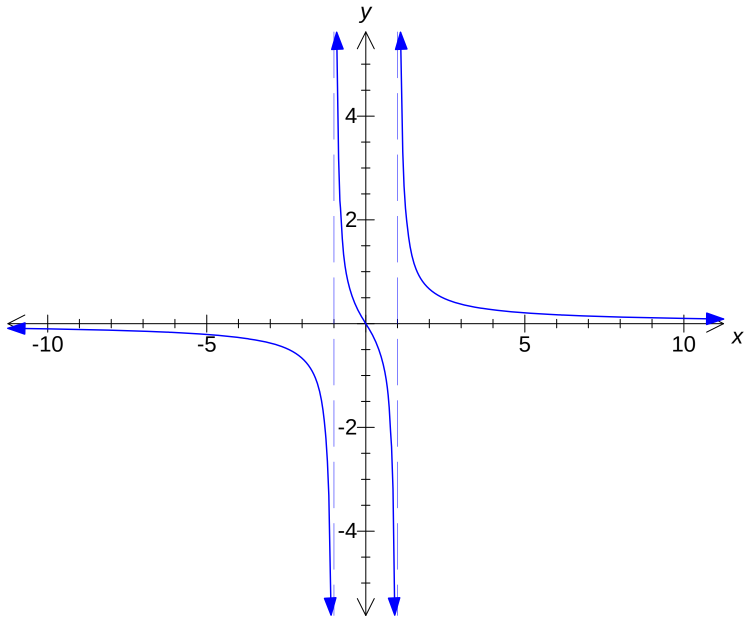
2(d)	$T_{k+1} = {}^{20}C_k x^k \left(\frac{1}{2}\right)^k \left(x^{-3}\right)^{20-k}$ $= {}^{20}C_k x^k \left(\frac{1}{2}\right)^k x^{3k-60}$ $= {}^{20}C_k x^{4k-60} \left(\frac{1}{2}\right)^k$ $4k-60=0$ $\therefore k=15$ ${}^{20}C_5\left(-\frac{1}{2}\right)^5 = \frac{-969}{2} = -485\frac{1}{2}$	**1** **1** **1**	

Question	Criteria	Marks	Bands
3(a)(i)	$Q\left(0,-ap^2\right)$	**1**	
3(a)(ii)	$SP = PM = a + ap^2$ definition of a parabola PM is perp. distance from directrix $SQ = OS + OQ = a + \left\|-ap^2\right\|$ $\therefore SP = SQ$	**1** **1**	
3(b)(i)	$R = 2$ $\alpha = \frac{\pi}{3}$ $2\cos\left(\theta + \frac{\pi}{3}\right)$	**1** **1**	
3(b)(ii)	 1 for y-intercept 1 for x-intercepts 1 for (shape) amp/shape		

3(c)	Test for $n=1$ $LHS=\frac{1}{3}$ $\qquad RHS=\frac{1}{2+1}=\frac{1}{3}$ $\therefore LHS=RHS$ true for $n=1$ Assume true for $n=k$ $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+.....+\frac{1}{(2k-1)(2k+1)}=\frac{k}{2k+1}$ Prove true for $n=k+1$ $i.e.\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+.....+\frac{1}{(2k+1)(2k+3)}=\left[\frac{k+1}{2k+3}\right]$ $LHS=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$ $=\frac{k(2k+3)+1}{(2k+1)(2k+3)}$ $=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$ $=\frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$ $=\frac{(k+1)}{(2k+3)}$ $\therefore true \;\; for \;\; n=k+1$ It has been proved true for $n=k+1$, and from above it was proved true for $n=1$ therefore it is true for $n=1+1$, $i.e. n=2$ and $n=3$ and so on. Therefore it is true for all integers $n\geq 1$	**1** **1** **1** **1**	

Mathematics Extension 1 HSC Task 3 2008 Solutions

Question	Criteria	Marks	Bands
4(a)(i)	$f(x) = x^3 - \ln(x+1)$ $f(0.8) = 0.8^3 - \ln(1.8)$ $= -0.075$ $f(0.9) = 0.9^3 - \ln(1.9)$ $= 0.087$ $\therefore 0.8 < root < 0.9 \quad due\ to\ change\ of\ sign$	1	
4(a)(ii)	$f(0.85) = 0.85^3 - \ln(1.85)$ $= -0.00106...$ $0.85 < root < 0.9 \quad due\ to\ change\ of\ sign$ $root \approx 0.9 \quad (to\ one\ decimal\ place)$	1 1	
4(b)(i)	$f(x) = \dfrac{x}{x^2-1}$ x is undefined for $x = \pm 1$	1	
4(b)(ii)	$f(-x) = \dfrac{-x}{(-x)^2-1}$ $= \dfrac{-x}{x^2-1}$ $= -f(x) \quad \therefore odd$	1	
4(b)(iii)	$f'(x) = \dfrac{(x^2-1)-2x^2}{(x^2-1)^2}$ $= \dfrac{-x^2-1}{(x^2-1)^2}$ $= \dfrac{-(x^2+1)}{(x^2-1)^2}$ ← neg ← pos which is negative for all $x \therefore$ always decreasing	1 1	

<table>
<tr><td>4(b)(iv)</td><td>
Asymptotes and symmetry

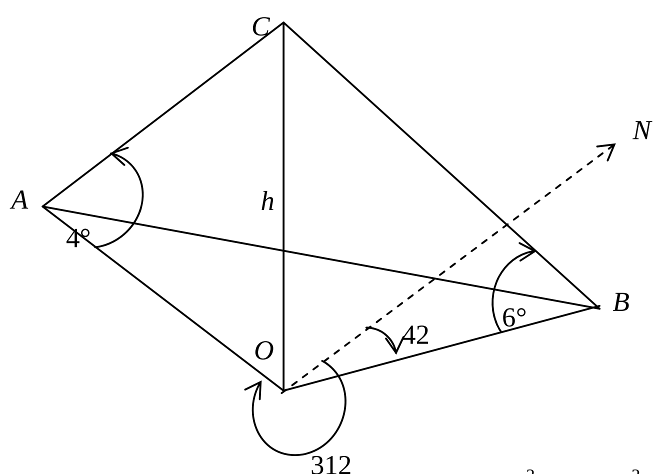
Shape</td><td>
1

1</td><td></td></tr>
<tr><td>4(c)(i)</td><td>$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$</td><td>1</td><td></td></tr>
<tr><td>4(c)(ii)</td><td>$if\ \ f(a) = a\cot a$
$LHS = 2a\cot 2a$
$= 2a \times \dfrac{1-\tan^2 a}{2\tan a}$
$= (1-\tan^2 a) \times \dfrac{a}{\tan a}$
$= (1-\tan^2 a) \times a\cot a$
$= (1-\tan^2 a)\, f(a)$
$= RHS$</td><td>1

1</td><td></td></tr>
</table>

Question	Criteria	Marks	Bands
5(a)	$\frac{d}{dx}(\sin^{-1} e^{-x}) = \frac{1}{\sqrt{1-(e^{-x})^2}} \times -e^{-x}$ $= \frac{-e^{-x}}{\sqrt{1-e^{-2x}}}$	**1** **1**	
5(b)(i)	let the roots of $x^3 + ax^2 + b = 0$ be $\alpha,\ \frac{1}{\alpha},\ \beta$ $\alpha \times \frac{1}{\alpha} \times \beta = \frac{-b}{1}$ $\therefore\ \beta = -b$	**1**	
5(b)(ii)	$\alpha + \frac{1}{\alpha} + \beta = -a$ *also* $\alpha \times \frac{1}{\alpha} + \alpha\beta + \frac{\beta}{\alpha} = 0$ $\alpha + \frac{1}{\alpha} = b - a$ ......A $1 - \alpha\beta - \frac{b}{\alpha} = 0$ $1 - b\left(\alpha + \frac{1}{\alpha}\right) = 0$.....$B$ substitute A into B $1 - b(b-a) = 0$ $1 - b^2 + ab = 0$ $ab = b^2 - 1$ $a = b - \frac{1}{b}$ as required $\boxed{\begin{aligned}(-b)^3 + a(-b)^2 + b &= 0\\ -b^3 + ab^2 + b &= 0\\ ab^2 &= b^3 - b\\ a &= b - \frac{1}{b}\end{aligned}}$	**1** **1** **1**	
5(c)(i)	*let* $y_1 = e^x - 1$ $y_2 = 2e^{-x}$ Point of intersection is when $y_1 = y_2$ *i.e.* $e^x - 1 = 2e^{-x}$ $e^x - 1 = \frac{2}{e^x}$ $e^{2x} - e^x - 2 = 0$ $(e^x + 1)(e^x - 2) = 0$ $e^x = -1$ $e^x = 2$ no solution $x = \ln 2$ $y = 2 - 1 = 1$ $\therefore$point of intersection $(\ln 2, 1)$	**1** **1** **1**	

5(c)(ii)	$y_1' = e^x$ $y_2' = -2e^{-x}$ $m_1 = e^{\ln 2} = 2$ $m_2 = -2e^{-\ln 2} = -1$ $\tan\theta = \dfrac{2 - -1}{1 + 2(-1)} = -3$ $\theta = 72°$ (nearest degree)	**2** **1**	

6(a)	$if \quad x = u^2$ $\frac{dx}{du} = 2u$ $dx = 2udu$ $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx = \int \frac{2udu}{u\sqrt{1-u^2}}$ $= \int \frac{2udu}{u\sqrt{1-u^2}}$ $= 2[\sin^{-1} u]$ $= 2\left[\sin^{-1}\sqrt{x}\right]_{\frac{1}{4}}^{\frac{1}{2}}$ $= 2\left[\sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}\frac{1}{2}\right]$ $= 2\left[\frac{\pi}{4} - \frac{\pi}{6}\right]$ $= \frac{\pi}{6}$ $\boxed{\frac{1}{2} = u^2 \Rightarrow u = \frac{1}{\sqrt{2}} \quad \frac{1}{4} = u^2 \Rightarrow u = \frac{1}{2}}$ $\boxed{2\left[\sin^{-1} u\right]_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} = 2\left[\frac{\pi}{4} - \frac{\pi}{6}\right]}$	1 1 1 1	
6(b)	 $\tan 6° = \frac{h}{OB} \Rightarrow OB = \frac{h}{\tan 6°}$ $\tan 4° = \frac{h}{OA} \Rightarrow OA = \frac{h}{\tan 4°}$ $AB^2 = \left(\frac{h}{\tan 6°}\right)^2 + \left(\frac{h}{\tan 4°}\right)^2$ $200^2 = \frac{h^2}{\tan^2 6°} + \frac{h^2}{\tan^2 4°}$ $h = \sqrt{\frac{200^2 \tan^2 6° \tan^2 4°}{\tan^2 4° + \tan^2 6°}}$ $= 11.6$ $= 12\,\text{metres}$ $h^2\left(\cot^2 6 + \cot^2 4\right) = 200^2$ Or $\quad h^2 = \frac{200^2}{\left(\cot^2 6 + \cot^2 4\right)}$ $h = 1.6\,\text{m}$	1 1 1	

6(c)(i)	$(7+3x)^{25} = \binom{25}{0}7^{25}(3x)^0 + \binom{25}{1}7^{24}(3x)^1 + \binom{25}{3}7^{23}(3x)^2 + \dots + \binom{25}{k}7^{25-k}(3x)^k + \dots + \binom{25}{25}7^0(3x)^{25}$ $T_{k+1} = \binom{25}{k}7^{25-k}(3x)^k$ $= \binom{25}{k}7^{25-k}3^k x^k$ $t_k = \binom{25}{k}7^{25-k}3^k$ $t_{k+1} = \binom{25}{k+1}7^{24-k}3^{k+1}$ $\frac{t_{k+1}}{t_k} = \frac{\binom{25}{k+1}7^{24-k}3^{k+1}}{\binom{25}{k}7^{25-k}3^k}$ $= \frac{3}{7}\left[\frac{\frac{25!}{(k+1)!(25-(k+1))!}}{\frac{25!}{k!(25-k)!}}\right]$ $= \frac{3}{7}\left[\frac{\cancel{25!}}{(k+1)\cancel{k!}(\cancel{24-k})!} \times \frac{\cancel{k!}(25-k)(\cancel{24-k})!}{\cancel{25!}}\right]$ $= \frac{3(25-k)}{7(k+1)}$ ♪ $T_{k+1} = {}^{25}C_k\, 7^{25-k}(3x)^k$ $T_k = {}^{25}C_{k-1}7^{25-(k-1)}(3x)$ . . $\frac{T_{k+1}}{T_k} = \frac{3(26-k)}{7k}$	1 1 1	
6(c)(ii)	Largest coefficient is when $\frac{t_{k+1}}{t_k} > 1$ $75-3k > 7k+7$ $68 > 10k$ $k < 6.8$ $k = 6$ $\therefore t_7 = {}^{25}C_7 7^{18}3^7$	1 1	

<table>
<tr><th>Question</th><th>Criteria</th><th>Marks</th><th>Bands</th></tr>
<tr><td>7(a)(i)</td><td>$-1 \le \sqrt{x} \le 1$
$\therefore x \le 1,$ but smallest x can be is 0
$\therefore 0 \le \text{x} \le 1$</td><td>1</td><td></td></tr>
<tr><td>7(a)(ii)</td><td>$0 \le x \le 1$
$\cos^{-1} 1 = 0$
$\cos^{-1} 0 = \frac{\pi}{2}$
$\therefore 0 \le y \le \frac{\pi}{2}$</td><td>1 (graph)

1</td><td></td></tr>
<tr><td>7(a)(iii)</td><td>

x	0	$\frac{1}{2}$	1
y	$\frac{\pi}{2}$	$\frac{\pi}{4}$	0

$A = \frac{0.5}{3}\left\{\frac{\pi}{2} + 4\frac{\pi}{4} + 0\right\}$
$= \frac{1}{6}\left\{\frac{3\pi}{2}\right\}$
$= \frac{\pi}{4}$</td><td>1

1</td><td></td></tr>
<tr><td>7(a)(iv)</td><td>$y = \cos^{-1}\sqrt{x} \;\Rightarrow \cos y = \sqrt{x}$
$x = \cos^2 y$
$\int_0^{\frac{\pi}{2}} \cos^2 y \; dy$
$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos 2y + 1)\; dy$
$= \frac{1}{2}\left[\frac{1}{2}\sin 2x + x\right]_0^{\frac{\pi}{2}}$
$= \frac{1}{2}\left[\sin \pi + \frac{\pi}{2} - (s\text{in}\, 0 + 0)\right]$
$= \frac{1}{2} \times \frac{\pi}{2}$
$= \frac{\pi}{4}$</td><td>1
1
1

1</td><td></td></tr>
<tr><td>7(b)(i)</td><td>$x(1+x)^n = x\left\{ {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + ... + {}^nC_n x^n \right\}$
$= {}^nC_0 x + {}^nC_1 x^2 + {}^nC_2 x^3 + {}^nC_3 x^4 + ... + {}^nC_n x^{n+1}$</td><td>1</td><td></td></tr>
</table>

7(b)(ii)	$\frac{d}{dx}\left\{ {}^nC_0 x + {}^nC_1 x^2 + {}^nC_2 x^3 + {}^nC_3 x^4 + ... + {}^nC_n x^{n+1} \right\}$ $= {}^nC_0 + 2\,{}^nC_1 x + 3\,{}^nC_2 x^2 + 4\,{}^nC_3 x^3 + ... + (n+1)\,{}^nC_n x^n$ $\frac{d}{dx} x(1+x)^n = (1+x)^n + nx(1+x)^{n-1}$ $\therefore (1+x)^n + nx(1+x)^{n-1} = {}^nC_0 + 2\,{}^nC_1 x + 3\,{}^nC_2 x^2 + 4\,{}^nC_3 x^3 + ..$ $+(n+1)\,{}^nC_n x^n$ let $x = 1$: $2^n + n.2^{n-1} = {}^nC_0 + 2\,{}^nC_1 + 3\,{}^nC_2 + 4\,{}^nC_3 + ... + (n+1)\,{}^nC_n$ $\therefore 2^{n-1}(2+n) = {}^nC_0 + 2\,{}^nC_1 + 3\,{}^nC_2 + 4\,{}^nC_3 + ... + (n+1)\,{}^nC_n$	**1** **1**	