KINCOPPAL–ROSE BAY
SCHOOL OF THE SACRED HEART

2009
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start a new booklet for each question

Total marks – 84

- Attempt Questions 1 – 7
- All questions are of equal value

Examiners : AD, ND and BW
11/8/09

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Total marks – 84
Attempt Questions 1–7
All questions are of equal value
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The polynomial $P(x) = x^3 - 2x + 7$ is divided by $x - 5$. What is the remainder? **1**

(b) Differentiate $y = \tan^{-1}\left(\frac{x}{3}\right)$ with respect to x. **2**

(c) Evaluate $\int_0^{\frac{3}{4}} \frac{1}{\sqrt{9 - 4x^2}}\, dx$ **3**

(d) If $x = 8\cos\theta, y = 5\sin\theta$ find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{4}$. **2**

(e) (i) Graph $y = 1 + \frac{3}{x - 2}$ showing all important features such as intercepts and asymptotes **2**

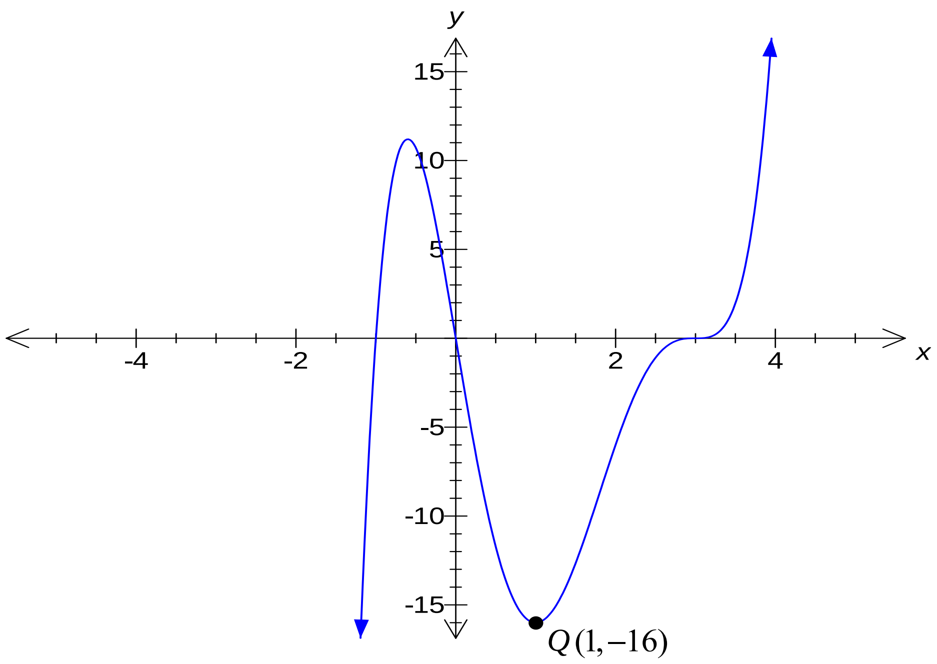
(ii) Hence solve $\frac{3}{x - 2} > -1$ **2**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\int \sin^2 4x \, dx$ **2**

(b)



(i) The above graph has equation $y = ax(x-b)(x+c)^d$. **3**
Write down values for *a*, *b*, *c* and *d*.

(ii) The approximate root was chosen as $x = 1$. Explain why this choice will not give a better approximation to the root using Newton's Method. **1**

(c) The function $f(x) = \sin x - \log_e x$ has a zero near $x = 2.5$ **2**
Use one application of Newton's method to obtain another approximation to this zero.
Give your answer correct to two decimal places.

(d) (i) How many numbers greater than 50000 can be formed with the digits is 2, 3, 5, 7, 9 if no digit repeated? **2**

(ii) What is the probability that if the numbers are selected at random and laid next to each other, that the number formed is greater than 50000 but less than 90000? **2**

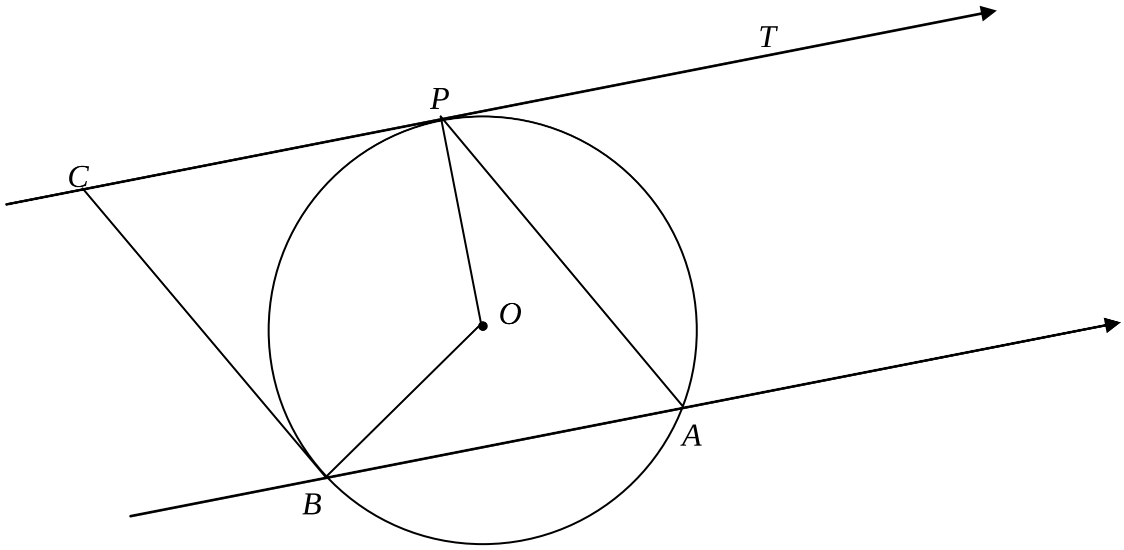
End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find, as an integer, the coefficient of x^3 in the expansion of $(2x-3)^9$. **2**

(b) The polynomial $x^3-15x^2+71x-105=0$ had roots α, β, γ which form an arithmetic sequence with a common difference of 2. Find the value of the three roots. **3**

(c) *CT* is a tangent to the circle, centre *O*, touching at *P*. Quadrilateral *PABC* is a rhombus and *CT* is parallel to *AB*.



(i) Let $\angle TPA = x$ and prove that $\angle POB = 2x$ **2**

(ii) Find the value of *x* such that *POBC* is a cyclic quadrilateral. You must support your answer with geometrical reasons. **2**

(d) In Poker 5 cards are dealt to each player. The deck has 4 suits with 13 cards in each suit. For the following questions leave your answer in unsimplified form.

(i) How many different hands are possible? **1**

(ii) Find the probability of a "flush" (all five cards are from the same suit). **2**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\sin x + \cos x = 1$, using "t" method, in the domain $0 \leq x \leq \pi$. **3**

(b) By using the substitution $u = \tan x$, or otherwise, find $\int \frac{\sec^2 x}{\sqrt{1+2\tan x}}\, dx$. **3**

(c) (i) Show that the equations of the tangent and normal to the parabola $x^2 = 4ay$, at the point $P(2ap, ap^2)$ are: **2**

Tangent: $y = px - ap^2$
Normal: $yp = -x + 2ap + ap^3$

(ii) T and G are the points where the tangent and normal meet the axis of the parabola. Find the coordinates of these points. **2**

(iii) If S is the focus of the parabola, explain why P, T and G all lie on the circle with centre S. **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Prove by mathematical induction that, for all integers $n \geq 1$ **4**

$$2 + 2^3 + 2^5 + \ldots + 2^{2n-1} = \frac{2(2^{2n} - 1)}{3}.$$

(b) If $f(x) = \tan^{-1} x + \tan^{-1} \frac{1}{x}$

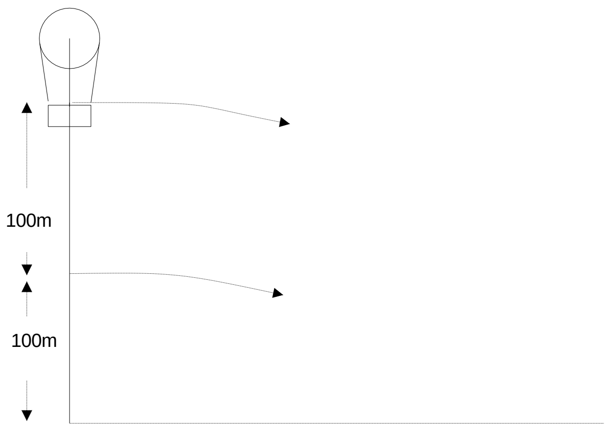
(i) Show that $f'(x) = 0$ **2**

(ii) State the domain of $f(x) = \tan^{-1} x + \tan^{-1} \frac{1}{x}$ **1**

(iii) Sketch $f(x)$ in its domain **2**

(c) A balloon rises vertically from level ground. Two projectiles are fired horizontally in the same direction from the balloon at a velocity of 80ms^{-1}. **3**
The first is fired at a point 100 m from the ground and the second when it has risen a further 100 m from the ground.

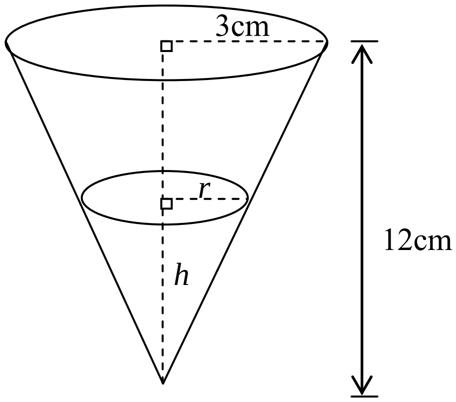
How far apart will the projectiles hit the ground (give your answer to the nearest metre)?
(Use $g = 10ms^{-2}$)



End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The diagram shows a conical drinking cup of height 12cm and radius 3cm. The cup is being filled with water at the rate of $2\,\text{cm}^3$ per second. The height of the water at time *t* seconds is *h* cm and the radius of the water's surface is *r* cm



(i) Show that $r = \frac{1}{4}h$ **1**

(ii) Find the rate at which the height is increasing when the height of the water is 8 cm. $\left(\text{Volume of a cone } = \frac{1}{3}\pi r^2 h\right)$ **3**

(b) A household iron is cooling in a room of constant temperature $23^\circ C$. At time *t* minutes its temperature *T* decreases according to the equation

$$\frac{dT}{dt} = -k\left(T - 23\right)$$ where *k* is a positive constant.

The initial temperature of the iron is $90^\circ C$ and it cools to $70^\circ C$ after 10 minutes.

(i) Verify that $T = 23 + Ae^{-kt}$ is a solution of this equation where *A* is a constant. **1**

(ii) Find the exact values of *A* and *k*. **2**

(iii) How long will it take for the iron to cool to a temperature of 40°? Give your answer to the nearest minute. **2**

(c) If $(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r$ prove $\sum_{r=1}^{n} r\,{}^nC_r = n.2^{n-1}$ **3**

End of Question 6

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(Question 7 continues on page 10)

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A projectile is fired from the origin O with velocity V and with angle of elevation θ, Where $\theta \neq \frac{\pi}{2}$. You may assume that:

$$x = Vt\cos\theta \text{ and } y = -\frac{1}{2}gt^2 + Vt\sin\theta$$

where x and y are the horizontal and vertical displacements of the projectile in metres from O at time t seconds after firing.

(i) Show that the equation of flight (trajectory) of the projectile can be written as: **3**

:

$$y = x\tan\theta - \frac{x^2}{4h}(1+\tan^2\theta) \qquad \text{where } h = \frac{V^2}{2g}$$

(ii) Show that the point (X, Y), where $X \neq 0$, can be hit by firing the projectile at two different angles θ_1 and θ_2 provided: **3**

$$X^2 < 4h(h - Y)$$

Question 7 continued on page 11

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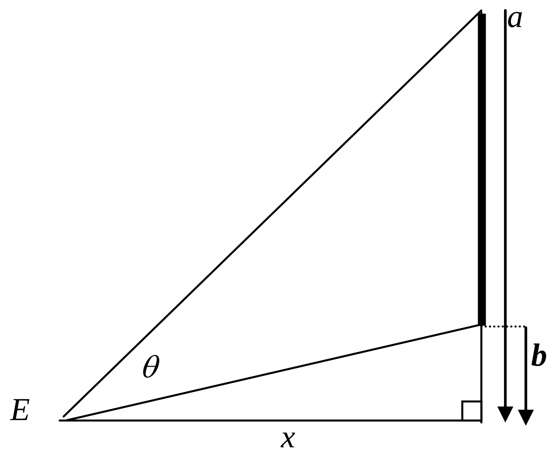
Question 7 (continued) **Marks**

(b) An observer's eye E looks up at a large painting on a vertical wall. The top of the painting is a metres above the level of E and the bottom of the painting is b metres above the level of E. θ is the angle subtended at the observer's eye by the top and bottom of the painting. E is x metres from the wall. The observer can move backwards and forwards changing x to find the position of best view when x is a maximum.



(i) Explain why $\theta = \tan^{-1}\dfrac{a}{x} - \tan^{-1}\dfrac{b}{x}$ **1**

(ii) Show that $\dfrac{d\theta}{dx} = \dfrac{(a-b)(ab-x^2)}{(a^2+x^2)(b^2+x^2)}$ **3**

(iii) If $a = 3b$, find the maximum possible value for θ. **2**

End of Test

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ if\ n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a \neq 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln(x + \sqrt{x^2 - a^2}),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$

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Mathematics Extension 1 (SOLUTIONS)

General Instructions

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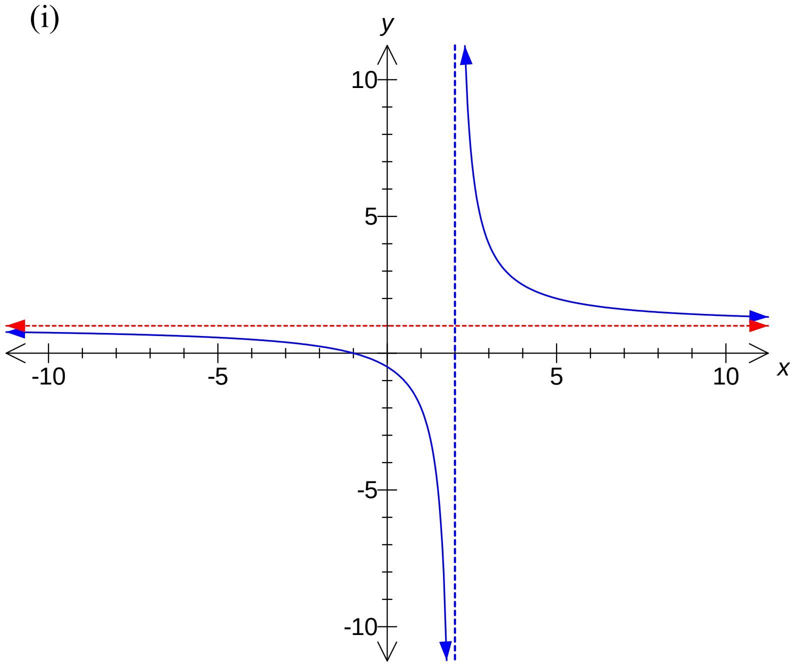
Question	Criteria	Marks	Bands
1(a)	$P(x)=x^3-2x+7$ Remainder is $P(5)=5^3-2\times5+7=122$	**1**	
1(b)	$y=\tan^{-1}\left(\frac{x}{3}\right)$ $\frac{dy}{dx}=\frac{1}{1+\left(\frac{x}{3}\right)^2}\times\frac{1}{3}$ ✓ $=\frac{3}{9+x^2}$ ✓	**2**	
1(c)	$\int_0^{\frac{3}{4}}\frac{1}{\sqrt{9-4x^2}}\,dx=\int_0^{\frac{3}{4}}\frac{1}{\sqrt{4\left(\frac{9}{4}-x^2\right)}}\,dx$ $=\int_0^{\frac{3}{4}}\frac{1}{2\sqrt{\left(\frac{9}{4}-x^2\right)}}\,dx$ ✓ $=\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{3}\right)\right]_0^{\frac{3}{4}}$ ✓ $=\frac{1}{2}\left[\sin^{-1}\left(\frac{2\times\frac{3}{4}}{3}\right)-\sin^{-1}(0)\right]$ $=\frac{1}{2}\sin^{-1}\frac{1}{2}$ $=\frac{1}{2}\times\frac{\pi}{6}$ ✓ $=\frac{\pi}{12}$	**3**	
1(d)	$\left.\begin{array}{l}\frac{dx}{d\theta}=-8\sin\theta\\ \frac{dy}{d\theta}=5\cos\theta\end{array}\right\}$ ✓ $when\ \theta=\frac{\pi}{4},\ \frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}=5\cos\frac{\pi}{4}\times\frac{1}{-8\sin\frac{\pi}{4}}$ $=\frac{-5}{8}$ ✓	**2**	

1(e)	(i)  ✓ hyperbola and intercepts ✓ asymptotes (ii) $\frac{3}{x-2} > -1$ is when $y = 0$ in equation $y = 1 + \frac{3}{x-2}$ That is, from the above graph where $x < -1$, $x > 2$	**2** **2**	

Question	Criteria	Marks	Bands
2(a)	$\cos 8x = 1 - 2\sin^2 4x$ $\sin^2 4x = \frac{1}{2}(1 - \cos 8x)$ $\int \sin^2 4x \, dx = \frac{1}{2}\int (1 - \cos 8x)dx$ ✓ $= \frac{x}{2} - \frac{\sin 8x}{16} + c$ ✓ $= \frac{8x - \sin 8x}{16} + c$	**2**	

2(b)(i)	$b=-1$ ✓ $\left.\begin{array}{l} c=-3 \\ d=3 \end{array}\right\}$ ✓ $when\, x=1,\ y=-16$ $-16=a(1+1)(1-3)^3$ $a=1$ ✓	**3**	
2(b)(ii)	A tangent to the curve at $x=1$ will be **horizontal** and will not therefore cut through the *x*-axis again to produce a new approx. to the root.	**1**	
2(c)(i)	$f(x)=\sin x-\log_e x,\ \ f'(x)=\cos x-\frac{1}{x}$ let the new approx. be x_1 $x_1=2.5-\frac{f(2.5)}{f'(2.5)}$ ✓ $=2.5-\frac{\sin 2.5-\log_e 2.5}{\cos 2.5-\frac{1}{2.5}}$ $=2.24$ ✓	**2**	
2(d)(i)	$\boxed{3} \times \underbrace{\boxed{4}\times\boxed{3}\times\boxed{2}\times\boxed{1}}_{\text{remainder}}=72$ ✓ $5,7,9$ ✓	**2**	
2(d)(ii)	$\frac{2\times 4!\ (✓)}{5!}=\frac{48}{120}$ $=\frac{2}{5}$ ✓	**2**	

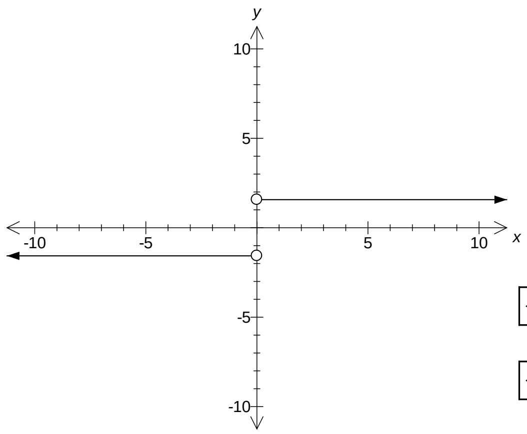
Question	Criteria	Marks	Bands
3(a)	${}^9C_6\left(2x\right)^3\left(-3\right)^6=489888x^3$ ✓✓ $\therefore 489888$	**2**	
3(b)	let the roots be $\alpha-2,\alpha,\alpha+2$ ✓ sum of the roots : $\alpha-2,+\alpha+\alpha+2=15$ ✓ $3\alpha=15$ $\alpha=5$ $\therefore$ roots are: 3, 5, 7 ✓	**3**	
3(c)(i)	$\angle PAB=x$ (alternate $\angle$'s; $CT \parallel BA$) ✓ $\angle POB=2x$ (angle at the centre is twice the angle on the circumference) ✓	**2** **Nb -1 for both reasons**	

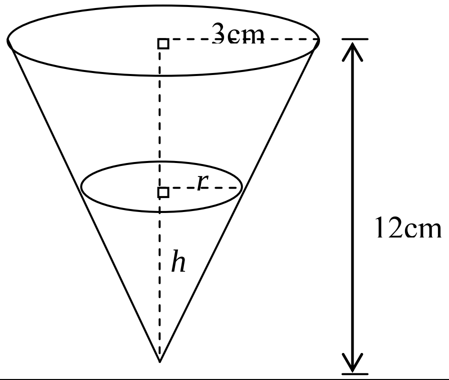
		wrong	
3(c)(ii)	$\angle PCB = x$ (opposite angles of a rhombus are equal) ✓ $x + 2x = 180$ (opposite angles of a cyclic quadrialteral add to 180°) ✓ $\therefore x = 60^\circ$	**2**	
3(d)(i)	${}^{52}C_5$ ✓	**1**	
3(d)(ii)	number of possible = ${}^{13}C_5 \times 4 = 5148$ ✓ $P(\text{flush}) = \frac{5148}{2598960} = \frac{33}{16660}$ ✓	**2**	

Question	Criteria	Marks	Bands
4(a)	$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$ ✓ $2t + 1 - t^2 = 1 + t^2$ $2t^2 - 2t = 0$ $2t(t-1) = 0$ $t = 0,1$ ✓ $\tan\frac{x}{2} = 0$ $\tan\frac{x}{2} = 1$ $\frac{x}{2} = 0$ $\frac{x}{2} = \frac{\pi}{4}$ $x = 0, \frac{\pi}{2}$ ✓ Test $x = \pi$; $\sin\pi + \cos\pi = -1$ $\therefore x \neq \pi$	**3**	
4(b)	$\int \frac{\sec^2 x}{\sqrt{1+2\tan x}} dx$ let $u = \tan x$ $du = \sec^2 x dx$ $\int \frac{du}{\sqrt{1+2u}}$ ✓ $= \int (1+2u)^{-\frac{1}{2}} du$ $= \frac{2(1+2u)^{\frac{1}{2}}}{2} du$ ✓ $= \sqrt{1+2\tan x} + c$ ✓	**3**	

4(c)(i)	$y=\frac{x^2}{4a}$ $\frac{dy}{dx}=\frac{2x}{4a}$; at $(2ap,ap^2)$ $\frac{dy}{dx}=p$ $y-ap^2=p(x-2ap)$ tangent: $y=px-ap^2$ ✓ $y-ap^2=-\frac{1}{p}(x-2ap)$ $yp-ap^3=-x+2ap$ normal: $yp=2ap+ap^3-x$ ✓	**2**	
4(c)(ii)	$(0,-ap^2)$ ✓ $(0,2a+ap^2)$ ✓	**2**	
4(c)(iii)	GT forms the diameter of a circle through P since $\angle GPT=90^\circ$ ✓ Midpoint of GT $x=0$ $y=\frac{2a+ap^2-ap^2}{2}=a$ $\therefore$ centre of circle is $S(0,a)$ ✓	**2**	

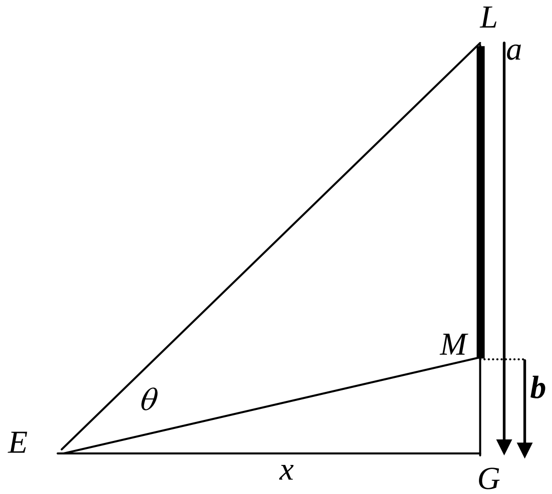
Question	Criteria	Marks
5(a)	$2+2^3+2^5+...+2^{2n-1}=\frac{2(2^{2n}-1)}{3}$ $Step 1:$ Pr$ove\ true\ \ for\ \ n=1$ $LHS=2^{2(1)-1}$ $\quad RHS=\frac{2(2^{2(1)}-1)}{3}$ $=2$ $\quad =\frac{2(3)}{3}$ $=2$ $\therefore\ LHS=RHS$ $true\ \ for\ \ n=1$ ☑ $proving\ true\ \ for\ \ n=1$ $Step 2: assume\ true\ \ for\ \ n=k$ $ie: 2+2^3+2^5+...+2^{2k-1}=\frac{2(2^{2k}-1)}{3}$ $Step 3:$ Pr$ove\ true\ \ for\ \ n=k+1$ $RTP: 2+2^3+2^5+...+2^{2(k+1)-1}=\frac{2(2^{2(k+1)}-1)}{3}$ $ie:\ 2+2^3+2^5+...+2^{2k+1}=\frac{2(2^{2k+2}-1)}{3}$ $LHS: 2+2^3+2^5+...+2^{2k-1}+2^{2(k+1)-1}$ $=2+2^3+2^5+...+2^{2k-1}+2^{2k+1}$ $=\frac{2(2^{2k}-1)}{3}+2^{2k+1}$ (using step 2) ☑ $sub\ step$ 2 step 3 $=\frac{2^{2k+1}-2+3.2^{2k+1}}{3}$ $=\frac{2^{2k+1}(1+3)-2}{3}$ $=\frac{2^{2k+1}(4)-2}{3}$ $=\frac{2^{2k+1}(2^2)-2}{3}$ $=\frac{2^{2k+3}-2}{3}$ $=\frac{2(2^{2k+2}-1)}{3}$ ☑ $proving\ RTP\ statement$ $\therefore\ true\ \ for\ \ n=k+1$ $step\ 4:$ if it is true $n=1$ and $n=1+2=2$, and it is true for $n=k$ and $n=k+1$ then it is true for integers $n\geq 1$ ☑	**4**

5(b)(i)	$f(x)=\tan^{-1}x+\tan^{-1}\dfrac{1}{x}$ $f'(x)=\dfrac{1}{1+x^2}+\dfrac{1}{1+\left(\dfrac{1}{x}\right)^2}\times\dfrac{-1}{x^2}$ ☑ *differentiation* $=\dfrac{1}{1+x^2}+\dfrac{1}{\dfrac{x^2+1}{x^2}}\times\dfrac{-1}{x^2}$ $=\dfrac{1}{1+x^2}+\dfrac{x^2}{x^2+1}\times\dfrac{-1}{x^2}$ $=\dfrac{1}{1+x^2}+\dfrac{-1}{x^2+1}$ ☑ *simplification* $=0$	**2**
5(b)(ii)	*all real* x $(x\in\mathbb{R})$ *except* $x=0$	**1**
5(b)(iii)	 ☑ $y=\dfrac{\pi}{2}\quad x>0$ *and* $y=\dfrac{-\pi}{2}\quad x<0$ ☑ $x\neq 0$	**2**
5(c)	*From* $100m$: $x=80t\cos(0)\quad y=-5t^2+80t\sin(0)+100\quad\Rightarrow\quad y=-5\left(\dfrac{x}{80}\right)^2+100\quad.....(1)$ $\therefore x=80t\qquad y=-5t^2+100$ *From* $200m$: $x=80t\cos(0)\quad y=-5t^2+80t\sin(0)+200\quad\Rightarrow\quad y=-5\left(\dfrac{x}{80}\right)^2+200\quad.....(2)$ $\therefore x=80t\qquad y=-5t^2+200$ ☑ *Hits ground when* $y=0$ (1) $-5\left(\dfrac{x}{80}\right)^2+100=0\quad\Rightarrow -5\left(\dfrac{x}{80}\right)^2=-100\qquad\therefore 358$ *metres* ☑ (2) $-5\left(\dfrac{x}{80}\right)^2+200=0\quad\Rightarrow -5\left(\dfrac{x}{80}\right)^2=-200\qquad\therefore 506$ *metres* ☑ $\therefore$ *difference in range is* $506-358=148$ *metres.*	**3**

Question	Criteria	Marks
6(a)(i)	 $\frac{h}{r}=\frac{12}{3} \quad \therefore r=\frac{h}{4}$ ✓	**1**
6(a)(ii)	$V=\frac{1}{3}\pi r^2 h \text{ and } r=\frac{h}{4} \quad \therefore V=\frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h \quad \Rightarrow V=\frac{\pi h^3}{48}$ $\frac{dV}{dh}=\frac{3\pi h^2}{48}=\frac{\pi h^2}{16}$ ✓ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ $\frac{dh}{dt}=\frac{16}{\pi h^2}\times 2$ ✓ $\frac{dh}{dt}=\frac{32}{\pi h^2}$ since $h=8$ $\frac{dh}{dt}=\frac{1}{2\pi}$ cm/s ✓	**3**
6(b)(i)	$T=23+Ae^{-kt} \Rightarrow Ae^{-kt}=T-23$ $\frac{dT}{dt}=-kAe^{-kt}$ $=-k(T-23)$ ✓	**1**
6(b)(ii)	$t=0 \text{ and } T=90 \quad \therefore 90=23+Ae^{-k(0)} \Rightarrow A=67$ ✓ $t=10 \text{ and } T=70 \quad \therefore 70=23+67e^{-k(10)} \Rightarrow k=-\frac{1}{10}\ln\left(\frac{47}{67}\right)$ ✓	**2**
6(b)(iii)	$T=23+67e^{-kt} \quad \text{where} \quad k=-\frac{1}{10}\ln\left(\frac{47}{67}\right)$ $40=23+67e^{-kt}$ $17=67e^{-kt}$ $\left(\frac{17}{67}\right)=e^{-kt}$ $t=\frac{\ln\left(\frac{17}{67}\right)}{-k}$ ✓ $t=38.68$ $t=39$ mins ✓	**2**

6(c)	If $(1+x)^n = \sum_{r=0}^{n} {}^nC_r\ x^r$ prove $\sum_{r=1}^{n} r\,{}^nC_r = n.2^{n-1}$ $(1+x)^n = {}^nC_0(1)^n(x)^0 + {}^nC_1(1)^{n-1}(x)^1 + {}^nC_2(1)^{n-2}(x)^2 + \ldots + {}^nC_n(1)^0(x)^n$ ☑ *Differentiate both sides:* $n(1+x)^{n-1} = {}^nC_1(1)^{n-1}(x)^0 + 2\,{}^nC_2(1)^{n-2}(x) + \ldots + n\,{}^nC_n(1)^0(x)^{n-1}$ ☑ *If* $x=1$ $n(2)^{n-1} = {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + \ldots + n\,{}^nC_n$ ☑ $n(2)^{n-1} = \sum_{r=1}^{n} r\,{}^nC_r$	**3**

Question	Criteria	Marks
7(a)(i)	$x = Vt\cos\theta \qquad y = -\frac{1}{2}gt^2 + Vt\sin\theta$ $t = \frac{x}{V\cos\theta} \qquad \therefore y = -\frac{1}{2}g\left(\frac{x}{V\cos\theta}\right)^2 + V\left(\frac{x}{V\cos\theta}\right)\sin\theta$ ☑ *sub of t* $y = \frac{-gx^2}{2V^2\cos^2\theta} + x\tan\theta$ ☑ *rearranging and substitution* $y = \frac{-gx^2}{2V^2}\sec^2\theta + x\tan\theta$ $y = x\tan\theta - \frac{x^2}{4h}(1+\tan^2\theta)$ ☑ *substitution of h and* $(\sec^2\theta = 1+\tan^2\theta)$	**3**
7(a)(ii)	$y = x\tan\theta - \frac{x^2}{4h}(1+\tan^2\theta)$ $X\tan\theta - \frac{X^2}{4h}\left(\tan^2\theta+1\right) - y = 0$ $X^2\tan^2\theta - 4hX\tan\theta + X^2 + 4hY = 0$ ☑ *Hits at two different angles* θ_1 *and* θ_2 *when* $\Delta > 0$ ☑ $\Delta = \left(4hX\right)^2 - 4(X^2)(X^2+4hY) > 0$ $16h^2X^2 - 4X^4 - 16X^2hY > 0$ $4X^4h < 16h^2X^2 - 16X^2hY$ $X^2 < 4h^2 - 4hY$ $X^2 < 4h(h-Y)$ ☑	**3**

7(b)(i)	 $\tan(\angle LEG) = \frac{a}{x} \quad \Rightarrow \tan^{-1}\left(\frac{a}{x}\right) = \angle LEG$ $\tan\left(\angle MEG\right) = \frac{b}{x} \quad \Rightarrow \tan^{-1}\left(\frac{b}{x}\right) = \angle MEG$ $\sin ce\ \theta = \angle LEG - \angle MEG$ $\therefore \theta = \tan^{-1}\left(\frac{a}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$ ✓	**1**
7(b)(ii)	$\theta = \tan^{-1}\left(\frac{a}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$ $\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{a}{x}\right)^2} \times \frac{-a}{x^2} - \frac{1}{1+\left(\frac{b}{x}\right)^2} \times \frac{-b}{x^2}$ ✓ differentiation $\frac{d\theta}{dx} = \frac{-a}{x^2+a^2} - \frac{-b}{x^2+b^2}$ $\frac{d\theta}{dx} = \frac{-ax^2 - ab^2 + bx^2 + ba^2}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$ ✓ common denominator $\frac{d\theta}{dx} = \frac{-x^2(a-b) + ab(a-b)}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$ $\frac{d\theta}{dx} = \frac{(a-b)(-x^2+ab)}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$ ✓ grouping method	**3**

7(b)(iii)	$\frac{d\theta}{dx}=\frac{(a-b)(-x^2+ab)}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$ $\frac{d\theta}{dx}=\frac{(3b-b)(-x^2+(3b)b)}{\left(x^2+(3b)^2\right)\left(x^2+b^2\right)}$ $\quad if\ \ a\ =\ 3b$ $\frac{d\theta}{dx}=\frac{2b(-x^2+3b^2)}{\left(x^2+9b^2\right)\left(x^2+b^2\right)}$ $\max\ occurs\ when\ \frac{d\theta}{dx}=0$ $\therefore \frac{2b(-x^2+3b^2)}{\left(x^2+9b^2\right)\left(x^2+b^2\right)}=0$ $2b(-x^2+3b^2)=0$ $-x^2+3b^2=0$ $x^2=3b^2$ $x=\pm\sqrt{3}b$ ☑ since x is a distance then $x>0$ $\quad then\ x=\sqrt{3}b$ $\theta=\tan^{-1}\left(\frac{3b}{x}\right)-\tan^{-1}\left(\frac{b}{x}\right) \quad if\ \ a=3b$ $\theta=\tan^{-1}\left(\frac{3b}{\sqrt{3}b}\right)-\tan^{-1}\left(\frac{b}{\sqrt{3}b}\right)$ $\theta=\tan^{-1}\left(\frac{3b}{\sqrt{3}b}\right)-\tan^{-1}\left(\frac{b}{\sqrt{3}b}\right)$ $\theta=60^\circ-30^\circ$ $\theta=30^\circ$ ☑	