Kinross Wolaroi School
Mathematics Extension 1
Trial Higher School Certificate
2012

Total Marks - 70

Section I	10 marks Attempt questions 1-10 Allow about 15 mins
Section II	60 marks Attempt quest., 11-14 Allow about 105 mins

General Instructions

- Reading Time – 5 Minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in questions 11-14

Section I

10 Marks
Attempt questions 1-10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross in the appropriate space on the grid attached at the back of this examination paper.

You may detach the grid from the examination paper after first writing your examination number in the space provided.

1. What is the value of the term independent of x in the expansion of

$$\left(x^2 + \frac{3}{x}\right)^9$$

(A) 4410
(B) 84
(C) 61236
(D) 126

2. Which of the following is an expression for

$$\int \frac{2x}{\sqrt{1+x^2}}\,dx$$

(A) $\log_e(1+x^2)+C$

(B) $\log_e\sqrt{1+x^2}+C$

(C) $\sqrt{1+x^2}+C$

(D) $2\sqrt{1+x^2}+C$

3. If $t = tan\frac{x}{2}$ which of the following is an expression for $\frac{dx}{dt}$

(A) $\frac{1}{1+t^2}$

(B) $\frac{2}{1+t^2}$

(C) $\frac{1}{2}\ sec^2\frac{x}{2}$

(D) $1 + t^2$

4. If $y = e^{x^3}$, which is an expression for $\frac{d^2y}{dx^2}$

(A) $6x + 3x^2e^{x^3}$

(B) $6xe^{3x^2}$

(C) $6xe^{x^3}$

(D) $3xe^{x^3}(2 + 3x^3)$

5. Which of the following is an expression for

$$\int sin^2 3x\ \ dx$$

(A) $\frac{x}{2} - \frac{\sin 3x}{6} + C$

(B) $\frac{x}{2} + \frac{\sin 3x}{6} + C$

(C) $\frac{x}{2} + \frac{\sin 3x}{12} + C$

(D) $\frac{x}{2} - \frac{\sin 6x}{12} + C$

6. A class consists of 10 girls and 12 boys. How many ways are there of selecting a committee of 3 girls and 2 boys from this class?

(A) 26 334

(B) 95 040

(C) 7920

(D) 110

7. A circular plate of radius r is heated so that the area of the plate expands at a constant rate of 3.2 cm^2min^{-1}. At what rate does r increase when $r = 10cm$? [Answer to 2 decimal places.]

(A) 0.03 cm/min

(B) 0.04 cm/min

(C) 0.05 cm/min

(D) 0.06 cm/min

8. The point $P(-3,\ 8)$ divides the interval AB externally in the ratio $k : 1$. If A is the point $(6, -4)$ and B is the point $(0, 4)$, find the value of k

(A) -3

(B) 3

(C) -2

(D) 2

9. For what values of x is

$$|x - 2| < x$$

(A) $x > 1$

(B) $x < 1$

(C) $x \geq -1$

(D) $x \leq -1$

10. A meeting room consists of a round table surrounded by ten chairs. These chairs are high backed, leather upholstered but indistinguishable from each other and equally spaced around the table. A committee of ten people includes three teenagers, why you would seek the opinion of a teenager I don't know. How many seating arrangements are there where all three teenagers sit together?

(A) 30 240

(B) 40 320

(C) 120 960

(D) 3 628 800

Section II

60 marks

Attempt questions 11-14

Allow about 105 minutes for this section

Start each question on a new page.

All necessary working should be shown in every question

Question 11

(a) Write down an indefinite integral of **[3]**

(i) $\dfrac{1}{1+x^2}$

(ii) $\dfrac{1}{(1+x)^2}$

(iii) $\dfrac{x}{1+x^2}$

(b) Find the size of the acute angle between the lines with equations **[3]**

$$x - 2y + 1 = 0$$

$$3x - y - 2 = 0$$

(c) Using the expansion of $cos(A + B)$ or otherwise, find the exact value of

[3]

$$cos\frac{5\pi}{12}$$

(d) Sketch the graph of $y = x - x^3$, indicating the x intercepts only and hence or otherwise solve the inequation **[3]**

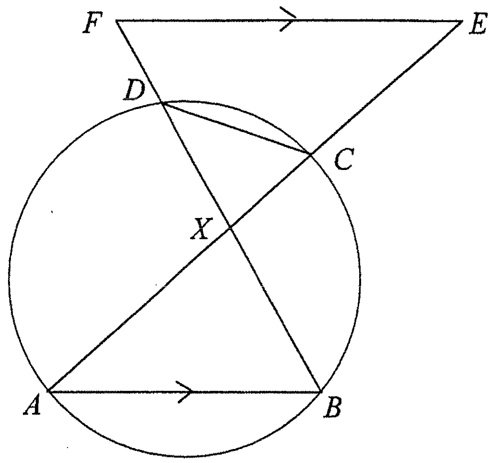
$$x - x^3 < 0$$

(e) Solve the inequality **[3]**

$$\frac{1}{x-1} < 1$$

Question 12 [Start a new page]

(a) **[3]**



AC and BD are two chords of a circle which intersect at a point X inside the circle. E is a point on AC produced and F is a point on BD produced such that FE$||AB$. Show that DCEF is a cyclic quadrilateral.

(b) Use mathematical induction to show that for all positive integers $n \geq 2$,

[3]

$$2 \times 1 + 3 \times 2 + 4 \times 3 + \ldots\ldots\ldots + n(n-1) = \frac{1}{3}n(n^2 - 1)$$

(c) Use the substitution $u = x + 1$ to evaluate **[3]**

$$\int_1^3 \frac{x+2}{(x+1)^2}\, dx$$

(d) (i) Find the domain and range of the function **[2]**

$$y = 2\, sin^{-1} 3x$$

(ii) Hence make a neat sketch of this function **[1]**

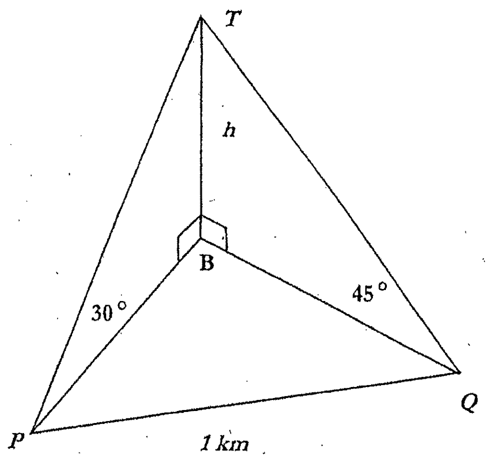
(e) Find all solutions for the equation [3]

$$\sin 2x = \sin x$$

For $0 \leq x \leq 2\pi$

Question 13 **[start a new page]**

(a)



The angle of elevation from a boat at P to a point T at the top of a vertical cliff is 30°. The boat sails 1 km to a second point Q, from which the angle of elevation of T is 45°. B is the point at the base of the cliff directly below point T and h is the height of the cliff in metres. The bearings of B from P and Q are 50° and 290° respectively.

(i) Show that angle PBQ = 120° **[1]**

(ii) By finding expressions for PB and QB in terms of h, show that **[4]**

$$h = \frac{10\sqrt{10}}{\sqrt{4+\sqrt{3}}} \text{ metres}$$

(b) Solve:

$$(n-2)! = 56(n-4)!$$

(c) (i) Show that the equation of the normal to the parabola $x^2 = 4y$ at the point $P(2p, p^2)$ is given by **[2]**

$$x + py = p(p^2 + 2)$$

(ii) If this normal cuts the axis of the parabola at Q, find the equation of the mid-point of PQ. **[3]**

(d) For the cubic equation **[2]**

$$2x^3 - 3x^2 + 5x - 2 = 0$$

With roots α, β and γ, find the value of

$$\alpha^2 + \beta^2 + \gamma^2$$

(e) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line, velocity $v\ ms^{-1}$ given by $v = \frac{2}{3\sqrt{x}}$ and acceleration $a\ ms^{-2}$. Initially the particle is 1 metre to the right of O.

(i) Show that $a = \frac{-2}{9x^2}$ **[1]**

(ii) Show that $x = (t+1)^{\frac{2}{3}}$ **[2]**

Question 14 **[start a new page]**

(a) A particle is moving in a straight line and performing Simple Harmonic Motion. At time t seconds it has displacement x metres from a fixed point O on the line, given by

$$x = 2\cos\left(2t - \frac{\pi}{4}\right)$$

Velocity $v\ ms^{-1}$ and acceleration $\ddot{x}\ ms^{-2}$

(i) Show that **[2]**

$$v^2 - x\ddot{x} = 16$$

(ii) Sketch the graph of x as a function of time for $0 \le t \le \pi$ showing clearly the coordinates of the end points. **[2]**

(iii) Show that the particle first returns to its starting point after one quarter of its period. **[2]**

(iv) Find the time taken by the particle to travel the first 100 metres of its motion. **[1]**

(b) A projectile is launched with an initial velocity of 40 ms^{-1} with an angle of elevation of θ^o from the top of a 50 metre high building. Acceleration due to gravity is assumed to be 10 ms^{-2}.

(i) Show that its horizontal and vertical displacements after t seconds are

$$x = 40tcos\theta$$

$$y = -5t^2 + 40tsin\theta + 50$$

[2]

(ii) The projectile lands at ground level 200 metres from the building. Find the two possible values for θ. Give your answers to the nearest degree.

[3]

(c) (i) Show that **[1]**

$$4tan^{-1}x + x - 4 = 0$$

has a root close to $x = 1$.

(ii) Use Newton's Method once to find an approximation to it to 3 decimal places. **[2]**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

STUDENT NUMBER/NAME: ……………………..

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

1) $\binom{9}{k}(x^2)^{9-k}\left(\frac{3}{x}\right)^k$

x^0

$x^{2(9-k)} \cdot x^{-k} = x^0$

$18 - 2k - k = 0$

$k = 6$

$\binom{9}{6}3^6 = 61236$

[C]

2) $\frac{d}{dx}\left[2(1+x^2)^{\frac{1}{2}} + C\right]$

$= (1+x^2)^{-\frac{1}{2}} \cdot 2x$

$= \frac{2x}{\sqrt{1+x^2}}$

[D]

3) $t = \tan\frac{x}{2}$

$\frac{x}{2} = \tan^{-1} t$

$x = 2\tan^{-1} t$

$\frac{dx}{dt} = \frac{2}{1+t^2}$

[B]

4) $y = e^{x^3}$

$\frac{dy}{dx} = 3x^2 e^{x^3}$ $\left\{ u = 3x^2,\ u' = 6x,\ v = e^{x^3},\ v' = 3x^2 e^{x^3} \right.$

$\frac{d^2y}{dx^2} = 6xe^{x^3} + 9x^4 e^{x^3}$

$= 3xe^{x^3}(2 + 3x^3)$

[D]

5) $\cos 6x = 1 - 2\sin^2 3x$

$2\sin^2 3x = 1 - \cos 6x$

$\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$

$\frac{1}{2}\int 1 - \cos 6x \, dx$

$= \frac{1}{2}\left[x - \frac{\sin 6x}{6}\right] + C$

$= \frac{x}{2} - \frac{\sin 6x}{12} + C$

[D]

6) $\binom{11}{k}(2x)^{11-k}\left(-\frac{1}{x}\right)^k$

x^3 term

$x^{11-k} \cdot x^{-k} = x^3$

$11 - 2k = 3$

$2k = 8$

$k = 4$

$\binom{11}{4}2^4$

[A]

7) $\frac{dA}{dt} = 3.2$ $\quad A = \pi r^2$, $\frac{dA}{dr} = 2\pi r$

$\frac{dr}{dt} = \frac{dr}{dA} \cdot \frac{dA}{dt}$

$= \frac{3.2}{2\pi r}$

let $r = 10$

$= 0.05$

[C]

8) $(6,-4)$ $(0,4)$

$-k : 1$

$\frac{6}{1-k} = -3$

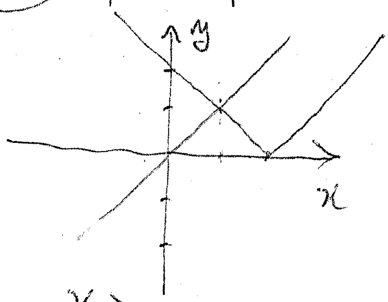
$6 = -3 + 3k$

$3k = 9$

$k = 3$

[B]

9) $|x - 2| < x$



$x > 1$

[A]

10) 8 GROUPS

TABLE

$7! \cdot 3!$

$= 30240$

[A]

A	3
B	2
C	2
D	3

11) a) (i) $\int \frac{1}{1+x^2}\,dx$

$= \tan^{-1} x + C$

(ii) $\int (1+x)^{-2}\,dx$

$= -(1+x)^{-1} + C$

$= \frac{-1}{1+x}$

(iii) $\frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

$= \frac{1}{2}\ln(1+x^2) + C$

$= \ln\sqrt{1+x^2} + C$

b) $m_1 = \frac{1}{2}$

$m_2 = 3$

$\tan\alpha = \left|\frac{3 - \frac{1}{2}}{1 + \frac{3}{2}}\right|$

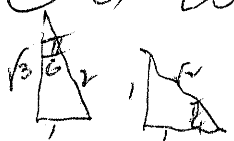
$= \frac{\frac{5}{2}}{\frac{5}{2}}$

$\tan\alpha = 1$

$\alpha = 45^\circ$

(11) c) $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$= \cos\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)$



$\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$

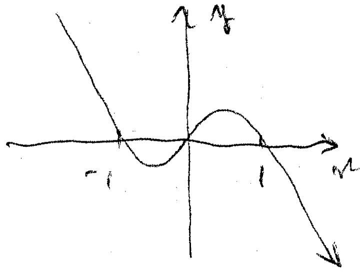
$= \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\times\frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

d) $y = x - x^3$

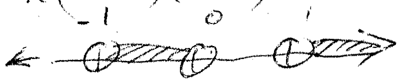
$= x(1-x^2)$

$= x(1-x)(1+x)$



$x - x^3 < 0$

$x(1-x)(1+x) < 0$



$-1 < x < 0$

$x > 1$

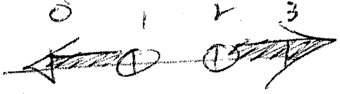
e) $\frac{1}{x-1} < 1$ (multiplying both sides by $(x-1)^2$)

$(x-1) < (x-1)^2$

$0 < (x-1)^2 - (x-1)$

$0 < (x-1)[x-1-1]$

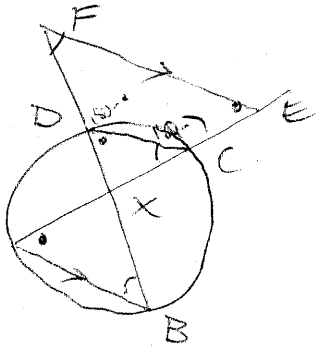
$(x-1)(x-2) > 0$



$x < 1$

$x > 2$

(12) a)



$\angle ABD = \angle BFE$ (alt angles)

$\angle ABD = \angle ACD$ (angles off the same arc)

$\therefore \angle BFE = \angle ACD$

(12) b)

$2\times1 + 3\times2 + 4\times3 + \ldots + n(n-1) = \frac{1}{3}n(n^2-1)$

FIRST TERM $n = 2$

LHS $= 2\times1$

$= 2$

RHS $= \frac{1}{3}\times2(4-1)$

$= 2$

$\therefore$ TRUE FOR $n = 2$

ASSUME TRUE FOR $n = k$

$2\times1 + 3\times2 + 4\times3 + \ldots + k(k-1) = \frac{1}{3}k(k^2-1)$

PROVE TRUE FOR $n = k+1$

$2\times1 + 3\times2 + 4\times3 + \ldots + k(k-1) + (k+1)k = \frac{1}{3}(k+1)[(k+1)^2-1]$

$= \frac{1}{3}(k+1)(k^2+2k)$

LHS $= \frac{1}{3}k(k^2-1) + k(k+1)$

$= \frac{1}{3}k(k+1)(k-1) + k(k+1)$

$= (k+1)\left[\frac{1}{3}k(k-1) + k\right]$

$= \frac{1}{3}(k+1)\left[k^2 - k + 3k\right]$

$= \frac{1}{3}(k+1)(k^2+2k)$

$\therefore$ LHS = RHS

$\therefore$ TRUE FOR ALL $n \geq 2$ BY MATHEMATICAL INDUCTION

(12) c) $\int_1^3 \frac{x+2}{(x+1)^2}\,dx$

$u + 1 = x + 2$

$u = x + 1$

$\frac{du}{dx} = 1$

$du = dx$

$\int_2^4 \frac{u+1}{u^2}\,du$

$= \int_2^4 u^{-2}(u+1)\,du$

$= \int_2^4 u^{-1} + u^{-2}\,du$

$= \left[\ln u - \frac{1}{u}\right]_2^4$

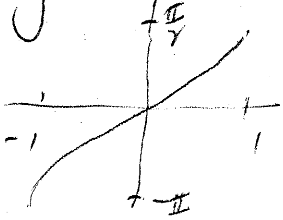
$= \left(\ln 4 - \frac{1}{4}\right) - \left(\ln 2 - \frac{1}{2}\right)$

$= \ln 4 - \ln 2 - \frac{1}{4} + \frac{1}{2}$

$= \frac{1}{4} + \ln 2$

(12) d) (i) $y = 2\sin^{-1} 3x$

$y = \sin^{-1} x$

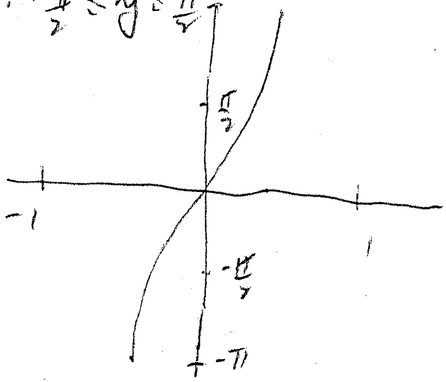


DOMAIN: $-\frac{1}{3} \le x \le \frac{1}{3}$

RANGE: $-\pi \le y \le \pi$

DOMAIN: $-1 \le x \le 1$

RANGE: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

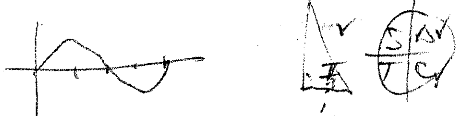


(12) e) $\sin 2x = \sin x$

$2\sin x \cos x = \sin x$

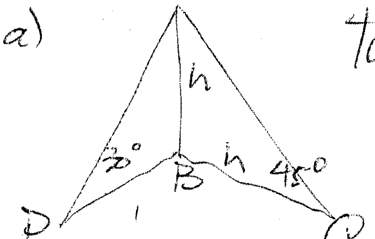
$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \quad \cos x = \frac{1}{2}$



$x = 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

(13) a)

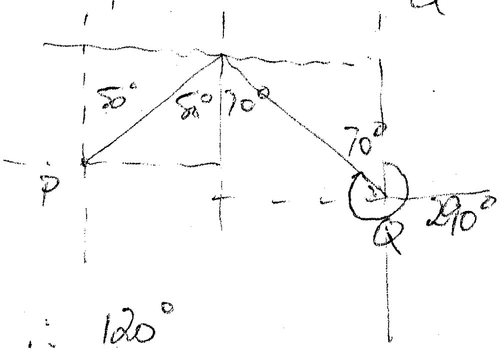


$\tan 30° = \frac{h}{PB}$

$PB = \frac{h}{\tan 30°}$

$= \sqrt{3}h$.

$QB = h$.



$\therefore 120°$

$a^2 = b^2 + c^2 - 2bc\cos 120°$

$1 = 3h^2 + h^2 - 2\sqrt{3}h^2 \times -\frac{1}{2}$

$1 = 4h^2 + \sqrt{3}h^2$

$h^2 = \frac{1}{4+\sqrt{3}}$ km

$= \frac{1000}{\sqrt{4+\sqrt{3}}}$ m

$h = \frac{1000}{[illegible]} = \frac{10\sqrt{10}}{[illegible]}$ METRES

(13) b) $(n-2)! = 56(n-4)!$

$\frac{(n-2)!}{(n-4)!} = 56$

$(n-2)(n-3) = 56$

$n^2 - 5n + 6 = 56$

$n^2 - 5n - 50 = 0$

$(n-10)(n+5) = 0$

$n = 10, -5$

$\therefore n = 10$

(13) c) i) $y = \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{2x}{4}$

let $x = 2p$

$m = \frac{4p}{4}$

$= p$

$\therefore m_{\perp} = -\frac{1}{p}$

$y - y_1 = m(x - x_1)$

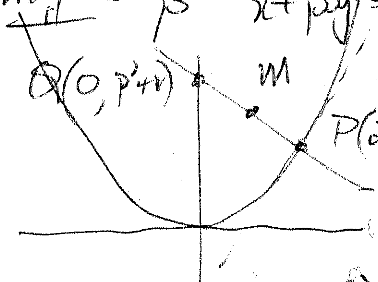
$y - p^2 = -\frac{1}{p}(x - 2p)$

$py - p^3 = -x + 2p$

$x + py = p^3 + 2p$

$x + py = p(p^2 + 2)$

(ii)



$x + py = p(p^2 + 2)$

let $x = 0$ TO FIND Q.

$y = p^2 + 2$

MIDPOINT

$M = \frac{2p+0}{2}$

$x = p \quad -(1)$

$y = \frac{p^2+2+p^2}{2}$

$= p^2 + 1 \quad -(11)$

$\therefore y = x^2 + 1$

(5) d) $2x^3 - 3x^2 + 5x - 1 = 0$

$\alpha+\beta+\gamma = \frac{3}{2}$

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{5}{2}$

$(\alpha+\beta+\gamma)^2 = (\alpha+\beta+\gamma)(\alpha+\beta+\gamma)$

$= \alpha^2+\alpha\beta+\alpha\gamma+\beta\alpha+\beta^2+\beta\gamma + \gamma\alpha+\gamma\beta+\gamma^2$

$= \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\quad)$

$= \left(\frac{3}{2}\right)^2 - 2\times\frac{5}{2}$

$= \frac{9}{4} - 5$

$= -\frac{11}{20}$

(13) e) $v = \frac{2}{3\sqrt{x}}$

(i) $v^2 = \frac{4}{9x}$

$\frac{1}{2}v^2 = \frac{2}{9x}$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(\frac{2x^{-1}}{9}\right)$

$= -\frac{2}{9}x^{-2}$

$a = \frac{-2}{9x^2}$

(ii) $\frac{dx}{dt} = \frac{2}{3\sqrt{x}}$

$\frac{dt}{dx} = \frac{3\sqrt{x}}{2}$

$t = \frac{3}{2}\int x^{\frac{1}{2}}\,dx$

$= \frac{\frac{3}{2}x^{\frac{3}{2}}}{\frac{3}{2}} + C$

$t = x^{\frac{3}{2}} + C$

when $t=0$, $x=1$

$\therefore C = -1$

$t = x^{\frac{3}{2}} - 1$

$t+1 = x^{\frac{3}{2}}$

$x = (t+1)^{\frac{2}{3}}$

(14) a) $x = 2\cos\left(2t - \frac{\pi}{4}\right)$

(i) $\dot{x} = -4\sin\left(2t - \frac{\pi}{4}\right)$

$\ddot{x} = -8\cos\left(2t - \frac{\pi}{4}\right)$

Show that

$v^2 - x\ddot{x} = 16$

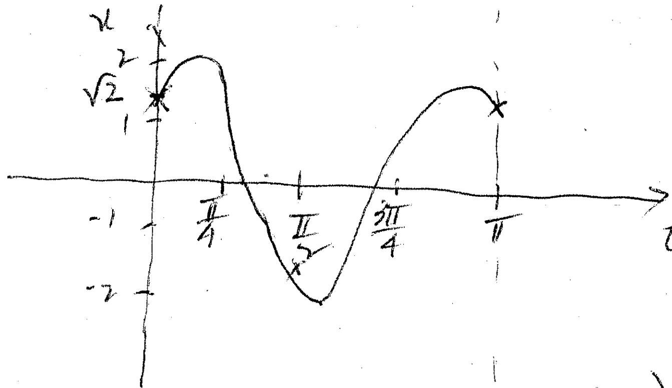
LHS $= 16\sin^2\left(2t-\frac{\pi}{4}\right) + 16\cos^2\left(2t-\frac{\pi}{4}\right)$

$= 16\left[\sin^2\left(2t-\frac{\pi}{4}\right) + \cos^2\left(2t-\frac{\pi}{4}\right)\right]$

$= 16\times 1$

$\therefore$ LHS = RHS

(ii)



when $t=0$ $x = 2\cos\left(-\frac{\pi}{4}\right)$

$= 2\cos\frac{\pi}{4}$

$= 2\times\frac{1}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

$t = \pi$

$= 2\cos\left(2\pi - \frac{\pi}{4}\right)$

$= 2\cos\frac{\pi}{4}$

$= \sqrt{2}$

$t = \frac{\pi}{2}$

$2\cos\left(\pi - \frac{\pi}{4}\right)$

$= 2\times\frac{1}{\sqrt{2}}$

$= -\sqrt{2}$

(iii) period $= \frac{2\pi}{2}$

$= \pi$

let $x = \sqrt{2}$

$\sqrt{2} = 2\cos\left(2t - \frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2} = \cos\left(2t - \frac{\pi}{4}\right)$

$\frac{1}{\sqrt{2}} = \cos\left(2t - \frac{\pi}{4}\right)$

$\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$2t - \frac{\pi}{4} = \frac{\pi}{4}$

$2t = \frac{\pi}{2}$

$t = \frac{\pi}{4}$

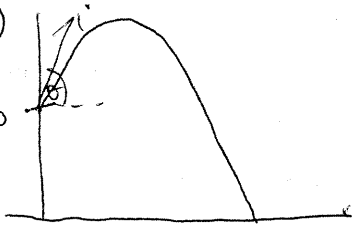
$\therefore \frac{1}{4}$ OF PERIOD

(iv) EACH OSCILLATION TRAVELS 8m.

$100 \div 8 = 12\frac{1}{2}\times$ PERIOD

$\therefore$ TIME $\frac{25\pi}{2}$ seconds

b) i)



$x = 40\cos\theta$

$\dot{y} = 40\sin\theta$

$t = 0$

$\ddot{x} = 0$

$\dot{x} = c$

Initially $\dot{x} = 40\cos\theta$

$\dot{x} = 40\cos\theta$

$x = 40t\cos\theta + c$ ✓

$t = 0 \quad x = 0 \quad c = 0$ ✓

$x = 40t\cos\theta$

$\ddot{y} = -10$

$\dot{y} = -10t + c$

$t = 0 \quad \dot{y} = 40\sin\theta$

$c = 40\sin\theta$

$\dot{y} = -10t + 40\sin\theta$

$y = -5t^2 + 40t\sin\theta + c$ ✓

$t = 0 \quad y = 50 \quad c = 50$

$y = -5t^2 + 40t\sin\theta + 50$

ii) $x = 40t\cos\theta$

$t = \dfrac{x}{40\cos\theta}$ ✓

$y = -5\left(\dfrac{x}{40\cos\theta}\right)^2 + 40 \times \left(\dfrac{x}{40\cos\theta}\right)\sin\theta + 50$

at $y = 0 \quad x = 200$ ✓

$0 = -5\left(\dfrac{200}{40\cos\theta}\right)^2 + 200\tan\theta + 50$

$125\tan^2\theta - 200\tan\theta + 75 = 0$

$5\tan^2\theta - 8\tan\theta + 3 = 0$

$\tan\theta = 1, \frac{3}{5}$ ✓

$\theta = 45^\circ \left(\frac{\pi}{4}\right), 30^\circ 58'$ ✓

c) i) $4\tan^{-1}x + x - 4 = 0$

at $x = 1 \quad 4\tan^{-1}1 + 1 - 4 = \pi - 3$ ✓

≈ 0.14 close to 0 $\therefore$ root near 1

ii) $x_1 = 1 - \dfrac{f(1)}{f'(1)}$

$f'(x) = \dfrac{4}{1+x^2} + 1$ ✓

$f'(1) = \frac{4}{2} + 1 = 3$

$= 1 - \dfrac{\pi - 3}{3}$ ✓

≈ 0.953 ✓