Student Name: ________________________

2014

Year 12 Extension 1 Mathematics
Trial Examination

Teacher Setting Paper: Mrs Northam
Head of Department: Mrs Hill

General Instructions
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen (Black pen is preferred)
- Board approved calculator may be used
- Write your answers for Section I on the multiple answer sheet provided
- Write your answers to Section II on the paper provided. Start a new sheet for each question
- Write your student number only at the top of each page
- A table of standard integrals is provided at the back of this paper
- In Questions 11- 14, show relevant mathematical reasoning and/or calculations.

Total marks – 70

Section I – Multiple Choice

10 marks

Attempt Questions 1-10

Allow 15 minutes for this section

Section II – Extended Response

60 marks

Attempt questions 11 - 14

Allow 1 hours and 45 minutes for this section

This examination paper does not necessarily reflect the content or format of the Higher School Certificate Examination in this subject

Section I

10 marks
Attempt Questions 1 – 10

Use the multiple-choice answer sheet for Questions 1 - 10.

1. What is the value of $\lim_{x\to 0}\dfrac{\sin 3x}{2x}$

(A) 0

(B) $\dfrac{2}{3}$

(C) 1

(D) $\dfrac{3}{2}$

2. The remainder obtained when $P(x) = x^3 - 3x^2 - 5x + 6$ is divided by $x - 3$ will be

(A) -34

(B) -9

(C) 6

(D) 21

3. How many distinct permutations of the letters of the word 'ATTAINS' are possible in a straight line when the word begins and ends with the letter T?

(A) 60

(B) 120

(C) 360

(D) 1260

4. What is the sixth term in the expansion of $(2x-3y)^9$?

(A) ${}^9C_3 \times 2^6 \times (-3)^3 x^6 y^3$

(B) ${}^9C_4 \times 2^5 \times (-3)^4 x^5 y^4$

(C) ${}^9C_5 \times 2^4 \times (-3)^5 x^4 y^5$

(D) ${}^9C_6 \times 2^3 \times (-3)^6 x^3 y^6$

5. The derivative of $\tan^{-1} 2x$ is

(A) $\dfrac{2}{4+x^2}$

(B) $\dfrac{2}{1+4x^2}$

(C) $\dfrac{1}{4+x^2}$

(D) $\dfrac{1}{1+4x^2}$

6. Which of the following is the range of the function $y = 4\cos^{-1} 3x$

(A) $-\dfrac{1}{3} \le y \le \dfrac{1}{3}$

(B) $-\dfrac{1}{4} \le y \le \dfrac{1}{4}$

(C) $0 \le y \le \dfrac{\pi}{4}$

(D) $0 \le y \le 4\pi$

7. Consider the function $f(x) = x^2 - 6x$. Which of the following gives the correct domain of $f(x)$ for which there exists an inverse function, $f^{-1}(x)$.

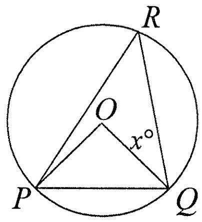
(A) All real x

(B) $0 \le x \le 6$

(C) $x \le 3$ *or* $x \ge 3$

(D) $x \ge 0$

8. Consider the diagram below.



PQ is a chord of a circle, centre O, and R is a point on the major arc.

If $\angle OPR = 5°, \angle OQP = 40°$, then the value of x is:

(A) 30

(B) 35

(C) 40

(D) 45

9. A particle is moving in a straight line with $v^2 = 36 - 4x^2$ and undergoing simple harmonic notion. If the particle is initially at the origin, which of the following is the correct equation for its displacement in terms of t?

(A) $x = 2\sin 3t$

(B) $x = 3\sin 2t$

(C) $x = 2\sin 9t$

(D) $x = 3\sin 4t$

10. Eden, Toby and four friends arrange themselves at random in a circle. What is the probability that Eden and Toby are *not* together?

(A) $\dfrac{1}{120}$

(B) $\dfrac{2}{5}$

(C) $\dfrac{3}{5}$

(D) $\dfrac{119}{120}$

END OF SECTION I

Section II

90 marks
Attempt Questions 11-14

Write you answers on the paper provided

Question 11 (15 marks) **Start a new page.**

(a) Find the coordinates of the point P, which divides the interval from A(-1,8) to B(13,3) internally in the ratio 5:2. **2**

(b) Find to the nearest minute, the acute angles between the lines **2**

$y = -\frac{x}{3} + 4$ and $y = x + 1$

(c) Using the substitution $u = e^{2x}$ find the value of $\int \frac{e^{2x}}{1 + e^{4x}} dx$ **3**

(d) Solve $\frac{3}{2x - 4} > -2$ **3**

(e) Differentiate $e^x \tan^{-1} 3x$ **2**

(f) The polynomial equation $2x^3 - 3x^2 + 4x - 7 = 0$ has roots α, β and γ. **3**

Find the exact value of $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$.

Question 12 (15 marks) **Start a new page.**

(a) Consider the function $f(x)=2\cos^{-1}\frac{x}{3}$

(i) Evaluate $f(0)$. **1**

(ii) State the domain and range of this function. **2**

(ii) Sketch the graph of $f(x)=2\cos^{-1}\frac{x}{3}$ over the stated domain. **3**

(b) Find $\int \sin^2 2x\,dx$ **2**

(c) Find the value of the term independent of x in the expansion of $\left(x^2-\frac{1}{x}\right)^{12}$ **3**

(d) The chord PQ of the parabola $x^2=4y$ subtends a right angle at the origin O.
If the coordinates of P and Q are $(2p,p^2)$ and $(2q,q^2)$ respectively:

(i) Find the gradients of PO and QO. **1**

(ii) Show that $pq=-4$. **1**

(iii) Find the equation of the locus of the midpoint M of PQ. **2**

Question 13 (15 marks) **Start a new page.**

(a) Find $\int \frac{1}{\sqrt{3-4x^2}}\,dx$ **2**

(b) At time t minutes, the temperature $T°$ Celsius of an object is given by $T = 24 - 22e^{-kt}$ where k is a constant.

After 5 minutes the temperature of the object has risen from $2°C$ to $13°C$.

(i) Find the exact value of k **2**

(ii) Find the temperature of the object after 10 minutes. **1**

(c) When a particle is x metres from the origin, its velocity, $v\ ms^{-1}$, is given by **2**

$$v = \sqrt{8-2x^2}\ .$$

Find the acceleration when the particle is 2 metres to the right of the origin.

(d) A computer animation shows the sides of a cube increasing at a rate of 3mm/s. **3**

Find the rate at which the volume V is increasing when the cube has a side length of 5mm.

(e) Find the exact value of $\cos\left(\sin^{-1}(-\frac{1}{3})\right)$ **2**

(f) Use the Principle of Mathematical Induction to prove that, for all positive integers, n, **3**

$$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{n}{4n+1}$$

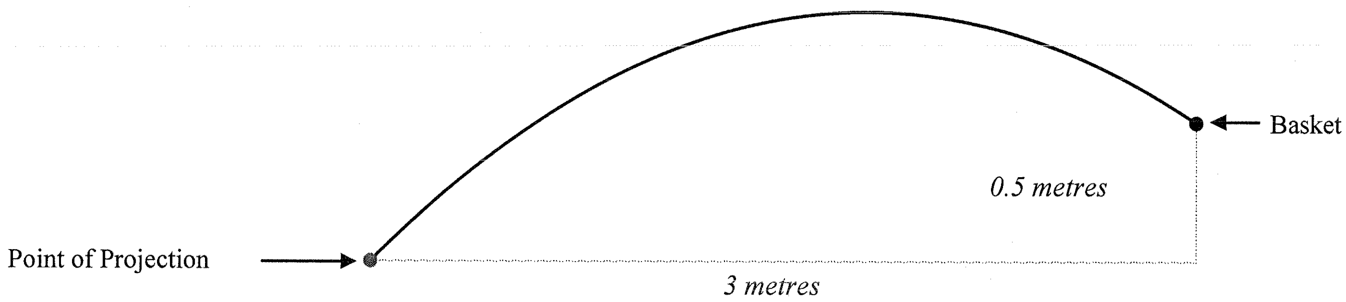
Question Fourteen (15 marks) **Start a new page.**

(a) Greg is about to have a shot at goal in a game of basketball.

From the point where the ball leaves his hand, the distance to the top of the basket is 3 metres horizontally and 0.5m vertically. Greg shoots at the optimal angle of 45°.

You may assume the equations of motion are

$$x = vt\cos 45^o \text{ and } y = vt\sin 45^o - 5t^2$$ (Do NOT prove this)



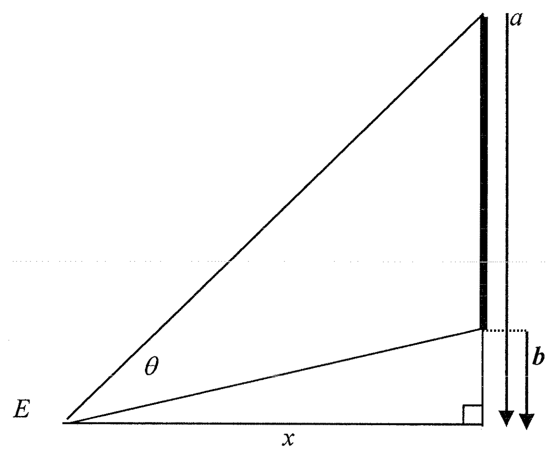
(i) Find the velocity of projection, v, required by Greg for the centre of the ball to land in the centre of the basket. **2**

(ii) Find the maximum vertical height above the basket that the ball reaches during Greg's shot. **2**

(iii) Find the speed of the ball on entry into the basket. **2**

(b) If $(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r$ prove $\sum_{r=1}^{n} r\,{}^nC_r = n.2^{n-1}$ **3**

(c) An observer's eye E looks up at a large painting on a vertical wall. The top of the painting is a metres above the level of E and the bottom of the painting is b metres above the level of E. θ is the angle subtended at the observer's eye by the top and bottom of the painting. E is x metres from the wall. The observer can move backwards and forwards changing x to find the position of best view when θ is a maximum.



(i) Explain why $\theta = \tan^{-1}\frac{a}{x} - \tan^{-1}\frac{b}{x}$ **1**

(ii) Show that $\frac{d\theta}{dx} = \frac{(a-b)(ab-x^2)}{(a^2+x^2)(b^2+x^2)}$ **3**

(iii) If $a = 3b$, find the maximum possible value for θ. **2**

END OF SECTION II

Student's Name/Number: ______________________________

2014

Year 12 Extension 1 Mathematics

Yearly Examination

MULTIPLE CHOICE ANSWER SHEET

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Ⓐ Ⓑ Ⓒ Ⓓ

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Ⓐ Ⓑ Ⓒ Ⓓ

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. Ⓐ Ⓑ Ⓒ Ⓓ

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Ⓐ Ⓑ Ⓒ Ⓓ

9. Ⓐ Ⓑ Ⓒ Ⓓ

10. Ⓐ Ⓑ Ⓒ Ⓓ

Standard Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} coxax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

2014 Extension 1 Trial Paper Solutions.

1. $\lim_{x \to 0} \frac{\sin 3x}{2x}$

$= \frac{3}{2} \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= \frac{3}{2}$ (D)

2. $P(3) = 3^3 - 3\times 3^2 - 5\times 3 + 6$

$= -9$ (B)

3. [T] $\frac{5!}{2!}$ [T]

$= 60$ (A)

4. ${}^9C_5$ ...... (C)

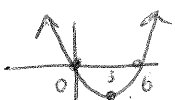
5. $\frac{1}{1+(2x)^2} \times 2$

$= \frac{2}{1+4x^2}$ (B)

6. $0 \le \frac{y}{4} \le \pi$

$0 \le y \le 4\pi$. (D)

7. $f(x) = x^2 - 6x$

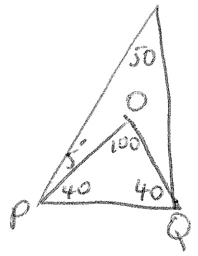


(C)

$x \le 3$ $\quad$ $x \ge 3$

Horizontal Straight line test

8.



$45 + 50 + 40 + x = 180$

$x = 45$

(D)

9. $v^2 = 36 - 4x^2$ $\quad$ $t=0$ $x=0$

$v^2 = 4(9 - x^2)$ $\quad$ $v^2 = n^2(a^2 - x^2)$

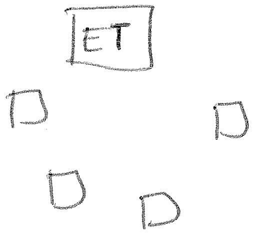
$\therefore n = 2 \quad a = 3$

$\therefore x = a\sin nt$

$x = 3\sin 2t$ (B)

10. All ways in a circle $5! = 120$

Eden + Toby together



$4! \times 2$.

$\therefore$ Eden + Toby not together

$120 - 4! \times 2 = 72$

$\frac{72}{120} = \frac{3}{5}$ (C)

1. D
2. B
3. A
4. C
5. B
6. D
7. C
8. D
9. B
10. C

Question 11

(a) $(-1,8)$ $(13,3)$

$5:2$

$$\left(\frac{5\times13+2\times-1}{5+2}, \frac{5\times3+2\times8}{5+2}\right)$$

$$\left(9, \frac{31}{7}\right) \text{ or } \left(9, 4\tfrac{3}{7}\right)$$

(b) $\tan\theta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$

$m_2 = -\frac{1}{3}$ $m_1 = 1$

$$\tan\theta = \left|\frac{1 - -\frac{1}{3}}{1 + -\frac{1}{3}\times1}\right|$$

$$\tan\theta = \left|\frac{\frac{4}{3}}{\frac{2}{3}}\right|$$

$$\tan\theta = 2$$

$63°26'5.82$

$= 63°26'$

(c) $u = e^{2x}$

$$\frac{du}{dx} = 2e^{2x}$$

$$\frac{1}{2}\int \frac{2e^{2x}}{1+e^{4x}}\,dx$$

$$\frac{1}{2}\int \frac{1}{1+u^2}\,du$$

$$= \frac{1}{2}\tan^{-1}u + C$$

$$= \frac{1}{2}\tan^{-1}e^{2x} + C$$

(d) $\frac{3}{2x-4} > -2$

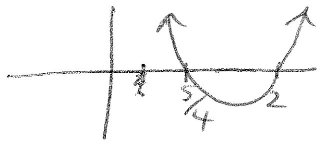
$$(2x-4)^2 \times \frac{3}{2x-4} > -2(2x-4)^2$$

$$3(2x-4) > -2(2x-4)^2$$

$$3(2x-4) + 2(2x-4)^2 > 0$$

$$(2x-4)[3+2(2x-4)] > 0$$

$$2(x-2)[4x-5] > 0$$



$x < \frac{5}{4}$ $x > 2$

(e) Product rule.

$$e^x \tan^{-1}3x + e^x \times \frac{1}{1+(3x)^2}\times 3$$

$$e^x \tan^{-1}3x + \frac{3e^x}{1+9x^2}$$

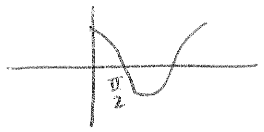
(f) $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$

$$= \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma}$$

$$= \frac{--\frac{3}{2}}{--\frac{7}{2}} = \frac{3}{7}$$

Question 12

(a)(i) $f(0) = 2\cos^{-1} 0$



$f(0) = 2 \times \frac{\pi}{2} = \pi$

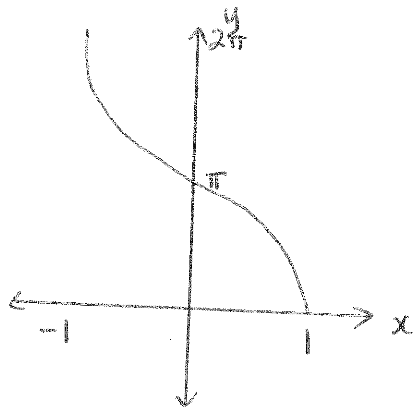
(ii) Domain: $-1 \le \frac{x}{3} \le 1$

$-3 \le x \le 3$

Range $0 \le \frac{y}{2} \le \pi$

$0 \le y \le 2\pi$

(iii)



(b) $\cos 4x = 1 - 2\sin^2 2x$

$2\sin^2 2x = 1 - \cos 4x$

$\sin^2 2x = \frac{1}{2}(1 - \cos 4x)$

$\therefore \int \sin^2 2x = \frac{1}{2}\int 1 - \cos 4x \, dx$

$\frac{1}{2}\left[x - \frac{\sin 4x}{4}\right] + C$

$= \frac{1}{2}x - \frac{\sin 4x}{8} + C$

(c) ${}^{12}C_k \left(x^2\right)^{12-k}\left(-\frac{1}{x}\right)^k$

$(x^2)^{12-k} \times (x^{-1})^k$

$24 - 2k - k = 0$

$24 - 3k = 0$

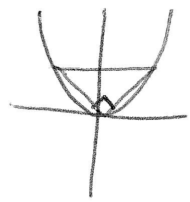
$k = 8.$

${}^{12}C_8 \, (-1)^8$

$= {}^{12}C_8 = 495$

(d)(i) $m_{PO} = \frac{p^2 - 0}{2p - 0}$

$= \frac{p}{2}$



$m_{QO} = \frac{q^2 - 0}{2q} = \frac{q}{2}$

(ii) If the chord subtends a right angle at the origin

$m_1 \times m_2 = -1$

$\frac{p}{2} \times \frac{q}{2} = -1$

$pq = -4$

(iii) $M\left(\frac{2p+2q}{2}, \frac{p^2+q^2}{2}\right)$

$\left(p+q, \frac{p^2+q^2}{2}\right)$

$x = p+q \qquad y = \frac{(p+q)^2 - 2pq}{2}$

$\therefore y = \frac{x^2}{2} - 2x - 4$

$y = \frac{x^2}{2} + 8$

$2y = x^2 + 16$

$x^2 = 2y - 16$

Question 13

(f) $\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{n}{4n+1}$

Step 1:
Prove true for $n=1$.

LHS $\frac{1}{(4\times1-3)(4\times1+1)} = \frac{1}{1\times5}$
$= \frac{1}{5}$

RHS $\frac{1}{4\times1+1} = \frac{1}{5}$

LHS = RHS
∴ true for $n=1$

Step 2: Assume true for $n=k$

$$\frac{1}{(4\times1-3)(4\times1+1)} + \frac{1}{(4\times2-3)(4\times2+1)} + \cdots \frac{1}{(4k-3)(4k+1)} = \frac{k}{4k+1}$$

Step 3: Prove true for $n=k+1$

$$\frac{1}{(4\times1-3)(4\times1+1)} + \cdots + \frac{1}{(4k-3)(4k+1)} + \frac{1}{(4(k+1)-3)(4(k+1)+1)} = \frac{k+1}{4(k+1)+1}$$

$$\frac{1}{(4\times1-3)(4\times1+1)} + \cdots + \frac{1}{(4k-3)(4k+1)} + \frac{1}{(4k+1)(4k+5)} = \frac{k+1}{4k+5}$$

LHS
$$\underbrace{\frac{1}{(4\times1-3)(4\times1+1)} + \cdots + \frac{1}{(4k-3)(4k+1)}} + \frac{1}{(4k+1)(4k+5)}$$

$$= \frac{k}{4k+1} + \frac{1}{(4k+1)(4k+5)}$$

$$= \frac{k(4k+5) + 1}{(4k+1)(4k+5)}$$

$$= \frac{4k^2+5k+1}{(4k+1)(4k+5)}$$

$$= \frac{(4k+1)(k+1)}{(4k+1)(4k+5)} = \frac{k+1}{4k+5} = \text{RHS}$$

Since $n=1$ is true then $n=2$ is true etc

∴ Proved by Mathematical Induction.

Question 13

(a) $\int \frac{1}{\sqrt{3-4x^2}}\,dx$

$\int \frac{1}{\sqrt{4(\frac{3}{4}-x^2)}}\,dx$

$\frac{1}{2}\int \frac{1}{\sqrt{\frac{3}{4}-x^2}}\,dx$

$\frac{1}{2}\sin^{-1}\frac{x}{\frac{\sqrt{3}}{2}} + C$

$\frac{1}{2}\sin^{-1}\frac{2x}{\sqrt{3}} + C$

(b)(i) $T = 24 - 22e^{-kt}$

$t = 5$ $\quad 13 = 24 - 22e^{-5k}$

$T = 13°C$ $\quad -11 = -22e^{-5k}$

$e^{-5k} = \frac{1}{2}$

$\ln e^{-5k} = \ln\frac{1}{2}$

$-5k = \ln\frac{1}{2}$

$k = \frac{\ln\frac{1}{2}}{-5}$

$k = \frac{\ln 2^{-1}}{-5}$

$k = \frac{-\ln 2}{-5}$

$k = \frac{\ln 2}{5}$

(ii) $T = 24 - 22e^{\frac{-\ln 2}{5}\times 10}$

$T = 18.5°C$

(c) $v = \sqrt{8-2x^2}$ $\quad v>0$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}(8-2x^2)\right)$

$\ddot{x} = \frac{d}{dx}\left(4-x^2\right)$

$\ddot{x} = -2x$

When $x = 2$

$\ddot{x} = -4\ m/s^2$

(d) Let the side of a cube be s

$\frac{ds}{dt} = 3 \qquad \frac{dV}{dt} = ?$

$\frac{dV}{dt} = \frac{ds}{dt} \times \frac{dV}{ds}$ ✓

$V = s^3$

$\frac{dV}{ds} = 3s^2$ ✓ when $s = 5$.

$= 3\times 5^2$

$= 75.$

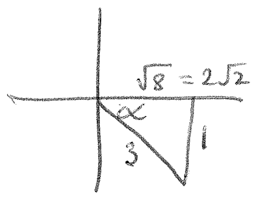
$\therefore \frac{dV}{dt} = 3\times 75$

$= 225\ mm^3/s$ ✓

(e) $\cos\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)$

$\cos\left(-\sin^{-1}\frac{1}{3}\right)$

$= \frac{2\sqrt{2}}{3}$



Question 14

(a) (i) $x = vt\cos 45$

$y = vt\sin 45 - 5t^2$

$x = 3 \quad y = 0.5$

$x = vt\cos 45$

$3 = vt \times \frac{1}{\sqrt{2}}$

$3\sqrt{2} = vt$

$t = \frac{3\sqrt{2}}{v}$

$y = vt\sin 45 - 5t^2$

$0.5 = v\left(\frac{3\sqrt{2}}{v}\right) \times \frac{1}{\sqrt{2}} - 5\left(\frac{3\sqrt{2}}{v}\right)^2$

$0.5 = 3 - 5 \times \frac{9 \times 2}{v^2}$

$-2.5 = -5 \times \frac{18}{v^2}$

$v^2 = \frac{-5 \times 18}{-2.5}$

$v^2 = 36$

$v = 6\ m/s$ since $v > 0$

(ii) Max height $\dot{y} = 0$

$y = vt\sin 45 - 5t^2$

$\dot{y} = v\sin 45 - 10t$

$0 = 6 \times \frac{1}{\sqrt{2}} - 10t$

$10t = \frac{6}{\sqrt{2}}$

$t = \frac{6}{10\sqrt{2}} = \frac{3}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{10}$ seconds.

(ii) continued.

$y = 6 \times \frac{3\sqrt{2}}{10} \times \frac{1}{\sqrt{2}} - 5\left(\frac{3\sqrt{2}}{10}\right)^2$

$y = 0.9m$

Which is $0.4m$ (40cm) above the basket.

(iii) When $x = 3$

$t = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$ seconds.

$\dot{x} = v\cos 45$

$\dot{x} = 6 \times \frac{1}{\sqrt{2}}$

$= \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= 3\sqrt{2}$

$\dot{y} = v\sin 45 - 10t$

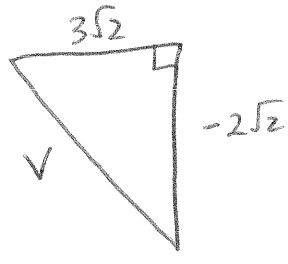
$\dot{y} = 6 \times \frac{1}{\sqrt{2}} - 10 \times \frac{\sqrt{2}}{2}$

$\dot{y} = \frac{6}{\sqrt{2}} - 5\sqrt{2}$

$\dot{y} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} - 5\sqrt{2}$

$\dot{y} = 3\sqrt{2} - 5\sqrt{2}$

$= -2\sqrt{2}$



$v^2 = (3\sqrt{2})^2 + (-2\sqrt{2})^2$

$v^2 = 18 + 8$

$v^2 = 26$

$v = \pm\sqrt{26}\ m/s$

$\therefore$ speed $= \sqrt{26}\ m/s$.

Question 14

(b) $(1+x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r$

$= {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$

Prove $\sum_{r=1}^{n} r\, {}^nC_r = n \times 2^{n-1}$

$\therefore {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + \ldots + n\,{}^nC_n = n2^{n-1}$

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$

Differentiate:

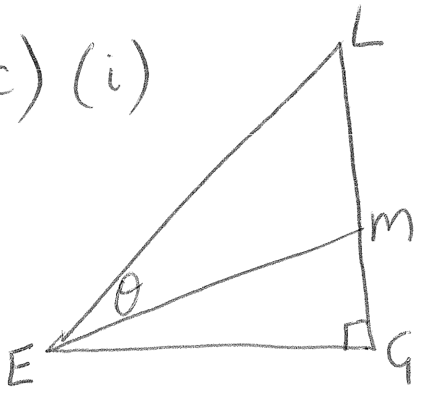
$n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + 3\,{}^nC_3 x^2 + \ldots + n\,{}^nC_n x^{n-1}$

Let $x=1$

$n \times 2^{n-1} = {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + 4\,{}^nC_4 + \ldots n\,{}^nC_n$

$\therefore$ proved.

(c) (i)



$\tan\angle LEG = \frac{a}{x}$ $\quad \therefore \angle LEG = \tan^{-1}\frac{a}{x}$

$\tan\angle MEG = \frac{b}{x}$ $\quad \angle MEG = \tan^{-1}\frac{b}{x}$

$\theta = \angle LEG - \angle MEG$

$= \tan^{-1}\frac{a}{x} - \tan^{-1}\frac{b}{x}$

(ii) $\theta = \tan^{-1}\frac{a}{x} - \tan^{-1}\frac{b}{x}$

$\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{a}{x}\right)^2} \times -ax^{-2} - \frac{1}{1+\left(\frac{b}{x}\right)^2} \times -bx^{-2}$

$$\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{a}{x}\right)^2} \times \frac{-a}{x^2} - \frac{1}{1+\left(\frac{b}{x}\right)^2} \times \frac{-b}{x^2}$$

$$= \frac{1}{1+\frac{a^2}{x^2}} \times \frac{-a}{x^2} - \frac{1}{1+\left(\frac{b}{x}\right)^2} \times \frac{-b}{x^2}$$

$$= \frac{1}{\frac{x^2+a^2}{\cancel{x^2}}} \times \frac{-a}{\cancel{x^2}} - \frac{1}{\frac{x^2+b^2}{\cancel{x^2}}} \times \frac{-b}{\cancel{x^2}}$$

$$= \frac{-a}{x^2+a^2} + \frac{b}{x^2+b^2}$$

$$= \frac{-a(x^2+b^2) + b(x^2+a^2)}{(x^2+a^2)(x^2+b^2)}$$

$$= \frac{-ax^2 - ab^2 + bx^2 + a^2b}{(x^2+a^2)(x^2+b^2)}$$

$$= \frac{-x^2(a-b) + ab(-b+a)}{(x^2+a^2)(x^2+b^2)}$$

$$= \frac{(a-b)\left[-x^2+ab\right]}{(x^2+a^2)(x^2+b^2)}$$

$$= \frac{(a-b)(ab-x^2)}{(x^2+a^2)(x^2+b^2)}$$