2015

Year 12 Extension 1 Mathematics

Trial Examination

Teacher Setting Paper: Mrs M Hill
Head of Department: Mrs M Hill

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board approved calculator may be used
- Write you answers to Section 1 on the multiple choice answer sheet provided
- Write your answers to Section 2 in the answer booklets provided. Start a new booklet for each question
- Write your student number only on the front of each booklet
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section 1 Pages 2-3
10 marks

- Attempt all questions 1-10
- Allow 15 minutes for this section

Section 2 Pages 4-8
60 marks

- Attempt all questions 11-14
- Allow 1 hour and 45 minutes for this section

This examination paper does not necessarily reflect the content or format of the Higher School Certificate Examination in this subject

Section 1

10 marks
Attempt Questions 1 -10
Allow 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 -10

1. Which of the following is an expression for $\int x\sqrt{1-x^2}\,dx$?
   Use the substitution $u = 1 - x^2$.

   (A) $-\frac{(1-x^2)^3}{3}+c$ (B) $\frac{(1-x^2)^3}{3}+c$

   (C) $-\frac{(1-x^2)^{\frac{3}{2}}}{3}+c$ (D) $\frac{(1-x^2)^{\frac{3}{2}}}{3}+c$

2. Which of the following is an expression for $\int \sin^2 6x\,dx$

   (A) $\frac{x}{2}-\frac{1}{12}\sin 6x+c$ (B) $\frac{x}{2}+\frac{1}{12}\sin 6x+c$

   (C) $\frac{x}{2}-\frac{1}{24}\sin 12x+c$ (D) $\frac{x}{2}+\frac{1}{24}\sin 12x+c$

3. A particle is moving along the x-axis. Its velocity v at position x is given by $v=\sqrt{8x-x^2}$.
   What is the acceleration when $x=3$?

   (A) 1 (B) 2

   (C) 3 (D) 4

4. A class consists of 15 students, of whom 5 are prefects. How many committees of 8 students can be formed if each committee contains exactly 2 prefects?

   (A) ${}^5C_2 \times {}^{10}C_6$ (B) ${}^5P_2 \times {}^{10}P_6$

   (C) ${}^{15}C_8$ (D) $2! \times {}^{10}C_6$

5. A particle moves in a straight line and its position at any time t is given by $x = 3\cos 2t + 4\sin 2t$. The motion is simple harmonic. What is the greatest speed?

   (A) 6 (B) 10

   (C) 12 (D) 20

6. If $f(x)=e^{x+2}$ what is the inverse function $f^{-1}(x)$?

(A) $f^{-1}(x)=e^{y-2}$ (B) $f^{-1}(x)=e^{y+2}$

(C) $f^{-1}(x)=\log_e x+2$ (D) $f^{-1}(x)=\log_e x-2$

7. What is the domain and range of $y=\cos^{-1}(\frac{3x}{2})$?

(A) Domain: $-\frac{2}{3}\le x\le\frac{2}{3}$; Range: $0\le y\le\pi$

(B) Domain: $-1\le x\le 1$; Range: $0\le y\le\pi$

(C) Domain: $-\frac{2}{3}\le x\le\frac{2}{3}$; Range: $-\pi\le y\le\pi$

(D) Domain: $-1\le x\le 1$; Range: $-\pi\le y\le\pi$

8. What is the exact value of the definite integral $\int_{\frac{2}{\sqrt{3}}}^{2\sqrt{3}}\frac{dx}{x^2+4}$?

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

9. What is the term independent of x in the expansion of $(x^2-\frac{2}{x})^9$

(A) ${}^9C_3(-2)^3$ (B) ${}^9C_6(-2)^6$

(C) ${}^9C_3(2)^3$ (D) ${}^9C_6(2)^6$

10. The function $f(x)=\sin x-\frac{2x}{3}$ has a real root close to $x=1.5$.

Let $x=1.5$ be a first approximation to the root.

What is the second approximation to the root using Newton's method?

(A) 1.495 (B) 1.496

(C) 1.503 (D) 1.504

Section 2

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

a) Solve for x: $\dfrac{4}{5-x} \geq 1$ 3

b) Find the coordinates of the point which divides the interval AB with $A(1,4)$ and $B(5,2)$ externally in the ratio $1:3$. 2

c) Let all the different arrangement of all the letters of DELETED be called a word.

i) How many words are possible? 1

ii) In how many words of these words will the D's be separated? 2

d) Find the acute angle between the straight lines $y = 2x$ and $x + y - 3 = 0$. Give answer to the nearest minute. 3

e) If $P(x) = x^3 - 2x^2 + ax + 4$ is divisible by $(x+2)$, what is the value of a? 2

f) The polynomial equation $3x^3 - 2x^2 + 3x - 4 = 0$ has roots α, β, γ. Find the exact value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\gamma\alpha}$. 2

Question 12 (15 marks) Use a SEPARATE writing booklet

a) i) Find $\frac{d}{dx}\ln(\cos 2x)$ 1

ii) Hence evaluate exactly $\int_0^{\frac{\pi}{6}} \tan 2x\, dx$ 2

b) For a series $T_{n+1} - T_n = 7$ and $T_1 = 3$. Find the value of S_{100}, where $S_n = T_1 + T_2 + \dots\dots + T_n$. 2

c) i) Write $\cos x - \sqrt{3}\sin x$ in the form $A\cos(x+\alpha)$ where $A > 0$, $0 < \alpha < \pi$. 2

ii) Hence or otherwise, solve $\cos x - \sqrt{3}\sin x = 1$ for all values of x. 2

d) Prove by mathematical induction that if n is a positive integer, then: 3

$$\frac{1}{1\times 5} + \frac{1}{5\times 9} + \frac{1}{9\times 13} + \dots\dots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1}$$

e) i) Sketch the graph of the function $y = \log_e(x-2)$ 1

ii) The region bounded by the curve $y = \log_e(x-2)$, the y-axis, $y = 0$ and $y = h$ is rotated about the y-axis to create a bowl. Show that the volume of the bowl, V, is given by: $V = \pi(\frac{e^{2h}}{2} + 4e^h + 4h - \frac{9}{2})$ 2

Question 13 (15 marks) Use a SEPARATE writing booklet

a)

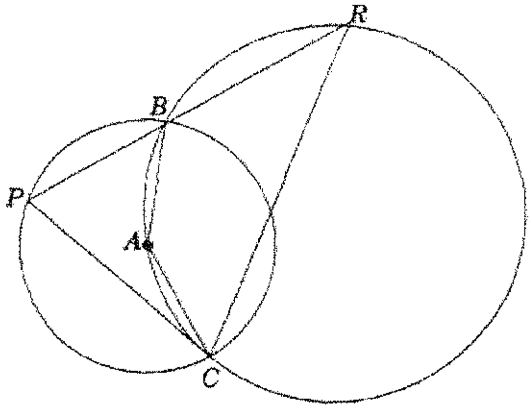


Diagram is not to scale

A is the centre of the circle BCP. The point A lies on another circle BAC. The two circles intersect in B and C as shown in the diagram. PBR is a straight line. 3

Copy or trace this diagram into your writing booklet.

Prove, with reasons, that $RP = RC$.

b) From the top of a mountain 200 metres above ground an observer sights two landmarks A and B. Point A has a bearing of $300°T$ at an angle of depression of $10°$. Point B has a bearing of $040°T$ at an angle of depression of $15°$. Both points A and B are at ground level.

i) Draw and label a diagram showing all the information 1

ii) Calculate the distance from A to B (give answer to the nearest metre) 3

c) Let T be the temperature inside a room at time t hours and let A be the constant outside air temperature. Newton's Law of Cooling states that the rate of change of the temperature T is proportional to $(T-A)$.

i) Show that $T = A + Ce^{kt}$ where C and k are constants satisfies Newton's Law of Cooling $\frac{dT}{dt} = k(T-A)$. 1

ii) The outside temperature is $5°C$ when a system failure causes inside room temperature to drop from $20°$ to $17°$ in half an hour. After how many hours is the inside room temperature equal to $10°C$? Give answer correct to 1 decimal place. 3

d) Let point $P(4p, 2p^2)$ be an arbitrary point on the parabola $x^2 = 8y$ with parameter p.

i) Show that the equation of the tangent at P is $y = px - 2p^2$ 1

ii) The tangent intersects the y-axis at C. The point Q divides CP, internally, in the ratio 1:3. Find the locus of all the points Q as parameter p varies. 3

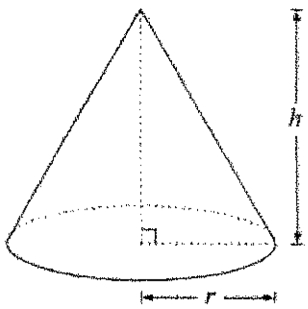
Question 14 (15 marks) Use a SEPARATE writing booklet

a) Steven borrows \$50 000 to pay for a new car. He plans to repay the loan by making 60 equal monthly instalments. Interest is charged at the rate of 0.6% per month on the balance owing.

i) Show that immediately after making two monthly instalments of \$M, the balance owing is given by \$(50601.80 – 2.006M) 1

ii) Calculate the value of each monthly instalment. 2

b) Grain is poured at a constant rate of 0.5 cubic metres per second. It forms a conical pile, with the angle at the apex of the cone equal to $60°$. The height of the pile is h metres, and the radius of the base is r metres.



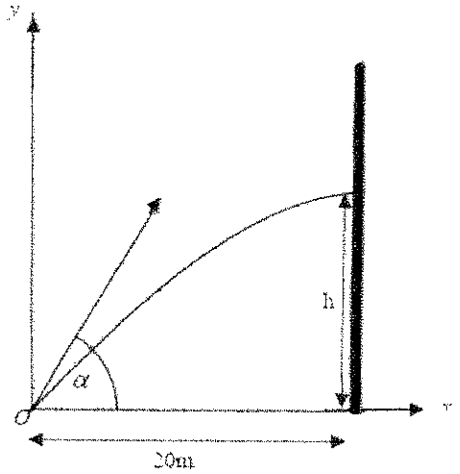
i) Show that $r = \dfrac{h}{\sqrt{3}}$ 1

ii) Show that V, the volume of the pile, is given by $V = \dfrac{\pi h^3}{9}$. 1

iii) Hence find the rate at which the height of the pile is increasing when the height of the pile is 3 metres. 2

Question 14 continued on next page

c) A softball player hits the ball from ground level with a speed of 20 m/s and an angle of elevation α . It flies towards a high wall 20 m away on level ground. Taking the origin at the point where the ball is hit, the derived expressions for the horizontal and vertical components of x and y of displacement at the time t seconds, taking $g = 10$ m/s^2, are $x = 20t\cos\alpha$ and $y = -5t^2 + 20t\sin\alpha$



i) Hence find the equation of the path of the ball in flight in terms of x, y and α . 1

ii) Show that the height h at which the ball hits the wall is given by $h = 20\tan\alpha - 5(1+\tan^2\alpha)$ 1

iii) Using part ii) above, show that the maximum value of h occurs when $\tan\alpha = 2$ and find this maximum height. 2

d) Using the expansion of $(1+x)^n$ prove:

i) $10^n = \binom{n}{0} + 3^2\binom{n}{1} + 3^4\binom{n}{2} + ... + 3^{2n}\binom{n}{n}$ 1

ii) Hence show that $1 + 3^4\binom{n}{2} + 3^8\binom{n}{4} + ... + 3^{2n}\binom{n}{n} = 2^{n-1}(5^n + 4^n)$, where n is an even integer. 3

End of Examination

2015 **Number: ____________**

Year 12 Extension 1 Mathematics

Trial Examination: Multiple Choice Answer Sheet

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓐ Ⓑ Ⓒ Ⓓ
7. Ⓐ Ⓑ Ⓒ Ⓓ
8. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ
10. Ⓐ Ⓑ Ⓒ Ⓓ

Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x > 0$

Extension 1 2015 Trial

Section 1 multiple choice

(1) $\int x\sqrt{1-x^2}\,dx$ $u = 1-x^2$, $du = -2x\,dx$, $-\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\int u^{1/2}\,du$

$= -\frac{1}{2} \times \frac{2}{3}u^{3/2} + c$

$= -\frac{1}{3}u^{3/2} + c$

$= -\frac{1}{3}\sqrt{(1-x^2)^3} + c$ (C)

(2) $\int \sin^2 6x\,dx$

$= \int \frac{1}{2} - \frac{1}{2}\cos 12x\,dx$

$= \frac{x}{2} - \frac{1}{24}\sin 12x + c$ (C)

(3) $v = \sqrt{8x - x^2}$

$v^2 = 8x - x^2$

$\frac{1}{2}v^2 = 4x - \frac{1}{2}x^2$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4 - x$

$\ddot{x} = 4 - x$

when $x = 3$

$\ddot{x} = 1$ (A)

(4) ${}^5C_2 \times {}^{10}C_6$

There are 5 prefects to chose 2 from ${}^5C_2$

leaving 10 students to choose the other 6 ${}^{10}C_6$

(A)

(5) $x = 3\cos 2t + 4\sin 2t$

greatest speed occurs at centre of motion

$x = 5\cos\left(2t - \tan^{-1}\left(\frac{4}{3}\right)\right)$

$\dot{x} = -10\sin\left(2t - \tan^{-1}(4/3)\right)$

$\therefore$ max speed is 10 (B)

(6) $y = e^{x+2}$

$x = e^{y+2}$

$\ln x = y + 2$

$\ln x - 2 = y$. (D)

(7) $y = \cos^{-1}\frac{3x}{2}$

Domain $-1 \le \frac{3x}{2} \le 1$

$-\frac{2}{3} \le x \le \frac{2}{3}$

Range $0 \le y \le \pi$ (A)

(8) $\int_{2/\sqrt{3}}^{2\sqrt{3}} \frac{dx}{x^2+4}$

$\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_{2/\sqrt{3}}^{2\sqrt{3}}$

$= \frac{1}{2}\left[\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right]$

$= \frac{1}{2}\left[\frac{\pi}{3} - \pi/6\right]$

$= \frac{1}{2} \cdot \pi/6$

$= \pi/12$ (A)

(9) $(x^2 - 2/x)^9$

${}^9C_k (x^2)^{9-k}\left(\frac{-2}{x}\right)^k$

${}^9C_k\, x^{18-2k} \, -2^k \cdot x^{-k}$

${}^9C_k\, x^{18-3k} \cdot (-2)^k$

$\therefore 18 - 3k = 0$

$k = 6$

${}^9C_6\,(-2)^6$ (B)

(10) $f(x) = \sin x - \frac{2x}{3}$

$f'(x) = \cos x - \frac{2}{3}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1.5 - \frac{\sin 1.5 - \frac{2}{3}\times 1.5}{\cos 1.5 - \frac{2}{3}}$

$= 1.49579\ldots$

≈ 1.496 (B)

Section 2

Question 11

a) $\frac{4}{5-x} \geq 1$

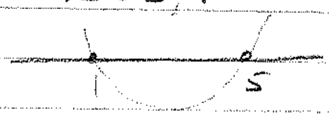
$4(5-x) \geq (5-x)^2$

$0 \geq (5-x)^2 - 4(5-x)$

$0 \geq (5-x)(5-x-4)$

$0 \geq (5-x)(1-x)$

c.v. $x = 5, 1$ ✓✓



✓

$1 \leq x < 5$

b) $A(1,4)$ $B(5,2)$

$-1:3$

$x = \frac{3(1) + (5)-1}{2}$ $y = \frac{3(4) + (-1)2}{2}$

$= -1$ $= 5$

$\therefore (-1,5)$

c) i) DELETED

$\frac{7!}{3!\times 2!}$

$= 420$

ii) Consider the D's together

(DD) _ _ _ _

$\frac{6!}{3!} = 120$

$\therefore$ the number of words with the D's seperated is

$420 - 120 = 300$

d) $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ $m_1 = 2$, $m_2 = -1$

$= \left|\frac{2+1}{1-2}\right|$

$= \left|\frac{-3}{1}\right|$

$= 3$

$\theta = 71^\circ 34'$

e) $P(-2) = 0$

$(-2)^3 - 2(-2)^2 + -2a + 4 = 0$

$-8 - 8 - 2a + 4 = 0$

$a = 6$

f) $\alpha + \beta + \gamma = \frac{2}{3}$

$\alpha\beta + \beta\gamma + \alpha\gamma = 1$

$\alpha\beta\gamma = \frac{4}{3}$

$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma}$

$= \frac{2/3}{4/3}$

$= \frac{1}{2}$

Question 12:

a) i) $\frac{d}{dx} \ln(\cos 2x)$

$= \frac{1}{\cos 2x} \cdot -2\sin 2x$

$= -2\tan 2x$ ✓

ii) $\therefore \int -2\tan 2x\, dx = \ln(\cos 2x) + c$

$-2\int_0^{\pi/6} \tan 2x\, dx = \left[\ln(\cos 2x)\right]_0^{\pi/6}$ ✓

$= -\frac{1}{2}\left(\ln(\cos\frac{\pi}{3}) - \ln(\cos 0)\right)$

$= -\frac{1}{2}\left(\ln\left(\frac{1}{2}\right) - \ln 1\right)$

$= -\frac{1}{2}\ln\left(\frac{1}{2}\right) = \frac{1}{2}\ln\left(\frac{1}{2}\right) = \frac{1}{2}\ln 2$

b) $T_{n+1} - T_n = 7 \quad T_1 = 3$

$T_2 - T_1 = 7$

$T_2 = 10$

$\therefore$ AP $a = 3$ $d = 7$ ✓

$S_n = \frac{n}{2}(a + l) \quad T_{100} = 3 + 99 \times 7$

$= 696$

$S_{100} = 50(3 + 696)$

$= 34950$ ✓

c) i) $\cos x - \sqrt{3}\sin x$

$r = \sqrt{3+1} \quad \tan\alpha = \sqrt{3}$

$= 2 \quad \alpha = \pi/3$

✓ ✓

$\therefore \cos x - \sqrt{3}\sin x = 2\cos\left(x + \frac{\pi}{3}\right)$

ii) $2\cos\left(x + \frac{\pi}{3}\right) = 1$

$\cos\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

$x + \pi/3 = 2n\pi \pm \cos^{-1}\left(\frac{1}{2}\right)$

$= 2n\pi \pm \frac{\pi}{3}$ ✓

$= 2n\pi$ or $2n\pi - \frac{2\pi}{3}$

✓

d) show true for $n = 1$

LHS $= \frac{1}{5}$ RHS $= \frac{1}{4+1}$

$= 1/5$ ✓

$\therefore$ true for $n=1$

assume true for $n = k$

$S_k = \frac{k}{4k+1}$

Prove true for $n = k+1$

$S_k + T_{k+1} = \frac{k+1}{4(k+1)+1}$

$= \frac{k+1}{4k+5}$ ✓

LHS $= \frac{k}{4k+1} + \frac{1}{(4(k+1)-3)(4(k+1)+1)}$

$= \frac{k}{4k+1} + \frac{1}{(4k+1)(4k+5)}$

$= \frac{k(4k+5)+1}{(4k+1)(4k+5)}$

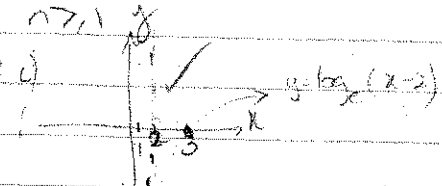
$= \frac{4k^2+5k+1}{(4k+1)(4k+5)}$

$= \frac{(4k+1)(k+1)}{(4k+5)(4k+1)}$ ✓

$= \frac{k+1}{4k+5}$

$\therefore$ True for $n=1$, whenever true for $n=k$ it is true for $n=k+1$ $\therefore$ true for all integers $n \geq 1$

e) i)



✓

ii) $V = \pi\int_0^h x^2\, dy \quad e^y = x - 2$

$e^y + 2 = x$

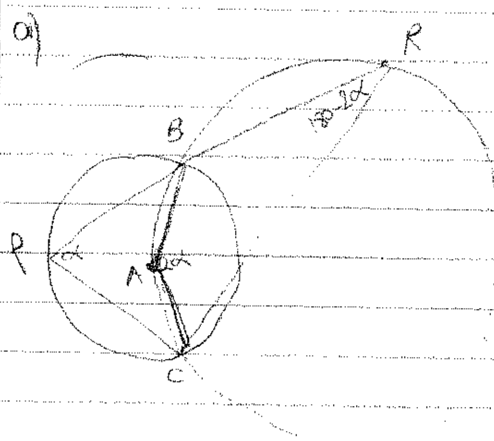
$V = \pi\int_0^h e^{2y} + 4e^y + 4\, dy$ ✓

$= \pi\left[\frac{e^{2y}}{2} + 4e^y + 4y\right]_0^h$ ✓

[illegible]

Question 13:

a)



Let $\angle PBC = \alpha$

$\therefore \angle BAC = 2\alpha$ (angle at centre is twice the angle on the circumference)

$\angle BRC = 180 - 2\alpha$ (ACBR is cyclic $\therefore$ opposite angles supplementary)

In $\triangle PRC$

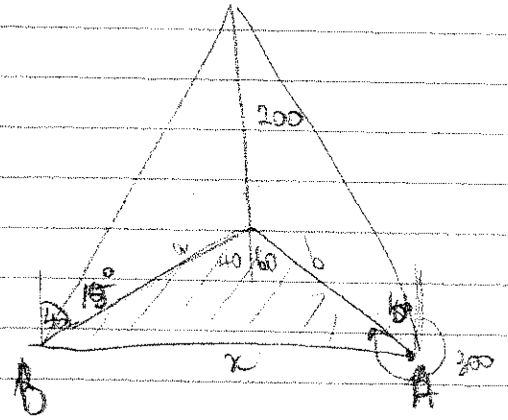
$\angle PCR = 180 - \alpha - (180 - 2\alpha)$

$= \alpha$

$\therefore \triangle PRC$ is isosceles (base angles equal)

$\therefore PR = PC$ (equal sides of isosceles $\triangle$)

b)



$\tan 15° = \frac{200}{a}$ $\quad$ $\tan 10 = \frac{200}{b}$

$a = \frac{200}{\tan 15}$ $\quad$ $b = \frac{200}{\tan 10}$

using cosine rule

$x^2 = \frac{200^2}{\tan^2 15} + \frac{200^2}{\tan^2 10} - 2 \times \frac{200}{\tan 15} \times \frac{200}{\tan 10} \cos 100$

$= 2137693.834$

$x = 1462$ m

c) i) $T = A + Ce^{kt}$

$\frac{dT}{dt} = Cke^{kt}$

$= k(Ce^{kt} - A + A)$

$= k(T - A)$

ii) $T = A + Ce^{kt}$

$T = 5 + Ce^{kt}$

when $t = 0$, $T = 20$

$\therefore C = 15$

$T = 5 + 15e^{kt}$

when $T = 17$ $t = 0.5$

$17 = 5 + 15e^{0.5k}$

$\log_e\left(\frac{12}{15}\right) = 0.5k$

$-0.446287 = k$

let $T = 10$

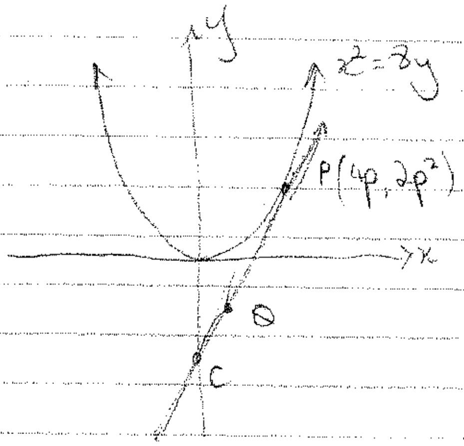
$10 = 5 + 15e^{kt}$

$\log_e\left(\frac{5}{15}\right) = kt$

$\therefore t = 2.46$

$\doteqdot 2.5$ hours

d)



i) $y = \frac{x^2}{8}$

$y' = \frac{x}{4}$

$m = \frac{4p}{4}$

$= p$

$y - 2p^2 = p(x - 4p)$

$y - 2p^2 = px - 4p^2$

$y = px - 2p^2$ as required

ii) $C(0, -2p^2)$ $(4p, 2p^2)$

1:3

$x = \frac{4p}{4}$

$= p$

$y = \frac{-6p^2 + 2p^2}{4}$

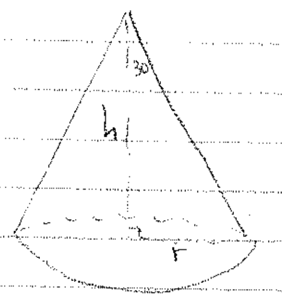
$= \frac{-4p^2}{4}$

$= p^2$

$\therefore$ locus is $y = x^2$

Question 14: a) is on next pg.

b)



i $\tan 30 = \frac{r}{h}$

$h \times \frac{1}{\sqrt{3}} = r$

$\frac{h}{\sqrt{3}} = r$

ii) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \frac{h^2}{3} \times h$

$= \frac{1}{9}\pi h^3$

iii) $\frac{dV}{dt} = 0.5$

$V = \frac{1}{9}\pi h^3$

$\frac{dV}{dh} = \frac{1}{3}\pi h^2$

Find $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{3}{\pi h^2} \; 0.5$

$= \frac{3}{\pi \times 9} \times 0.5$

$= \frac{1}{6\pi}$

c)(i) $x = 20t\cos\alpha \quad y = -5t^2 + 20t\sin\alpha$

$y = -5\left(\frac{x}{20\cos\alpha}\right)^2 + 20\left(\frac{x}{20\cos\alpha}\right)\sin\alpha$

$= \frac{-5x^2}{400\cos^2\alpha} + x\tan\alpha$

$= \frac{-x^2}{80}\sec^2\alpha + x\tan\alpha$

(ii) when $x = 20 \quad y = h$

$h = \frac{-20^2}{80}(1+\tan^2\alpha) + 20\tan\alpha$

$= -\frac{400}{80}(1+\tan^2\alpha) + 20\tan\alpha$

$= 20\tan\alpha - 5(1+\tan^2\alpha)$

(iii) $h = -5\tan^2\alpha + 20\tan\alpha - 5$

maximum value of h occurs at the turning point

$\tan\alpha = -b/2a$

$= \frac{-20}{2(-5)}$

$= 2$

maximum height is

$-5(2)^2 + 20(2) - 5$

$=$ 15 metres

d)

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \dots + {}^nC_n x^n$

let $x = 9$

$10^n = {}^nC_0 + {}^nC_1 9 + {}^nC_2 9^2 + {}^nC_3 9^3 + \dots {}^nC_n 9^n$

$= {}^nC_0 + {}^nC_1 3^2 + {}^nC_2 3^4 + \dots + {}^nC_n 3^{2n}$

$= \binom{n}{0} + 3^2\binom{n}{1} + 3^4\binom{n}{2} + \dots + 3^{2n}\binom{n}{n}$ (1)

(ii)

let $x = -9$ (2)

$8^n = {}^nC_0 - {}^nC_1 3^2 + {}^nC_2 3^4 - {}^nC_3 3^6 \dots + 3^{2n}\binom{n}{n}$

Add (1) + (2)

$10^n + 8^n = 2\,{}^nC_0 + 2\,{}^nC_2 3^4 + \dots 2\times 3^{2n}\,{}^nC_n$

$2^n5^n + 2^n4^n = 2[\binom{n}{0} + 3^4\binom{n}{2} + \dots + 3^{2n}\binom{n}{n}]$

$\frac{2^n(5^n+4^n)}{2} = \binom{n}{0} + 3^4\binom{n}{2} + \dots + 3^{2n}\binom{n}{n}$

$2^{n-1}(5^n+4^n) = 1 + 3^4\binom{n}{2} + 3^8\binom{n}{4} + \dots 3^{2n}\binom{n}{n}$

a) $A_1 = 50000 \times 1.006 - M$

$A_2 = (50000 \times 1.006 - M) \times 1.006 - M$

$= 50000 \times 1.006^2 - M(1.006) - M$

$= 50601.80 - M \times 2.006$

$A_3 = 50000 \times 1.006^3 - M(1.006^2 + 1.006 + 1)$

$\vdots$

$A_{60} = 50000 \times 1.006^n - M(1 + 1.006 + \dots 1.006^{n-1})$

but $A_n = 0$ when paid off. $n = 60$

$0 = 50000 \times 1.006^{60} - M(1 + 1.006 + \dots + 1.006^{59})$

$= 50000 \times 1.006^{60} - M\left(\frac{1(1.006^{60} - 1)}{1.006 - 1}\right)$

$M = \frac{50000 \times 1.006^{60} \times (.006)}{(1.006^{60} - 1)}$

$= \$994.78$