**Question 1** Use a SEPARATE Writing Booklet. **Marks**

(a) Differentiate $x^2 \sin^{-1} x$. **2**

(b) Find the value of $k$ if $x + 3$ is a factor of $P(x) = 2x^3 - 5kx + 9$. **2**

(c) The interval $AB$ has end points $A(-3, 5)$ and $B(3, 2)$. Find the coordinates of the point $P$ which divides the interval $AB$ externally in the ratio 2 : 5. **2**

(d) Find the acute angle, to the nearest degree, between the lines $x + y = 5$ and $2y = 3x + 5$. **2**

(e) Find $\lim_{\theta \to 0} \left( \frac{\sin^2 \theta}{\theta} \right)$. **2**

(f) Use the table of standard integrals to find the exact value of **2**

$$\int_0^{\frac{3}{2}} \frac{dx}{\sqrt{x^2 + 4}}.$$

**Question 2** Use a SEPARATE Writing Booklet. **Marks**

(a) Find the quotient, $Q(x)$, and the remainder, $R(x)$, when the polynomial $P(x) = 2x^4 - 3x^3 + 2x + 1$ is divided by $x^2 + 2x - 1$. **3**

(b) Prove the identity $\dfrac{1 - \tan^2 A}{1 + \tan^2 A} = \cos 2A$. **2**

(c) Find the value of $x$ if $\dfrac{d}{dx}\left(\dfrac{x+2}{\sqrt{x-1}}\right) = 0$. **3**

(d) Use the substitution $u = x - 1$ to evaluate $\displaystyle\int_2^5 \frac{x+1}{\sqrt{x-1}}\,dx$. **4**

**Question 3** Use a SEPARATE Writing Booklet. **Marks**

(a) Solve $\frac{x}{x^2 - 4} \leq 0$. **3**

(b) Solve $|x - 3| \leq 2x + 1$ **3**

(c) Consider the function $f(x) = x \log_e x$.

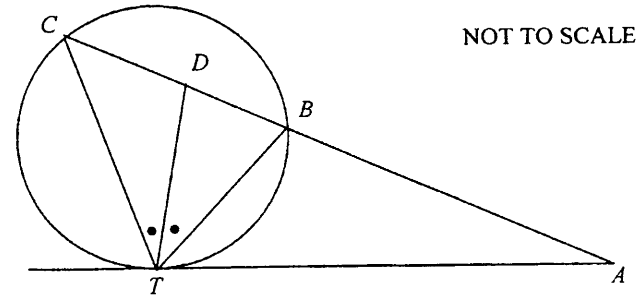
(i) Show that $y = f(x)$ has a minimum turning point at $(\frac{1}{e}, -\frac{1}{e})$. **4**

(ii) Hence sketch the curve of $y = f(x)$ for $x \geq \frac{1}{e}$. **1**

(iii) On the same set of axes as part (ii), draw the graph of the inverse function of $y = f(x)$, $x \geq \frac{1}{e}$. **1**

**Question 4** Use a SEPARATE Writing Booklet. **Marks**

(a) 3



*TA* is a tangent to a circle. Line *ABDC* intersects the circle at *B* and *C*. Line *TD* bisects angle *BTC*.

Prove $AT = AD$.

(b) $P(2ap, ap^2)$ is any point on the parabola $x^2 = 4ay$. The line $l$ is parallel to the tangent at P and passes through the focus, S, of the parabola.

i) Find the equation of the line $l$. 1

ii) The line $l$ intersects the $x$-axis at the point $Q$. Find the coordinates of the midpoint, $M$, of the interval $QS$. 2

iii) What is the equation of the locus of $M$? 1

(c) Equipment being delivered by a parachute drop is falling at a speed of $v$ m s$^{-1}$. When the parachute opens, the equipment is falling at 50 m s$^{-1}$, and thereafter its acceleration is given by $\frac{dv}{dt} = k(2 - v)$, where $k$ is a constant.

(i) Show that this equation for $\frac{dv}{dt}$ is satisfied by $v = 2 + Ae^{-kt}$, where $A$ is a constant. 1

(ii) Find the value of $A$. 1

(iii) One second after the parachute opens, the speed of the equipment has fallen to 35 m s$^{-1}$. Determine the value of $k$ correct to 4 decimal places. 2

(iv) After a period of time, the equipment continues to fall with a speed which is very nearly constant, and which is called the "terminal speed". Find the terminal speed for this particular parachute drop. 1

**Question 5** Use a SEPARATE Writing Booklet. **Marks**

(a) (i) Express $\sqrt{3}\cos 2t - \sin 2t$ in the form $R\cos(2t+\alpha)$, where $0 < \alpha < \frac{\pi}{2}$. **2**

(ii) Hence or otherwise find all positive solutions of $\sqrt{3}\cos 2t - \sin 2t = 0$. **2**

(b) A particle moves in straight line and is $x$ metres from a fixed point $O$ after $t$ seconds, where $x = 5 + \sqrt{3}\cos 2t - \sin 2t$.
Note The results of part (a) may be useful in answering this part.

(i) Prove that the acceleration of the particle is $-4(x-5)$. **2**

(ii) Between which two points does the particle oscillate? **1**

(iii) At what time does the particle first pass through the point $x = 5$? **1**

(c) Use mathematical induction to prove that **4**

$$1^2 + 3^2 + \ldots\ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

for all integers $n = 1, 2, 3, \ldots\ldots\ldots$ .

**Question 6** Use a SEPARATE Writing Booklet. **Marks**

(a) When a particle moving in a straight line has displacement $x$ metres from a fixed point $O$, its acceleration in metres per second per second is given by $\ddot{x} = \sqrt{3x+4}$.

(i) Show that $v^2 = \frac{4}{9}(3x+4)^{\frac{3}{2}} + c$, where $v$ is the velocity of the particle in metres per second, and $c$ is a constant. **1**

(ii) Given that the particle starts from rest at $O$, evaluate $c$. **1**

(iii) Explain why the motion of the particle is always in the positive direction. **1**

(b) A spherical map of the earth is being inflated at a constant rate of $25\text{cm}^3\text{s}^{-1}$. Find the rate at which the length of the equator is changing when the radius is 10cm. **3**

(c) (i) By using graphs or otherwise, show that the curves $y = \ln x$ and $y = 2 - x$ have a point of intersection for which the $x$ coordinate is close to 1.5. **1**

(ii) Use one application of Newton's method to find a better approximation for the $x$ coordinate of this point of intersection, correct to two decimal places. **2**

(d) A solid is formed by rotating the curve $y = 1 + \sqrt{2}\cos x$, between $x = 0$ and $x = \frac{\pi}{4}$, about the $x$-axis. Find the exact volume of the solid. **3**

**Marks**

**Question 7** Use a SEPARATE Writing Booklet.

(a) Write down the last digit of the expansion of $7^{2002}$. (You are only required to write the units digit). **2**

(b) (i) Differentiate $\sin^{-1} x + \cos^{-1} x$ **1**

(ii) Hence, or otherwise explain why $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ **1**

(iii) Find the exact values of $x$ and $y$ which satisfy the simultaneous equations **3**

$$\sin^{-1} x - \cos^{-1} y = \frac{\pi}{12}; \qquad \cos^{-1} x + \sin^{-1} y = \frac{5\pi}{12}$$

(c) A vertical tower subtends angles $\alpha$ and $\beta$ respectively at two points $A$ and $B$ in a horizontal plane through the base of the tower. $A$ is due south of the tower and $B$ is due west.

(i) Draw a diagram to illustrate this information. **1**

(ii) Show that the cosine of the angle subtended at the top of the tower by the line $AB$ is $\sin\alpha \sin\beta$. **4**

## END OF EXAMINATION

# TRIAL EXAM EXT 1 2002.

Q1. a) $y = x^2 \sin^{-1} x$

$\frac{dy}{dx} = \frac{x^2}{\sqrt{1-x^2}} + 2x \sin^{-1} x$

1 for use of product rule
1 for correct answer.

b) $P(x) = 2x^3 - 5kx + 9$

$P(-3) = 0$

$\therefore \ 0 = 2(-3)^3 - 5k(-3) + 9$ 1 for correct subst at this lin

$-54 + 15k + 9 = 0$

$15k = 45$

$k = 3$ 1 for correct answer

c) $A(-3,5)$ $B(3,2)$ $-2:5$

$x = \frac{-2\times 3 + 5\times -3}{-2+5}$

$= \frac{-6-15}{3}$

$= \frac{-21}{3}$

$= -7$

$y = \frac{-2\times 2 + 5\times 5}{3}$

$= \frac{-4+25}{3}$

$= \frac{21}{3}$

$= 7$

1 mark for each co-ordinate

$\therefore$ P is $(-7, 7)$

d) $x + y = 5$

$y = 5 - x$

$m_1 = -1$

$2y = 3x + 5$

$y = \frac{3}{2}x + \frac{5}{2}$

$m_2 = \frac{3}{2}.$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{-1 - \frac{3}{2}}{1 - 3/2}\right|$

$= 5$

1 for correct subst in formula
1 for answer
Allow 1st mark if 1 error in finding grad

Acute $\angle\theta = 79°$ to nearest degree

e) $\lim_{\theta\to 0} \frac{\sin^2\theta}{\theta} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta}\cdot\frac{\sin\theta}{\theta}\cdot\theta$ 1 mark

$= \lim_{\theta\to 0} \theta = 0$ 1mark

f) $\int_0^{3/2} \frac{dx}{\sqrt{x^2+4}} = \left[\ln\left(x + \sqrt{x^2+4}\right)\right]_0^{3/2}$ 1mark

$= \ln 2$ 1mark

Q2)

$$\begin{array}{r|l} & 2x^2 - 7x + 16 \\ \hline x^2+2x-1 & 2x^4 - 3x^3 \quad + 2x + 1 \\ & \underline{2x^4 + 4x^3 - 2x^2} \\ & -7x^3 + 2x^2 + 2x + 1 \\ & \underline{-7x^3 - 14x^2 + 7x} \\ & 16x^2 - 5x + 1 \\ & \underline{16x^2 + 32x - 16} \\ & -37x + 17 \end{array}$$

1 mark each step in the division

$Q(x) = 2x^2 - 7x + 16$, $R(x) = -37x + 17$

b) To prove that $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos 2A$.

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\tan^2 A}{\sec^2 A}$$

1 mark.

$$= \cos^2 A - \frac{\sin^2 A}{\cos^2 A} \times \cos^2 A$$

$$= \cos^2 A - \sin^2 A$$

$$= \cos 2A.$$

1 mark.

c) $$\frac{d}{dx}\left(\frac{x+2}{\sqrt{x-1}}\right) = 0$$

$$\frac{\sqrt{x-1} \times 1 - (x+2) \times \frac{1}{2}(x-1)^{-\frac{1}{2}}}{x-1} = 0$$

$$\frac{2(x-1) - (x+2)}{2(x-1)\sqrt{x-1}} = 0$$

$$\therefore \quad 2x - 2 - x - 2 = 0$$

$$\underline{x = 4}$$

1 for correct derivative.
1 for simplification
1 for answer

d) $$\int_2^5 \frac{x+1}{\sqrt{x-1}}\, dx$$

$u = x - 1$
$du = dx$
$x = 5, \ u = 4$
$x = 2, \ u = 1$

1 for correct s
1 for integral
1 for subst of correct limi
1 for answer

$$= \int_1^4 \frac{u+2}{\sqrt{u}}\, du$$

$$= \int_1^4 \left(\sqrt{u} + 2u^{-\frac{1}{2}}\right) du$$

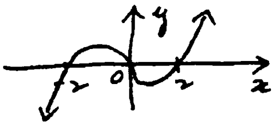
$$= \left[\frac{2}{3}u^{3/2} + 4u^{\frac{1}{2}}\right]_1^4$$

$$= \left(\frac{2}{3} \times 8 + 4 \times 2\right) - \left(\frac{2}{3} + 4\right)$$

$$= \frac{14}{3} + 4$$

$$= 8\frac{2}{3}$$

Q3. a) $\frac{x}{x^2-4} \le 0$

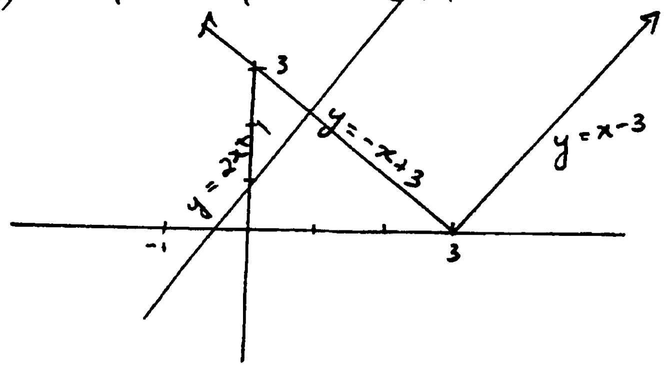


$x(x^2-4) \le 0$

$x(x-2)(x+2) \le 0$ 1 mark.

$\therefore x < -2, \quad 0 \le x < 2$ 2 marks Award 1 mark for $x \le -2, 0 \le$;

b) $|x-3| \le 2x+1$



1 mark each graph

1 correct solution

$2x+1 = -x+3$

$3x = 2$

$x = \frac{2}{3}.$

$\therefore |x-3| \le 2x+1$

$x \geq 2/3$

c) i) $y = x \log_e x$ $\quad x = \frac{1}{e}, \; y = \frac{1}{e}x - 1$

$\frac{dy}{dx} = 1 + \log_e x$

$\frac{d^2y}{dx^2} = \frac{1}{x}.$

For a turning pt. $\frac{dy}{dx} = 0$

$\therefore 1 + \log_e x = 0, \; \log_e x = -1$

$\therefore x = e^{-1}$

$x = \frac{1}{e}, \; \frac{d^2y}{dx^2} = e > 0 \; \therefore$ min tp at $(\frac{1}{e}, -\frac{1}{e})$

1 mark for deriva

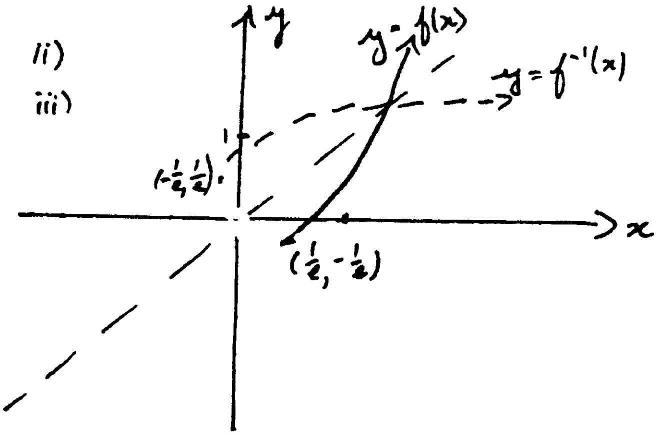
1 mark for $x = e$

1 mark for testing for minima.

1 mark for y-co-o

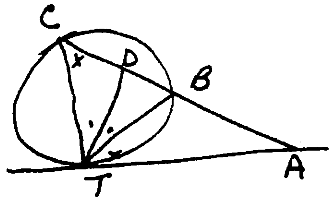
ii)

iii)



1 for each graph in (ii) & (iii)

Q 4. a)



$\angle ATB = \angle TCB$ (alt seg thm) 1 mark

$\angle CTD = \angle DTB$ (given)

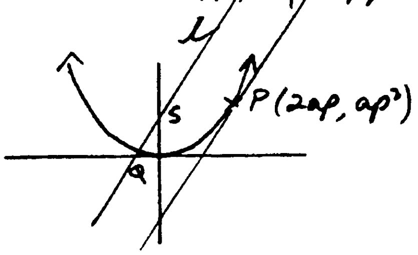
$\angle TDA = \angle TCD + \angle DTC$ (ext angle of $\triangle CDT$) 1 mark

$= \angle ATB + \angle DTB$

$= \angle DTA$

$\therefore AD = AT$ (opp equ angles of $\triangle ADT$) 1 mark.

b)



Grad of $l = p$

Equn $l$, $y - a = p(x - 0)$

$\therefore px - y + a = 0$ 1 mark.

ii) $y = 0$, $x = -\frac{a}{p}$ Q is $\left(-\frac{a}{p}, 0\right)$ 1 mark

M is $\left(-\frac{a}{2p}, \frac{a}{2}\right)$ 1 mark

iii) $\therefore$ Locus of M is $y = \frac{a}{2}$ 1 mark.

c) i) $\frac{dv}{dt} = k(2 - v)$

If $v = 2 + Ae^{-kt}$ $Ae^{-kt} = v - 2$

$\frac{dv}{dt} = -kAe^{-kt}$

$= k(-Ae^{-kt})$

$= k(2 - v)$ 1 mark

ii) $t = 0$, $v = 50$ $\therefore$ $50 = 2 + Ae^{0}$

$A = 48$ 1 mark.

iii) $t = 1$, $v = 35$ $\therefore 35 = 2 + 48e^{-k}$ 1 mark.

$e^{-k} = \frac{33}{48}$

$+k = -\ln \frac{33}{48} = \cdot 3747$ to 4dp. 1 mark

iv) $v = 2 + 48e^{-kt}$

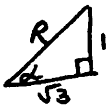
As t increases $e^{-kt}$ decreases & approaches 0

$\therefore$ Terminal velocity is 2m/sec 1 mark

5a) i) $\sqrt{3}\cos 2t - \sin 2t = R\cos(2t+\alpha)$

$= R\cos 2t\cos\alpha - R\sin 2t\sin\alpha$

$\therefore R\cos\alpha = \sqrt{3}, \quad R\sin\alpha = 1$



1 mark for R

1 mark for α

$\therefore R = 2, \ \alpha = \frac{\pi}{6}$

$\therefore \sqrt{3}\cos 2t - \sin 2t = 2\cos\left(2t + \frac{\pi}{6}\right)$

ii) if $\sqrt{3}\cos 2t - \sin 2t = 0$

$2\cos\left(2t + \frac{\pi}{6}\right) = 0$

$2t + \frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$ OR $2t + \frac{\pi}{6} = (2n+1)\frac{\pi}{2}$

1 mark here.

$2t = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}, \frac{10\pi}{3}, \ldots$

$t = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}, \ldots$ 1 mark answer.

OR $n\frac{\pi}{2} + \frac{\pi}{6}$ or equivalent.

b) i) $x = 5 + \sqrt{3}\cos 2t - \sin 2t$

$\dot{x} = -2\sqrt{3}\sin 2t - 2\cos 2t$ 1 for differentiations

$\ddot{x} = -4\sqrt{3}\cos 2t + 4\sin 2t$

$= -4(\sqrt{3}\cos 2t + \sin 2t)$ 1 for substitution

$= -4(x-5)$

ii) Centre of oscillation 5, amplitude 2

∴ Oscillates between 3 & 7 1 mark

iii) First passes $x = 5$ after $\frac{\pi}{6}$ secs 1 mark

c) $n = 1 \quad 1^2 + 3^2 + \cdots (2n-1)^2 = 1^2 \qquad \frac{1}{3}n(2n-1)(2n+1) = \frac{1}{3} \times 1 \times 3 = 1$

$\therefore 1^2 + 3^2 + \cdots (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ when $n = 1$ 1 mark.

Assume $1^2 + 3^2 + \cdots (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ when $n = k$, $k \geq n$ an integ

ie assume $1^2 + 3^2 + \cdots (2k-1)^2 = \frac{1}{3}k(2k-1)(2k+1)$

when $n = k+1$, $1^2 + 3^2 + \cdots (2n-1)^2 = 1^2 + 3^2 + \cdots (2k-1)^2 + (2k+1)^2$

$= \frac{1}{3}k(2k-1)(2k+1) + (2k+1)^2$ from assum

$= \frac{1}{3}(2k+1)\left[2k^2 - k + 6k + 3\right]$ 2 mark.

$= \frac{1}{3}(2k+1)(2k+3)(k+1)$

$= \frac{1}{3}(2n-1)(2n+1)n$ when $n = k+1$

If $1^2 + 3^2 + \cdots (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ when $n = k$

then $1^2 + 3^2 + \cdots (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ when $n = k+1$ 1 mark

Since $1^2 + 3^2 + \cdots (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ when $n = 1$

then $1^2 + 3^2 + \cdots (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ when $n = 2, 3, 4 \ldots$

Therefore $1^2 + 3^2 + \cdots (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ for all +ve integral values of n

Q6. a) $\ddot{x} = \sqrt{3x+4}$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \sqrt{3x+4}$

$\frac{1}{2}v^2 = \frac{1}{3}\int 3(3x+4)^{\frac{1}{2}}\,dx$

$= \frac{1}{3}(3x+4)^{3/2} \times \frac{2}{3} + c$

$v^2 = \frac{4}{9}(3x+4)^{3/2} + c$ 1 mark.

ii) $x=0,\ v=0,$ $\therefore$ $0 = \frac{4}{9} \times 4^{3/2} + c$

$c = -\frac{64}{27}$ 1 mark.

iii) Since $3x+4 \geq 0$, $x \geq -\frac{4}{3}$. Motion begins at 0 $\therefore$ must go in +ve direction
OR Accel is +ive $\therefore$ vel increases etc 1 mark.

b) $\frac{dV}{dt} = 25$ $\frac{dE}{dt} = ?$

$V = \frac{4}{3}\pi r^3$ $E = 2\pi r$

$\frac{dV}{dr} = 4\pi r^2$ $\frac{dE}{dr} = 2\pi$ 1 for correct info.

$\frac{dE}{dt} = \frac{dE}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$

$= 2\pi \times \frac{1}{4\pi r^2} \times 25$ 1 for this line

$r=10,$ $\frac{dE}{dt} = \frac{1}{200} \times 25$

$= \frac{1}{8}$

$\therefore$ Equator is increasing at $\frac{1}{8}$ cm/sec. 1 for answer.

c) i) $y = \ln 1.5 = 0.4$ $y = 2 - 1.5 = 0.5$

$\therefore$ Pt of intersection is close to $x = 1.5$ 1 mark.

ii) $\ln x - 2 + x = 0.$ Let $x_1 = 1.5$

$x_2 \doteq x_1 - \frac{f(x)}{f'(x)}$ $f'(x) = \frac{1}{x} + 1$

$f'(1.5) = \frac{2}{3} + 1 = 1\frac{2}{3}$

$= 1.5 - \frac{-0.1}{1\frac{2}{3}}$

$= 1.5 + 0.1 \times \frac{3}{5}$

$= 1.5 + 0.06$

$= 1.56$

d) $V = \pi\int_0^{\pi/4}(1+\sqrt{2}\cos x)^2\,dx = \pi\int_0^{\frac{\pi}{4}}(1 + 2\sqrt{2}\cos x + 2\cos^2 x)\,dx$

$= \pi\left[x + 2\sqrt{2}\sin x\right]_0^{\pi/4} + \pi\int_0^{\pi/4}(\cos 2x + 1)\,dx.$ 1 for subst for $2\cos^2 x$

$= \pi\left[x + 2\sqrt{2}\sin x + \frac{1}{2}\sin 2x + x\right]_0^{\pi/4}$ 1 for integration

$= \pi\left[\frac{\pi}{2} + 2 + \frac{1}{2}\right]$ 1 for answer

$\text{Vol} = \frac{\pi}{2}(\pi + 5)\ u^3$

Q7. a) Last digits are 7, 9, 3, 1, 7, 9, 3 ... 1 mark

2002 ÷ 4 leaves a remainder of 2.

∴ Last digit is 9 1 mark.

b) i) $\frac{d}{dx}(\sin^{-1}x + \cos^{-1}x) = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}} = 0$ 1 mark.

ii) Since the derivative is zero, $\sin^{-1}x + \cos^{-1}x = c$

Let $x = 0$, $\sin^{-1}0 + \cos^{-1}0 = 0 + \frac{\pi}{2}$ $\therefore c = \frac{\pi}{2}$. 1 mark.

i.e. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

iii) $\sin^{-1}x - \cos^{-1}y = \frac{\pi}{12}$.

Let $\alpha = \sin^{-1}x$ $\therefore \sin\alpha = x$

$\beta = \cos^{-1}y$ $\cos\beta = y$

$\cos^{-1}x + \sin^{-1}y = \frac{5\pi}{12}$

$\frac{\pi}{2} - \sin^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{5\pi}{12}$ 1 mark

$\sin^{-1}x + \cos^{-1}y = \frac{7\pi}{12}$.

$\therefore \alpha - \beta = \frac{\pi}{12}$.

$\alpha + \beta = \frac{7\pi}{12}$

$2\alpha = \frac{8\pi}{12} = \frac{2\pi}{3}$

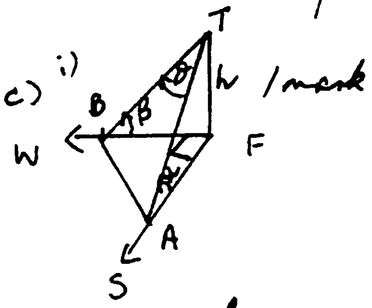
$\alpha = \frac{\pi}{3}$ –

$\therefore \beta = \frac{\pi}{4}$

$\therefore x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ 1 mark α,

$y = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ 1 mark x, y

c) i)



1 mark

ii) $\sin\beta = \frac{h}{BT}$ $BT = \frac{h}{\sin\beta}$

$\sin\alpha = \frac{h}{AT}$ $AT = \frac{h}{\sin\alpha}$

$\tan\beta = \frac{h}{BF}$ $BF = h\cot\beta$

$\tan\alpha = \frac{h}{AF}$ $AF = h\cot\alpha$. 1 mark

In $\triangle BFA$ $AB^2 = BF^2 + AF^2$

$= h^2\cot^2\beta + h^2\cot^2\alpha$. 1 mark

In $\triangle BTA$ by cosine rule $\cos\theta = \frac{BT^2 + AT^2 - AB^2}{2BT \times AT}$

$$= \frac{\frac{h^2}{\sin^2\beta} + \frac{h^2}{\sin^2\alpha} - \frac{h^2}{\tan^2\beta} - \frac{h^2}{\tan^2}}{2 \times \frac{h}{\sin\beta} \times \frac{h}{\sin\alpha}}$$

1 mark using Cosine Rule

$$= \frac{\sin^2\alpha + \sin^2\beta - \cos^2\beta\sin\alpha - \cos^2\alpha}{2\sin\alpha\sin\beta}$$

1 mark for correct answer

$$= \frac{\sin^2\alpha(1-\cos^2\beta) + \sin^2\beta(1-\cos^2\alpha)}{2\sin\alpha\sin\beta}$$

$$= \frac{\sin^2\alpha\sin^2\beta + \sin^2\beta\sin^2\alpha}{2\sin\alpha\sin\beta}$$

$= \sin\alpha\sin\beta$