Total marks (84)
Attempt questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) Find $\frac{d}{dx}\left(x\tan^{-1}2x\right)$ **2**

(b) The parametric equations of a curve are given by $x=t^2$, $y=t^3+t$. **2**
Find the Cartesian equation of the curve (that is y in terms of x).

(c) Write down the general solution of $\sin x=\frac{1}{2}$. **2**

(d) The interval AB has end points A (5, 4) and B (x, y). The point P (−1, 3) divides AB internally in the ratio 2:3. Find the coordinates of B. **2**

(e) Evaluate $\lim_{x\to 0}\left(\frac{\sin 3x}{4x}\right)$. **2**

(f) Use the table of standard integrals to find the exact value of **2**

$$\int_0^1 \frac{1}{\sqrt{4+x^2}}\,dx$$

Question 2 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) Find, correct to the nearest degree, the obtuse angle between the lines $x + y - 4 = 0$ and $y = 2x + 1$. **2**

(b) Solve $\dfrac{2x+3}{x-4} \leq 1$. **3**

(c) Use the substitution $u = 2 - x$ to evaluate $\displaystyle\int_0^1 x\sqrt{2-x}\;dx$. **4**

(d) (i) Write down the domain and range of the function $y = \dfrac{\pi}{2} - \sin^{-1}\dfrac{x}{2}$. **2**

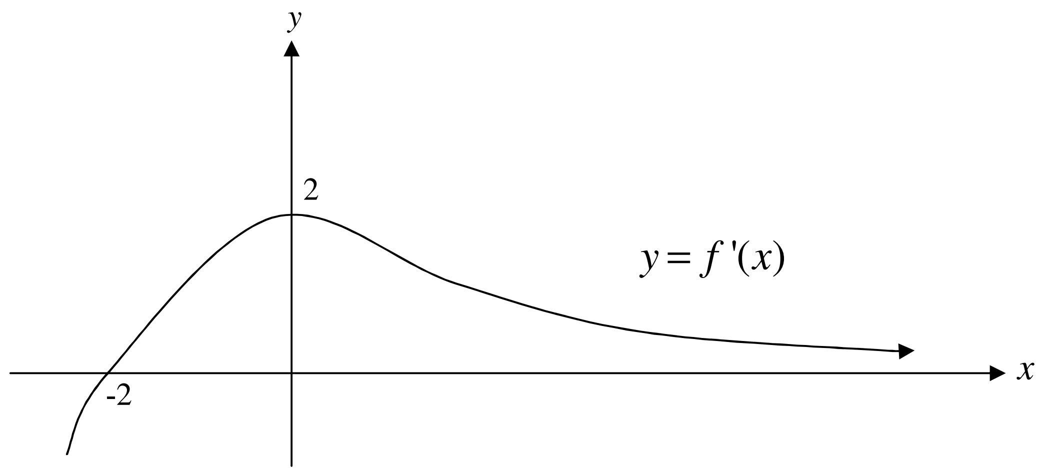
(ii) Hence sketch the function. **1**

.

Question 3 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$ in simplest surd form, with a rational denominator. **3**

(b) **2**



The diagram above shows a sketch of the gradient function of the curve $y=f(x)$.

Copy this diagram into your writing booklet.

On the same diagram, draw a possible sketch of the function $y=f(x)$, given that $f(0)=3$ and $\lim_{x\to\infty} f(x) = 6$.

(c) Consider the point $P\left(2ap, ap^2\right)$ on the parabola $x^2 = 4ay$.

(i) Show that the equation of the normal to the parabola $x^2 = 4ay$ at the point P is given by $x + py = 2ap + ap^3$. **2**

(ii) Find the equation of the line which passes through the focus $S(0, a)$ and is perpendicular to the normal. **1**

(iii) If the line found in part (ii) meets the normal at N, find the coordinates of N. **2**

(iv) Show that the locus of N is a parabola and find its vertex. **2**

Question 4 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) Determine the exact value of $\cos\left(2\sin^{-1}\left(\frac{12}{13}\right)\right)$. **2**

(b) (i) Show that the equation $x - \tan^{-1} 3x = 0$ has a root lying between $x = 1$ and $x = 2$. **1**

(ii) By taking $x = 1.5$ as an initial approximation to the root of $x - \tan^{-1} 3x = 0$, in the interval $1 < x < 2$, use one application of Newton's method to find a second approximation to this root. **2**

(c) The velocity of a particle moving in a straight line is given by **2**

$$v = 4x + 1,$$

where x is the displacement (in metres) from a fixed point *0*, and v is the velocity in metres per second.

Find the acceleration of the particle when it is 5 metres to the right of the origin.

(d) Newton's law of cooling states that the rate of cooling of a body is proportional to the excess of the temperature of a body above the surrounding room temperature. The temperature of a cup of chocolate drink satisfies an equation of the form $T = B + Ae^{kt}$ where T is the temperature of the drink, t is time in minutes, A and k are constants and B is the temperature of the surroundings.

The drink cools from 85°C to 70°C in three minutes in a room of temperature of 22°C.

(i) Find the value of A. **1**

(ii) Find the value of k, correct to 3 decimal places. **2**

(iii) Find the temperature of the cup of chocolate, to the nearest degree, after a further 9 minutes have passed. **2**

Question 5 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

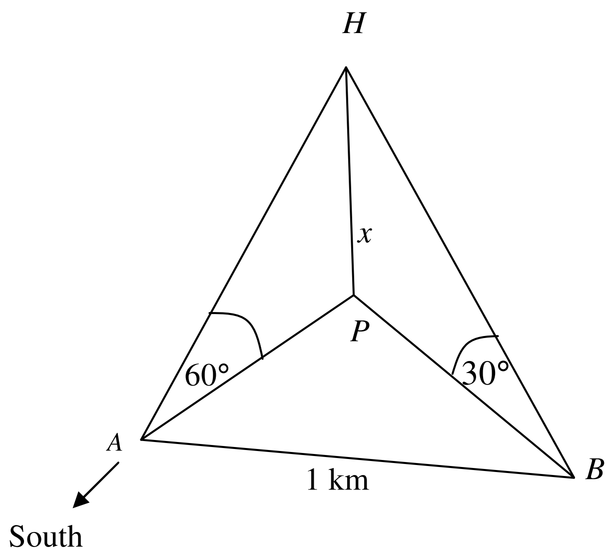
(a) Suppose $\frac{\alpha}{r}$, α and αr are the real roots of the cubic equation $2x^3 - 3x^2 - 3x + 2 = 0$.

(i) Write down the value of the sum of all three roots. **1**

(ii) Write down the value of the product of all three roots. **1**

(iii) Deduce that r can take on two real non-zero values and find them. **2**

(b) Anna (A) is standing due south of Phillip (P) who is assisting an injured bush walker. A rescue helicopter (H) is hovering directly over P and lowering a stretcher. Anna measures the angle of elevation of the helicopter to be 60° from her position. Belinda (B) is 1 kilometre due east of A and measures the angle of elevation of the helicopter to be 30°. The height of the helicopter above P is x metres.



NOT TO SCALE

(i) Write expressions for both AP and BP in terms of x. **1**

(ii) Hence or otherwise find the height of the helicopter correct to the nearest 10 m. **3**

(c) Use the Principle of Mathematical Induction to show that $9^{n+2} - 4^n$ is divisible by 5 for all positive integers n. **4**

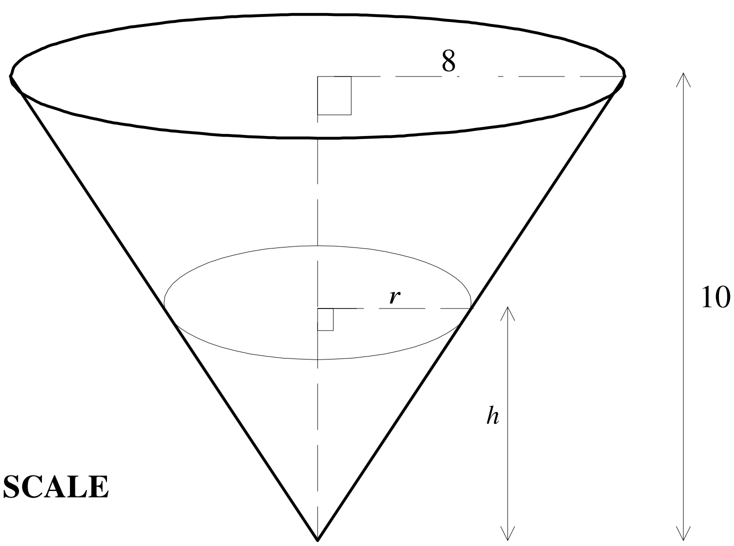
Question 6 **(12 Marks)** Use a SEPARATE Writing Booklet. **Marks**

(a) (i) State the domain and range for $f(x) = 4 - \sqrt{x-1}$. **2**

(ii) Find the inverse function $f^{-1}(x)$ and state the domain and range for which it exists. **3**

(iii) Sketch the graph of $f(x) = 4 - \sqrt{x-1}$ and its inverse function $f^{-1}(x)$ on the same number plane. **2**

(b) **5**



NOT TO SCALE

A bulk container for emptying grain into rail trucks is in the shape of an inverted cone with base radius 8 metres and height 10 metres. The grain is released from the apex of the cone at a constant rate of 35 m^3/s. The depth of grain in the container at any given time is h metres and the radius of the circle formed by the top of the grain at that same time is r metres.

If the grain is released continuously until the container is empty, calculate the rate at which the radius (r) is decreasing when the depth (h) is 0.65 metres.

Question 7 **(12 Marks)** Use a SEPARATE Writing Booklet.

Marks

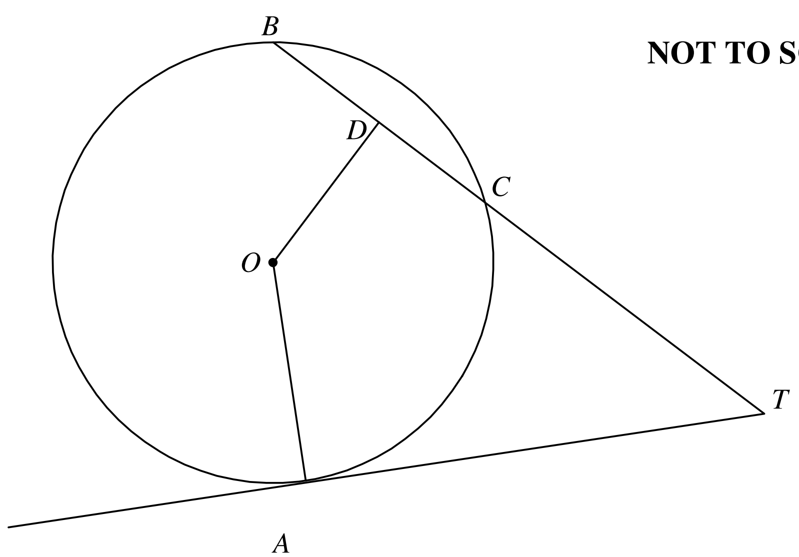
(a) By using the t – method (that is, let $t = \tan\frac{x}{2}$) solve the equation **4**

$$\cos x + \frac{1}{\sqrt{2}}\sin x = -1,$$

for x such that $0° \le x \le 360°$

(b) Find the volume of the solid formed by rotating about the x axis, the region bounded by $y = \cos 2x$, the x axis, from $x = 0$ to $x = \frac{\pi}{2}$. **4**

(c)



NOT TO SCALE

In the diagram above A, B and C are three points on a circle, centre O. The tangent at A meets BC produced at T. D is the midpoint of BC.

Copy this diagram into your writing booklet.

(i) Prove that $AODT$ is a cyclic quadrilateral. **3**

(ii) Explain why $\angle AOT = \angle ADT$. **1**

End of Paper

BLANK PAGE

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \, , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

Question 1   Ext. 1 (3U) - Solutions TRIAL 2003.

(a) $\frac{d}{dx}\left(x\tan^{-1}2x\right) = \tan^{-1}2x + x \times \frac{2}{1+4x^2}$

$= \tan^{-1}2x + \frac{2x}{1+4x^2}$ (2)

(b) $x = t^2$, $y = t^3 + t$

$t = \pm\sqrt{x}$: $y = x\sqrt{x} + \sqrt{x}$ or $y = -x\sqrt{x} - \sqrt{x}$ (2)

(c) $\sin x = \frac{1}{2}$

$x = n\pi + (-1)^n \frac{\pi}{6}$ (2)

(d) $A\overset{x_1\ y_1}{(5,4)}$ $B\overset{x_2\ y_2}{(x,y)}$ $\overset{m_1\ m_2}{2:3}$ $P(-1,3)$

$\frac{2x + 3\times5}{5} = -1$ $\qquad$ $\frac{2y + 3\times4}{5} = 3$

$2x + 15 = -5$ $\qquad$ $2y + 12 = 15$

$x = -10$ $\qquad$ $y = 1\frac{1}{2}$ (2)

(e) $\lim_{x\to0} \frac{\sin 3x}{4x} = \lim_{x\to0} \frac{3}{4}\frac{\sin 3x}{3x}$

$= \frac{3}{4}$ (2)

(f) $\int_0^1 \frac{1}{\sqrt{4+x^2}}\,dx = \left[\ln\left(x + \sqrt{4+x^2}\right)\right]_0^1$

$= \ln(1+\sqrt{5}) - \ln 2$ (2)

Question 2

(a) $x + y - 4 = 0$ $\qquad$ $y = 2x + 1$

$m = -1$ $\qquad$ $m = 2$

$\tan\theta = \frac{2-(-1)}{1+2\times(-1)}$

$= -3$

$\theta = 180^\circ - 72^\circ$

$= 108^\circ$ (2)

(b) $\frac{2x+3}{x-4} \le 1$ $\qquad$ $x \ne 4$

$(2x+3)(x-4) \le (x-4)^2$

$(x-4)\left[(x-4) - (2x+3)\right] \ge 0$

$(x-4)(-x-7) \ge 0$

$-7 \le x < 4$ $\quad (x \ne 4)$ (3)

(c) $\int_0^1 x\sqrt{2-x}\,dx = \int_2^1 (2-u)\sqrt{u}\,(-du)$ $\qquad$ $u = 2-x$, $x = 2-u$, $x=0, u=2$, $x=1, u=1$, $dx = -du$

$= \int_1^2 \left(2u^{\frac{1}{2}} - u^{\frac{3}{2}}\right)du$

$= \left[2\times\frac{2}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}}\right]_1^2$

$= \left(\frac{4}{3}\times2\sqrt{2} - \frac{2}{5}\times4\sqrt{2}\right) - \left(\frac{4}{3} - \frac{2}{5}\right)$

$= 8\sqrt{2}\left(\frac{1}{3} - \frac{1}{5}\right) - \frac{14}{15}$

$= \frac{16\sqrt{2} - 14}{15}$ (4)

(d) (i) $y = \frac{\pi}{2} - \sin^{-1}\frac{x}{2}$

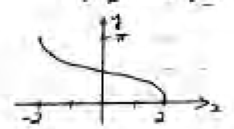
Domain: $-1 \le \frac{x}{2} \le 1$ $\therefore$ $-2 \le x \le 2$

Range: $-\frac{\pi}{2} \le \sin^{-1}\frac{x}{2} \le \frac{\pi}{2}$

$\frac{\pi}{2} \ge -\sin^{-1}\frac{x}{2} \ge -\frac{\pi}{2}$

$\pi \ge \frac{\pi}{2} - \sin^{-1}\frac{x}{2} \ge 0$ (2)

(ii) (1)



Question 3

(a) $\cos\frac{7\pi}{12} = \cos 105^\circ = \cos(45^\circ + 60^\circ)$

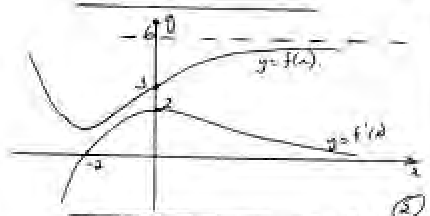
$= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}-\sqrt{6}}{4}$ (2)

(b)



(2)

(c) $x^2 = 4ay$

(i) $\frac{dy}{dx} = \frac{2x}{4a}$

At $x = 2ap$, $\frac{dy}{dx} = \frac{2 \times 2ap}{4a} = p$

Eqn of normal: $y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$x + py = 2ap + ap^3$ (A) (2)

(ii) SN: $y = px + a$ (1)

(iii) Sub $y = px + a$ into (A)

$x + p(px + a) = 2ap + ap^3$

$x + p^2x + ap = 2ap + ap^3$

$x(1 + p^2) = ap(1 + p^2)$

$x = ap$, $y = p(ap) + a$

N: $x = ap$, $y = ap^2 + a$ (2)

(iv) $p = \frac{x}{a}$ ∴ $y = a \times \frac{x^2}{a^2} + a$

$y = \frac{x^2}{a} + a$ which is a parabola

$ay = x^2 + a^2$

$x^2 = a(y - a)$

Vertex is $(0, a)$. (2)

Question 4.

(a) $\cos\left[2\sin^{-1}\left(\frac{12}{13}\right)\right]$

Let $\theta = \sin^{-1}\frac{12}{13}$

$\sin\theta = \frac{12}{13}$

$= \cos 2\theta$

$= 1 - 2\sin^2\theta$

$= 1 - 2 \times \frac{144}{169}$

$= -\frac{119}{169}$ (2)

(b) (i) Let $f(x) = x - \tan^{-1} 3x$

$f(1) = 1 - \tan^{-1} 3 = -0.249$

$f(2) = 2 - \tan^{-1} 6 = 0.594$

Since $f(1)$, $f(2)$ have opposite signs & $f(x)$ is continuous there is a root between $x = 1$, $x = 2$ (1)

(ii) $f'(x) = 1 - \frac{3}{1+9x^2}$

$f'(1.5) = 1 - \frac{3}{1+9\times1.5^2} = 0.8588$

$f(1.5) = 1.5 - \tan^{-1} 4.5 = 0.1479$.

Second approx'n $= 1.5 - \frac{0.1479}{0.8588}$

$= 1.33$ (2 dp) (2)

(c) $v = 4x + 1$

accel $= \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\frac{1}{2}(4x+1)^2$

$= 4(4x+1)$

when $x = 5$, accel^n = 84 m/s^2 (2)

(d) $T = 22 + Ae^{kt}$

(i) when $t = 0$, $T = 85$ ∴ $85 = 22 + Ae^0$ $A = 63$ (1)

(ii) when $t = 3$, $T = 70$ ∴ $70 = 22 + 63e^{3k}$

$e^{3k} = \frac{48}{63}$

$k = \frac{1}{3}\ln\left(\frac{48}{63}\right)$

$= -0.091$ (3 dp) (2)

(iii) when $t = 12$, $T = 22 + 63e^{(-0.091 \times 12)}$

$= 43$ degrees (nearest degree) (2)

Question 5

$x^3 - 3x^2 - 3x + 2 = 0$ $\qquad \frac{\alpha}{r}, \alpha, \alpha r$

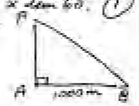
(i) Sum of roots $= \frac{3}{2}$ (1)

(ii) Product of roots $= -\frac{2}{2} = -1$ (1)

(iii)
$$\frac{\alpha}{r} \times \alpha \times \alpha r = -1$$
$$\alpha^3 = -1$$
$$\alpha = -1$$
$$\frac{-1}{r} + (-1) + (-1)r = \frac{3}{2}$$
$$-\frac{1}{r} - r = \frac{5}{2}$$
$$-2 - 2r^2 = 5r$$
$$2r^2 + 5r + 2 = 0$$
$$(2r+1)(r+2) = 0$$
$$r = -\frac{1}{2}, -2$$
(2)

(i) $AP = x\tan 30^\circ$ $\qquad BP = x\tan 60^\circ$ (1)

(ii)
$$x^2\tan^2 60^\circ - x^2\tan^2 30^\circ = 1000^2$$
$$3x^2 - \frac{x^2}{3} = 1000000$$
$$\frac{8x^2}{3} = 1000000$$
$$x^2 = \frac{3000000}{8}$$
$$x = 610 \text{ m (nearest 10 m)}$$
(3)



Prove $9^{n+2} - 4^n$ is divisible by 5.

When $n = 1$, $9^{n+2} - 4^n = 9^3 - 4 = 725$

∴ True for $n = 1$

Assume true for $n = k$, i.e. assume $9^{k+2} - 4^k = 5N$ (N integer)

When $n = k+1$, $9^{n+2} - 4^n = 9^{k+1+2} - 4^{k+1}$
$$= 9 \times 9^{k+2} - 4 \times 4^k$$
$$= 9(5N + 4^k) - 4 \times 4^k \text{ (by assumption)}$$
$$= 45N + 5 \times 4^k$$
$$= 5(9N + 4^k) \text{ - divisible by 5}$$

∴ If true for $n = k$, then true for $n = k+1$.
Since true for $n = 1$, then true for $n = 2, 3, \ldots$ (4)

Question 6

(a) (i) $f(x) = 4 - \sqrt{x-1}$

Domain: $x - 1 \geq 0$ i.e. $x \geq 1$

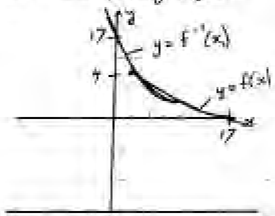
Range: $f(x) \leq 4$ (2)

(ii) $y = 4 - \sqrt{x-1}$

Inverse: $x = 4 - \sqrt{y-1}$
$$x - 4 = -\sqrt{y-1}$$
$$(x-4)^2 = y - 1$$
$$y = (x-4)^2 + 1$$

Domain $x \leq 4$, Range, $y \geq 1$ (3)

(iii)



(2)

(b)
$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi\left(\frac{4}{5}h\right)^2 h$$
$$= \frac{16}{75}\pi h^3$$

$\frac{r}{h} = \frac{8}{10}$

$r = \frac{4}{5}h$

$\frac{dr}{dt} = \frac{4}{5}\frac{dh}{dt}$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$
$$35 = \frac{16\pi}{25}h^2 \frac{dh}{dt}$$

when $h = 0.65$,
$$35 = \frac{16\pi}{25} \times 0.65^2 \frac{dh}{dt}$$
$$\frac{dh}{dt} = \frac{35 \times 25}{16\pi \times 0.65^2}$$
$$= 41.2$$
$$\frac{dr}{dt} = \frac{4}{5}\frac{dh}{dt}$$
$$= \frac{4}{5} \times 41.2$$
$$= 32.96$$

Radius is decreasing at 32.96 m/s. (5)

Question 5

(a) $2x^3 - 3x^2 - 3x + 2 = 0$ $\qquad \frac{\alpha}{r}, \alpha, \alpha r$.

(i) Sum of roots $= \frac{3}{2}$ (1)

(ii) Product of roots $= -\frac{2}{2} = -1$. (1)

(iii) $\frac{\alpha}{r} \times \alpha \times \alpha r = -1$

$\alpha^3 = -1$

$\alpha = -1$

$\frac{-1}{r} + (-1) + (-1)r = \frac{3}{2}$

$-\frac{1}{r} - r = \frac{5}{2}$

$-2 - 2r^2 = 5r$

$2r^2 + 5r + 2 = 0$

$(2r+1)(r+2) = 0$

$r = -\frac{1}{2}, -2$ (2)

(b) (i) $AP = x \tan 30^\circ$ $\qquad BP = x \tan 60^\circ$. (1)

(ii) $x^2 \tan^2 60^\circ - x^2 \tan^2 30^\circ = 1000^2$

$3x^2 - \frac{x^2}{3} = 1\,000\,000$

$\frac{8x^2}{3} = 1\,000\,000$

$x^2 = \frac{3\,000\,000}{8}$

$x = 610$ m (nearest 10 m) (3)

(c) Prove $9^{n+2} - 4^n$ is divisible by 5.

When $n = 1$, $9^{n+2} - 4^n = 9^3 - 4 = 725$

True for $n = 1$

Assume true for $n = k$, i.e. assume $9^{k+2} - 4^k = 5N$ (N integer)

When $n = k+1$, $9^{n+2} - 4^n = 9^{k+1+2} - 4^{k+1}$

$= 9 \times 9^{k+2} - 4 \times 4^k$

$= 9(5N + 4^k) - 4 \times 4^k$ (by assumption)

$= 45N + 5 \times 4^k$

$= 5(9N + 4^k)$ divisible by 5

∴ If true for $n = k$, then true for $n = k+1$.

Since true for $n = 1$, then true for $n = 2, 3, \ldots$ (4)

Question 6

(a) (i) $f(x) = 4 - \sqrt{x-1}$

Domain: $x - 1 \geq 0$ i.e. $x \geq 1$

Range: $f(x) \leq 4$. (2)

(ii) $y = 4 - \sqrt{x-1}$

Inverse: $x = 4 - \sqrt{y-1}$

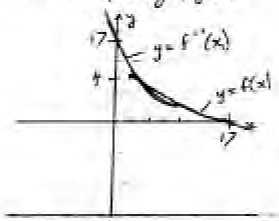
$x - 4 = -\sqrt{y-1}$

$(x-4)^2 = y - 1$

$y = (x-4)^2 + 1$

Domain: $x \leq 4$, Range: $y \geq 1$. (3)

(iii)



(2)

(b) $V = \frac{1}{3}\pi r^2 h$ $\qquad \frac{r}{h} = \frac{8}{10}$

$= \frac{1}{3}\pi \left(\frac{4}{5}h\right)^2 h$ $\qquad r = \frac{4}{5}h$

$= \frac{16}{75}\pi h^3$ $\qquad \frac{dr}{dt} = \frac{4}{5}\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$35 = \frac{16\pi}{25} h^2 \frac{dh}{dt}$

when $h = 0.65$

$35 = \frac{16\pi}{25} \times 0.65^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{35 \times 25}{16\pi \times 0.65^2}$

$= 41.2$

$\frac{dr}{dt} = \frac{4}{5}\frac{dh}{dt}$

$= \frac{4}{5} \times 41.2$

$= 32.96$

Radius is decreasing at 32.96 m/s. (5)

Question 7

$\cos x + \frac{1}{\sqrt{2}} \sin x = -1$

Let $t = \tan\frac{x}{2}$: $\frac{1-t^2}{1+t^2} + \frac{1}{\sqrt{2}} \times \frac{2t}{1+t^2} = -1$

$1 - t^2 + \sqrt{2}\,t = -1 - t^2$

$\sqrt{2}\,t = -2$

$t = -\sqrt{2}$

$\tan\frac{x}{2} = -\sqrt{2}$ ∴ $\frac{x}{2} = 180^\circ - 55^\circ$

$x = 250^\circ$

Test $x = 180^\circ$: $\cos x + \frac{1}{\sqrt{2}} \sin x = \cos(180) + \frac{1}{\sqrt{2}} \sin 180$

$= -1 + \frac{1}{\sqrt{2}} \times 0$

$= -1$

∴ Solutions are 180, 250° (nearest degree)

(4)

$V = \pi \int_0^{\pi/2} y^2 \, dx$

$= \pi \int_0^{\frac{\pi}{2}} \cos^2 2x \, dx$

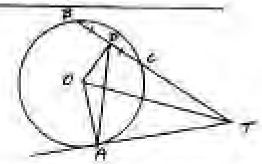
$= \pi \int_0^{\pi/2} \frac{1}{2}(1 + \cos 4x)\, dx$

$= \frac{\pi}{2} \left[x + \frac{1}{4} \sin 4x \right]_0^{\pi/2}$

$= \frac{\pi}{2} \left[\left(\frac{\pi}{2} + \frac{1}{4} \sin 2\pi\right) - (0 + 0) \right]$

Volume $= \frac{\pi^2}{4}$ unit^3

(4)



a) $\angle OAT = 90^\circ$ (angle between tangent & radius to pt. of contact)

$\angle ODT = 90^\circ$ (line from centre of circle to midpoint of a chord bisects the chord)

Since opp. angles of AODT are supplementary,
AODT is a cyclic quad

(3)

When a circle is drawn through AODT,
$\angle AOT$ and $\angle ADT$ are angles at circumference
standing on same chord AT

∴ $\angle AOT = \angle ADT$.

(1)