SET BY: JH

MV
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KNOX GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

2004
TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page of this paper
- All necessary working should be shown in every question

Total marks (84)

- Attempt Questions 1–7
- All questions are of equal value
- Use a **SEPARATE** Writing Booklet for each question
- Please write your **Board of Studies** Student Number and Teachers Initials on the front cover of each of your writing booklets.

NAME:____________________________ TEACHER:_________________________

Total marks (84)
Attempt questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find $\lim_{x \to 0} \frac{\sin 2x}{4x}$. **2**

(b) Find the exact value of $\int_2^3 \left(\frac{x^2}{x^3 - 7} \right) dx$. **3**

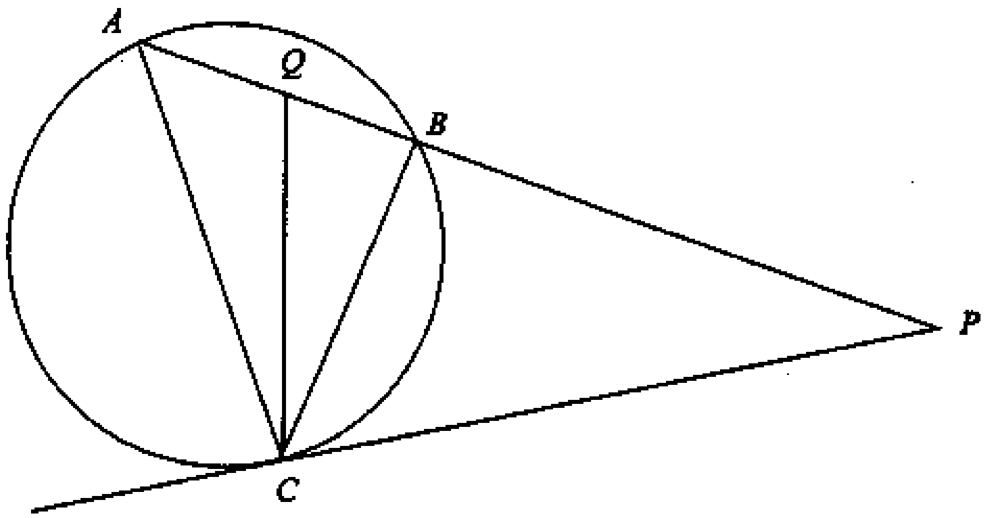
(c) Solve for x: $\frac{2x}{x-1} \leq 1$. **3**

(d) Find $\frac{d}{dx}\left(\tan^{-1} \frac{x}{3} \right)$. **1**

(e) The point $P(19, -15)$ divides an interval AB externally in the ratio 3:2. **3**
Find the coordinates of the point $B(x, y)$ given $A(-2, 3)$.

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



In the diagram above, PC is a tangent to the circle at C and QC bisects $\angle ACB$. 3

Copy the diagram into your writing booklet.

Prove, with reasons, that $PC = PQ$.

(b) Use the substitution $u = e^x$ to find: $\int \frac{dx}{e^x + 4e^{-x}}$ 3

(c) Evaluate $\int_0^{\frac{\pi}{2}} \cos^2 2x \; dx$. 3

(d) Find the exact value of $\cos^{-1}\left(\sin\frac{4\pi}{3}\right)$. 3

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the value of the term independent of x in the expansion of $\left(x-\dfrac{2}{x^3}\right)^{12}$. **2**

(b) (i) Find the equation of the tangent to the curve $y = x^2 - x$ at the point where $x = 2$. **2**

(ii) Find the obtuse angle between the line $\dfrac{x}{3}+\dfrac{y}{2}=1$ and the tangent found in part (i). Give your answer to the nearest degree. **2**

(c) (i) Express $\sqrt{12}\sin x + 2\cos x$ in the form $A\cos(x-\alpha)$; where $A > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. **2**

(ii) Hence, sketch the graph of $y = \sqrt{12}\sin x + 2\cos x$, in the domain $0 \le x \le 2\pi$. **2**

(iii) State the number of solutions that satisfy the equation $\sqrt{12}\sin x + 2\cos x = 1$ in the domain $0 \le x \le 2\pi$. **1**

(iv) Write down the general solution to $\sqrt{12}\sin x + 2\cos x = 1$ **1**

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Use one application of Newton's method to find a better approximation to the root of the equation $e^{-x} - \log_e x = 0$, given that there is a root near $x = 1.4$. Give your answer to 3 decimal places. **3**

(b) Use the Principle of Mathematical Induction to show that the expression $7^n + 5$ is divisible by 6 for all positive integers n. **4**

(c) (i) Find $\frac{d}{dx}\left(x\sin^{-1}\frac{x}{4} + \sqrt{16 - x^2}\right)$. **3**

(ii) Hence, evaluate $\int_0^4 \sin^{-1}\frac{x}{4}\ dx$. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Newton's Law of Cooling states that when an object at temperature T (°C) is placed in an environment at a temperature R (°C), then the rate of temperature loss is given by the equation $\dfrac{dT}{dt} = k(T-R)$; where t is the time in seconds and k is a constant.

A packet of peas, initially at $24\,°C$ is placed in a snap-freeze refrigerator in which the internal temperature is maintained at $-40\,°C$. After 5 seconds the temperature of the packet is $19\,°C$. Suppose $T = R + Ae^{kt}$, where A is a constant.

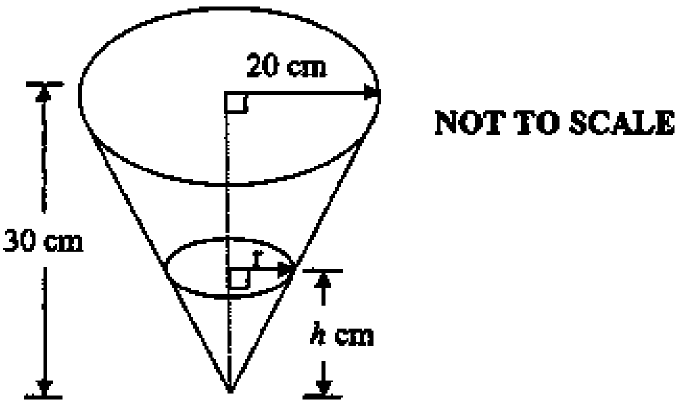
(i) State the value of A. **1**

(ii) Show that $k = \dfrac{1}{5}\log_e\left(\dfrac{59}{64}\right)$. **2**

(iii) Hence show that it will take approximately 29 seconds for the packet's temperature to reduce to $0\,°C$. **2**

(b) Prove that: $\tan\left(\dfrac{\pi}{4}+\theta\right) - \tan\left(\dfrac{\pi}{4}-\theta\right) = 2\tan 2\theta$ **3**

(c)



Water is poured into a conical vessel, of base radius 20 cm, and height 30 cm at a constant rate of 24 cm^3 per second. The depth of water is h cm at time t seconds and V is the volume of the water in the vessel at this time.

(i) Explain why $r = \dfrac{2h}{3}$. **1**

(ii) Hence show that the volume of water in the vessel at any time t is given by $V = \dfrac{4\pi h^3}{27}$. **1**

(iii) Find the rate of increase of the area (A) of the surface of the water, when the depth is 16cm. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ $(a > 0)$.

(i) By derivation, show that the equation of the chord is: **2**

$$y = \frac{1}{2}(p+q)x - apq.$$

(ii) If the chord PQ passes through the focus, S, show that $pq = -1$. **2**

(iii) Using the fact that $PQ = PS + SQ$, or otherwise, show that the chord PQ has length $a\left(p + \frac{1}{p}\right)^2$. **3**

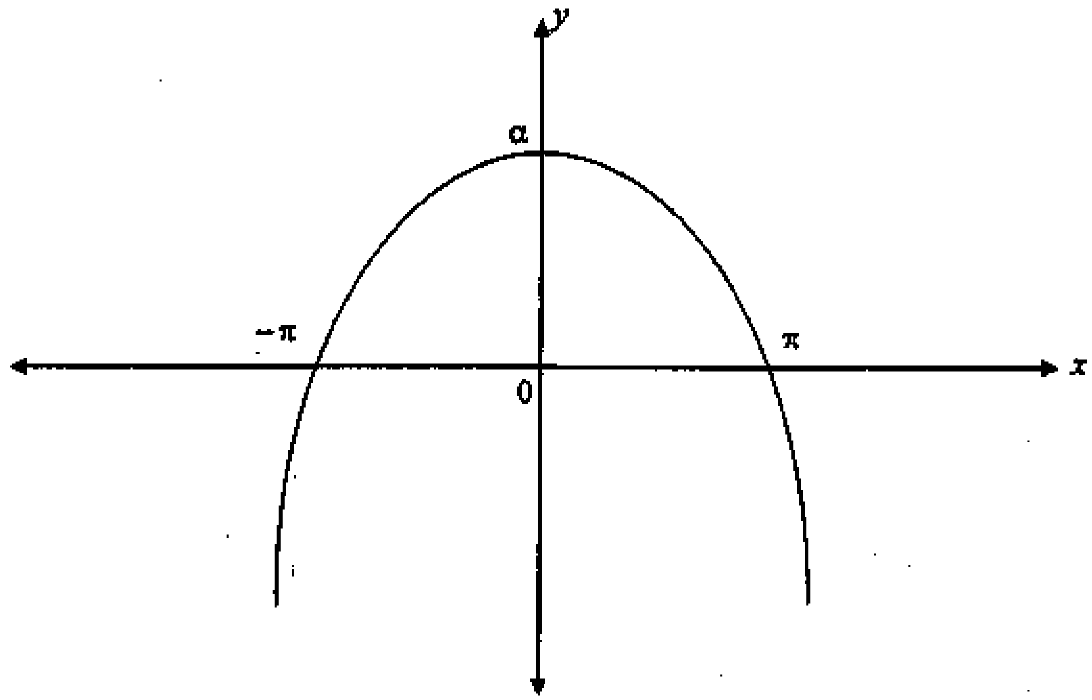
(b) A particle moves along a straight line such that its distance from the origin at time t (s) is x (m) and its velocity is v.

(i) Prove that $\frac{d^2x}{dt^2} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$. **2**

(ii) If the acceleration satisfies $\frac{d^2x}{dt^2} = -4\left(x + \frac{16}{x^3}\right)$ and the particle is initially at rest when $x = 2$, show that $v^2 = 4\left(\frac{16 - x^4}{x^2}\right)$. **3**

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



The diagram shows a parabola $y = f(x)$, with vertex $(0, \alpha)$ and $\alpha > 0$.
The parabola passes through the points $(-\pi, 0)$ and $(\pi, 0)$ as shown.

If a is the focal length of the parabola:

(i) Show that $4a = \dfrac{\pi^2}{\alpha}$. **2**

(ii) Show that $f(x)$ can be expressed in the form $f(x) = \alpha\left(1 - \dfrac{x^2}{\pi^2}\right)$. **2**

(iii) Find the exact value of α given that the area between $y = f(x)$ and the x axis from $x = -\pi$ to $x = \pi$ is 4 square units. **3**

(b) Assume that tides rise and fall in **Simple Harmonic Motion.** A ship needs 11 metres of water to pass down a channel safely. At low tide, the channel is 8m deep and at high tide 12 m deep. Low tide is at 10:00 am and high tide at 4:00 pm. **5**

Find the first time period during which the ship can safely proceed through the channel.

END OF PAPER

Suggested Solution (s)	Comments

1) a) $\lim_{x\to 0}\frac{\sin 2x}{4x}$

$=\frac{1}{2}\lim_{x\to 0}\frac{\sin 2x}{2x}$ ✓

$=\frac{1}{2}$ ✓ or aw 2

b) $\int_2^3\left(\frac{x^2}{x^3-7}\right)dx$

$=\frac{1}{3}\int_2^3\left(\frac{3x^2}{x^3-7}\right)dx$

$=\left[\frac{1}{3}\ln(x^3-7)\right]_2^3$ ✓

$=\frac{1}{3}\left(\ln(27-7)-\ln(8-7)\right)$ ✓

$=\frac{1}{3}\left(\ln 20-\ln 1\right)$

$=\frac{1}{3}\ln 20$ ✓

c) $\frac{2x}{x-1}\le 1$

$(x-1)^2\cdot\frac{2x}{x-1}\le(x-1)^2$ ✓

$2x(x-1)\le(x-1)^2$

$2x(x-1)-(x-1)^2\le 0$

$(x-1)(2x-x+1)\le 0$

$(x-1)(x+1)\le 0$ ✓

and $x\ne 1$.

$\therefore -1\le x<1$ ✓

d) $\frac{d}{dx}\left(\tan^{-1}\frac{x}{3}\right)$

$=\frac{3}{9+x^2}$ ✓

e) $x=\frac{mx_2+nx_1}{m+n}$

$19=\frac{-3(x)+2(-2)}{-3+2}$ ✓

$-19=-3x-4$

$-3x=-15$

$x=5$

and $y=\frac{my_2+ny_1}{m+n}$

$-15=\frac{-3y+2(3)}{-3+2}$ ✓

$15=-3y+6$

$3y=-9$

$y=-3$

$\therefore B(5,-3)$ ✓

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Suggested Solution (s)	Comments

2a) Let $\angle ACQ=\alpha$

$\therefore \angle QCB=\alpha$ (QC bisects $\angle ACB$).

Let $\angle BCP=\beta$.

$\therefore \angle CAB=\beta$ ($\angle$ between a tangent and a chord is equal to the $\angle$ in the alt. segment). ✓

So $\angle BQC=\alpha+\beta$ (ext. $\angle$ of ΔACQ). ✓

also $\angle QCP=\alpha+\beta$.

$\therefore \angle BQC=\angle QCP$ (both $=\alpha+\beta$) ✓

$\therefore PC=PQ$ (base $\angle$'s of Isos Δ are equal).

b) $\int\frac{dx}{e^x+4e^{-x}}$

$=\int\frac{dx}{\frac{e^x}{1}+\frac{4}{e^x}}$ $\quad u=e^x$, $du=e^x dx$ ✓

$=\int\frac{e^x\,dx}{e^{2x}+4}$ ✓

$=\int\frac{1\,du}{u^2+4}$

$=\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)+C$

$=\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)+C$. ✓

c) $\int_0^{\frac{\pi}{2}}\cos^2 2x\,dx$

aside: $\cos^2 x=\frac{1}{2}(\cos 2x+1)$

$\therefore \cos^2(2x)=\frac{1}{2}(\cos 4x+1)$

$=\frac{1}{2}\int_0^{\pi/2}(\cos 4x+1)dx$ ✓

$=\frac{1}{2}\left[\frac{\sin 4x}{4}+x\right]_0^{\pi/2}$

$=\frac{1}{2}\left[\left(\frac{\sin 2\pi}{4}+\frac{\pi}{2}\right)-(0)\right]$ ✓

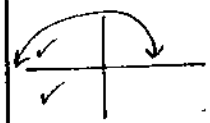
$=\frac{1}{2}\left(\frac{\pi}{2}\right)$

$=\frac{\pi}{4}$ ✓

d) let $\alpha=\cos^{-1}\left(\sin\frac{4\pi}{3}\right)$

$\therefore \alpha=\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ ✓

$\cos\alpha=\frac{-\sqrt{3}}{2}$



$0\le\alpha\le\pi$.

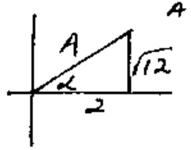
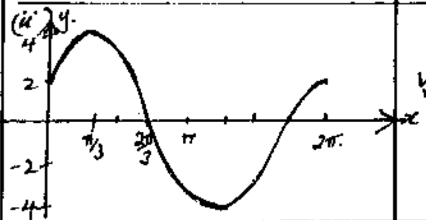
$\therefore \alpha$ is in the 2nd quad. ✓

Related $\measuredangle=\pi/6$.

$\therefore \alpha=\pi-\frac{\pi}{6}$

$\alpha=\frac{5\pi}{6}$ ✓

Suggested Solution (s)	Comments
1) $^{12}C_3 (x)^9 \left(\frac{-2}{x^3}\right)^3$	✓
$= {}^{12}C_3\, x^9 \cdot \frac{(-2)^3}{(x^3)^3}$	
$= {}^{12}C_3 \times (-2)^3$	
$= -1760.$	✓
(i) $y = x^2 - x$	
$\frac{dy}{dx} = 2x - 1$	
@ $x = 2$ $\frac{dy}{dx} = 3.$	✓
$\therefore y - y_1 = m(x - x_1)$	
$y - 2 = 3(x - 2)$	
$y = 3x - 6 + 2$	
$y = 3x - 4$	✓
) $\frac{x}{3} + \frac{y}{2} = 1$	
$2x + 3y = 6$	
$3y = 6 - 2x$	
$y = 2 - \frac{2}{3}x$	
$\therefore m_1 = -2/3.$	
$m_2 = 3.$	
$\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$	
$\tan\theta = \left\lvert \frac{3 - -2/3}{1 + 3(-2/3)} \right\rvert$	
$\tan\theta = 3\frac{2}{3}$	✓

Suggested Solution (s)	Comments
$\therefore \theta = 74°45'$	
$\therefore$ obtuse ∠ = 105	✓
c)(i) $\sqrt{12}\sin x + 2\cos x \equiv A\cos(x - \alpha)$	
$\equiv A\cos x \cdot \cos\alpha + A\sin x \sin\alpha$	
$\therefore A\cos\alpha = 2$, $\cos\alpha = \frac{2}{A}$; $A\sin\alpha = \sqrt{12}$, $\sin\alpha = \frac{\sqrt{12}}{A}$	
	
$A = 4.$	
$\tan\alpha = \frac{\sqrt{12}}{2}$	
$\tan\alpha = \sqrt{3}$	
$\therefore \alpha = \frac{\pi}{3}.$	
$\sqrt{12}\sin x + 2\cos x = 4\cos\left(x - \frac{\pi}{3}\right)$	
(ii) 	✓✓
(iii) 2	✓
(iv) $4\cos(x - \pi/3) = 1$	
$\cos(x - \pi/3) = 1/4.$	
$\therefore x = \frac{\pi}{3} + 2\pi n \pm \cos^{-1}\left(\frac{1}{4}\right)$	✓

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Suggested Solution (s)	Comments
4/ $e^{-x} - \log_e x = 0$	
a) $f(x) = e^{-x} - \log_e(x)$	
$f'(x) = -e^{-x} - \frac{1}{x}$	✓
$f(1.4) = e^{-1.4} - \log_e(1.4)$	
$f'(1.4) = -e^{-1.4} - \frac{1}{1.4}$	
hence $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$	
$= 1.4 - \left(\frac{e^{-1.4} - \ln(1.4)}{-e^{-1.4} - \frac{1}{1.4}}\right)$	✓
$\doteqdot 1.306$ (3dp)	✓
b) Test that the statement is true for $n = 1$; where n is a positive Integer.	✓
ie $7^1 + 5 = 12 = 6 \times 2$ ∴ divisible by 6.	
Assume that the statement is true for $n = k$, ie	
$7^k + 5 = 6P$; where P is a positive Integer.	—* ✓
Prove that the statement is true for $n = k+1$.	
ie $7^{k+1} + 5 = 6Q$; where Q is a positive Integer.	
So $7^{k+1} + 5$	
$= 7(7^k) + 5$	
$= 7(6P - 5) + 5$; from the assumption	—*

Suggested Solution (s)	Comments
$= 42P - 35 + 5$	
$= 42P - 30$	✓
$= 6(7P - 5)$	
$= 6Q$; where $Q = 7P - 5$ which is divisible by 6.	
If the statement is true for $n = k$, then the statement is true for $n = k+1$.	✓
Since the statement is true for $n = 1$, then it is true for $n = 1 + 1 = 2$, $2 + 1 = 3$, etc for all positive Integers n.	
note: students must have attempted steps 1, 2, 3 to be awarded marks for step 4.	
c) $\frac{d}{dx}\left(x\sin^{-1}\frac{x}{4} + \sqrt{16 - x^2}\right)$	
$= x \cdot \frac{1}{\sqrt{16 - x^2}} + \sin^{-1}\left(\frac{x}{4}\right) \cdot 1 + \frac{1}{2}(16 - x^2)^{-1/2} \times -2x$	✓
$= \frac{x}{\sqrt{16 - x^2}} + \sin^{-1}\left(\frac{x}{4}\right) + \frac{-x}{\sqrt{16 - x^2}}$	✓
$= \sin^{-1}\left(\frac{x}{4}\right).$	✓
(ii) $\int_0^4 \sin^{-1}\left(\frac{x}{4}\right) = \left[x\sin^{-1}\left(\frac{x}{4}\right) + \sqrt{16 - x^2}\right]_0^4$ — from (i) —*	✓
$= \left(4\sin^{-1}(1) + \sqrt{16 - 16}\right) - \left(0 + \sqrt{16}\right)$	
$= 4\sin^{-1}(1) - 4$	
$= 2\pi - 4.$	✓

Suggested Solution (s)	Comments

when $t=0$, $T=24°C$,
$C = -40°C$,
$\therefore 24 = -40 + Ae^{0}$
$\therefore A = 64$ ✓

) $t=5$, $T = 19°C$.
$19 = -40 + 64e^{5k}$ ✓
$e^{5k} = \frac{59}{64}$
$\ln(e^{5k}) = \ln(59/64)$
$\therefore 5k = \ln(59/64)$.
$k = \frac{1}{5}\ln(59/64)$ ✓

) when $T = 0°C$;
$0 = -40 + 64e^{kt}$
$\frac{40}{64} = e^{kt}$ ✓
$\ln\left(\frac{40}{64}\right) = \ln(e^{kt})$.
$\therefore kt = \ln\left(\frac{40}{64}\right)$
$t = \dfrac{\ln\left(\frac{40}{64}\right)}{\frac{1}{5}\ln\left(\frac{59}{64}\right)}$ ✓
$t = 28.889\ldots$
$\doteqdot 29$ seconds

Suggested Solution (s)	Comments

$LHS = \dfrac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\cdot\tan\theta} - \left(\dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\cdot\tan\theta}\right)$ ✓

$= \dfrac{1+\tan\theta}{1-\tan\theta} - \left(\dfrac{1-\tan\theta}{1+\tan\theta}\right)$

$= \dfrac{(1+\tan\theta)^2 - (1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}$

$= \dfrac{1 + 2\tan\theta + \tan^2\theta - (1 - 2\tan\theta + \tan^2\theta)}{(1-\tan\theta)(1+\tan\theta)}$ ✓

$= \dfrac{1 + 2\tan\theta + \tan^2\theta - 1 + 2\tan\theta - \tan^2\theta}{(1-\tan\theta)(1+\tan\theta)}$

$= \dfrac{4\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2(2\tan\theta)}{1-\tan^2\theta}$ ✓

$= \dfrac{2(\tan\theta + \tan\theta)}{1 - (\tan\theta)(\tan\theta)}$

$= 2(\tan 2\theta)$

$= RHS$.

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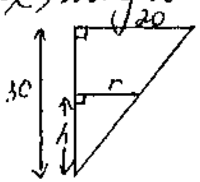
Suggested Solution (s)	Comments

5c(i) using similar triangles:



$\therefore \frac{r}{h} = \frac{20}{30}$
$\frac{r}{h} = \frac{2}{3}$
$\therefore r = \frac{2h}{3}$ ✓

(ii) $V = \frac{1}{3}\pi r^2 h$; $r = \frac{2}{3}h$
$= \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h$
$= \frac{1}{3}\pi\left(\frac{4h^2}{9}\right)h$ ✓
$= \frac{4}{27}\pi h^3$.

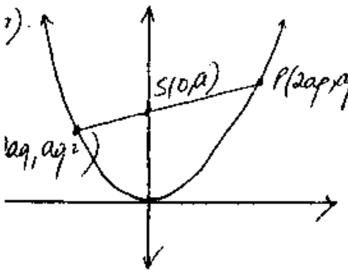
(iii) $\frac{dV}{dh} = \frac{4}{9}\pi h^2$
$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$
$= \frac{9}{4\pi h^2} \times 24$
$\frac{dh}{dt} = \frac{54}{\pi h^2}$

Suggested Solution (s)	Comments

$A = \pi r^2$
$A = \pi\left(\frac{2h}{3}\right)^2$
$= \frac{4\pi}{9}h^2$ ✓
$\frac{dA}{dh} = \frac{8}{9}\pi h$
$\therefore \frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$
$= \frac{8\pi h}{9} \times \frac{54}{\pi h^2}$
when $h = 16$ cm
$\frac{dA}{dt} = \frac{8\pi}{9} \times \frac{54}{\pi 16}$
$= 3\ cm^2/s$. ✓

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

i)



$m_{PQ} = \dfrac{ap^2-aq^2}{2ap-2aq}$

$= \dfrac{a(p-q)(p+q)}{2a(p-q)}$

$= \dfrac{p+q}{2}$ ✓

$y-ap^2 = \dfrac{p+q}{2}(x-2ap)$

$2y-2ap^2 = (p+q)(x-2ap)$ ✓

$2y-2ap^2 = px-2ap^2+qx-2apq$

$2y = (p+q)x-2apq$

$\therefore y = \frac{1}{2}(p+q)x-apq$

ii) Since PQ is a focal chord it passes through $S(0,a)$. ✓

$\therefore a = \frac{1}{2}(p+q)(0)-apq$ ✓

$a = -apq$

$\therefore pq = -1.$

iii) $PS = \sqrt{(2ap-0)^2+(ap^2-a)^2}$

$= \sqrt{(2ap)^2+a^2(p^2-1)^2}$

$= \sqrt{4a^2p^2+a^2(p^4-2p^2+1)}$

$= \sqrt{4a^2p^2+a^2p^4-2a^2p^2+a^2}$

$= \sqrt{a^2(p^4+2a^2p^2+1)}$

$= \sqrt{a^2(p^2+1)^2}$

$= a(p^2+1)$ ✓

Similarly $QS = a(q^2+1)$

$\therefore PQ = PS+SQ$

$= a(p^2+1)+a(q^2+1)$

$= a(p^2+q^2+2)$ ✓

Since $pq=-1$

$q = \dfrac{-1}{p}$

$\therefore PQ = a\left(p^2+\dfrac{1}{p^2}+2\right)$ ✓

$= a\left(p+\dfrac{1}{p}\right)^2.$

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

6b)(i) Using the chain Rule;

$\dfrac{d}{dx}\left(\frac{1}{2}v^2\right) = \dfrac{d}{dv}\left(\frac{1}{2}v^2\right)\times\dfrac{dv}{dx}$

$= v\times\dfrac{dv}{dx}$ ✓

$= \dfrac{dx}{dt}\times\dfrac{dv}{dx}$ ✓

$= \dfrac{dv}{dt}$

$= \dfrac{d^2x}{dt^2}$

ii) $\dfrac{d^2x}{dt^2} = -4\left(x+\dfrac{16}{x^3}\right)$

$\therefore -4\left(x+\dfrac{16}{x^3}\right) = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\frac{1}{2}v^2 = \int(-4x-64x^{-3})\,dx$ ✓

$\frac{1}{2}v^2 = \dfrac{-4x^2}{2}-\dfrac{64x^{-2}}{-2}+c$ ✓

$t=0,\ v=0,\ x=2,\ c=0.$

$\therefore v^2 = -4x^2+64x^{-2}$

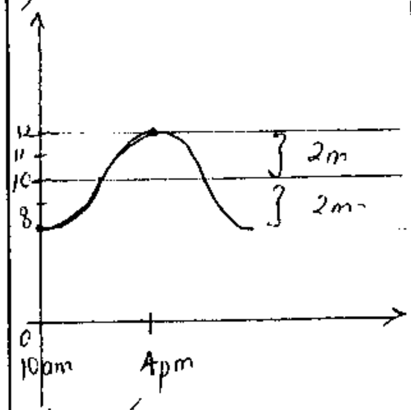
$v^2 = \dfrac{64}{x^2}-4x^2$ ✓

$v^2 = \dfrac{64-4x^4}{x^2}$

$v^2 = \dfrac{4(16-x^4)}{x^2}$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
i) $(x-h)^2 = -4a(y-k)$ vertex $(0, \alpha)$. $(x-0)^2 = -4a(y-\alpha)$ $x^2 = -4a(y-\alpha)$ function passes through $(\pi, 0)$ $\therefore \pi^2 = -4a(0-\alpha)$ $\pi^2 = -4a(-\alpha)$ $\pi^2 = 4a\alpha$ $\therefore 4a = \frac{\pi^2}{\alpha}$	✓ ✓	iii) $\int_{-\pi}^{\pi} \alpha\left(1-\frac{x^2}{\pi^2}\right)dx = 4$ $2\int_{0}^{\pi} \alpha\left(1-\frac{x^2}{\pi^2}\right)dx = 4$ $\int_{0}^{\pi} \left(\alpha-\frac{\alpha}{\pi^2}x^2\right)dx = 2$ $\left[\alpha x - \frac{\alpha}{\pi^2}\frac{x^3}{3}\right]_0^{\pi} = 2$ $\left(\alpha\pi - \frac{\alpha}{\pi^2}\frac{\pi^3}{3}\right) - (0-0) = 2$ $\alpha\pi - \frac{\alpha\pi}{3} = 2$. $\frac{2}{3}\alpha\pi = 2$. $\alpha\pi = 3$ $\alpha = \frac{3}{\pi}$.	✓ ✓ ✓
ii) $x^2 = -4a(y-\alpha)$. since $4a = \frac{\pi^2}{\alpha}$ $\therefore x^2 = -\frac{\pi^2}{\alpha}(y-\alpha)$ $x^2 = -\frac{\pi^2 y}{\alpha} + \pi^2$ $\alpha x^2 = -\pi^2 y + \alpha\pi^2$ $\pi^2 y = \alpha\pi^2 - \alpha x^2$ $y = \frac{\alpha\pi^2 - \alpha x^2}{\pi^2}$ $y = \alpha\left(1 - \frac{x^2}{\pi^2}\right)$	✓ ✓		

Year 12- 2004 Term 3 Mathematics Extension 1 HSC TRIAL EXAM

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
7 b) 6 hours. $\therefore$ wavelength = 12 h. period $= \frac{2\pi}{n} = 12$ $\therefore n = \frac{\pi}{6}$ Amplitude = 2m. $x = -2\cos\left(\frac{\pi}{6}t\right) + 10$ when $x = 11$ m; $11 = -2\cos\left(\frac{\pi}{6}t\right) + 10$ $-\frac{1}{2} = \cos\left(\frac{\pi}{6}t\right)$. $\therefore \frac{\pi}{6}t = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$ $t = \frac{2\pi}{3} \times \frac{6}{\pi}, \frac{4\pi}{3} \times \frac{6}{\pi}$ = 2 hours. t = 4 hours, 8 hours from 10am.	✓ ✓ ✓ ✓ ✓	ie, the first time period the ship can safely pass through would be between 2pm and 6pm.	