Total marks (84)
Attempt questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) **Marks**

(a) Find: $\frac{d}{dx}\tan^{-1}(2x)$ **1**

(b) A and B are the points $(-5,12)$ and $(4,9)$ respectively. P is the point which divides AB internally in the ratio 3 : 2. Find the coordinates of P. **2**

(c) Evaluate: $\lim_{x\to 0}\left(\frac{\tan 3x}{2x}\right)$ **1**

(d) Find the acute angle, in degrees correct to one decimal place, between the two curves $y = x^2$ and $y = x$ at the point of intersection $(1,1)$. **2**

(e) Find: $\int \sin^2 3x \; dx$ **2**

(f) Using the substitution $u = x - 1$, evaluate $\int_2^5 \frac{x}{\sqrt{x-1}}\,dx$. **4**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Let α, β and γ be the roots of the equation $x^3 - 5x^2 - 2x - 8 = 0$.

Without finding the actual roots, evaluate:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

(iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(b) Let $f(x) = \ln x - \sin x$. It is known that the real root of $f(x) = 0$ lies between $x = 2$ and $x = 2.5$. **3**

Using one application of the *'halving the interval'* method, determine whether the root of $f(x) = 0$ is closer to $x = 2$ or $x = 2.5$.

(c) Find the volume of the solid generated when the area under the curve $y = \dfrac{1}{(1-9x^2)^{\frac{1}{4}}}$, above the x-axis and between $x = 0$ and $x = \dfrac{1}{3\sqrt{2}}$, is rotated about the x-axis. **3**

(d) (i) Write down the value of the constant k in the equation $5^x = e^{kx}$, $x \neq 0$. **1**

(ii) Hence or otherwise, find $\dfrac{d}{dx}\left(5^x\right)$. **1**

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

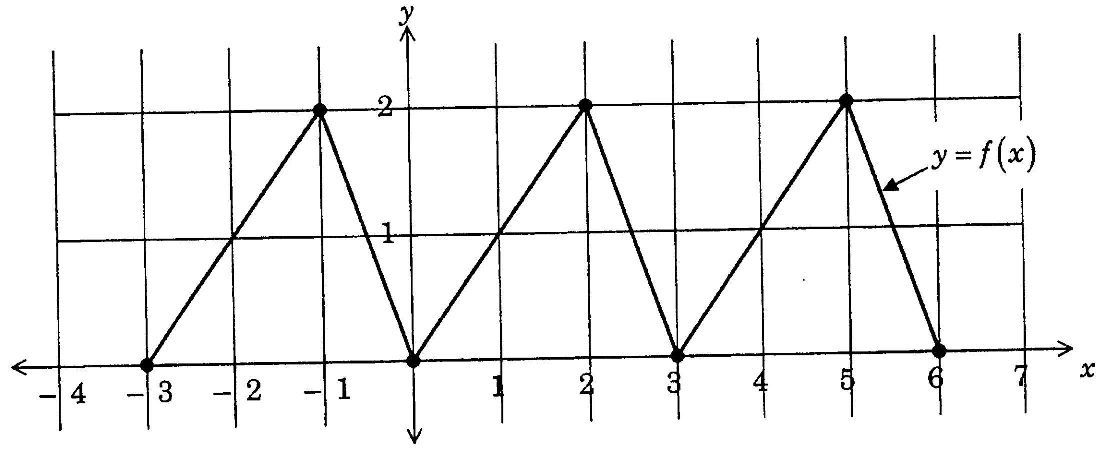
(a) Solve $2\sin x + \cos x = -1$, for $0 \le x \le 2\pi$, by first using the substitution, $t = \tan\frac{x}{2}$. **3**

(b) Prove by mathematical induction that if n is a positive integer, then: **3**

$$\frac{1}{1\times5}+\frac{1}{5\times9}+\frac{1}{9\times13}\cdots\cdots+\frac{1}{(4n-3)(4n+1)}=\frac{n}{4n+1}$$

(c) Without using a calculator, find the exact value of $\sin\left(\cos^{-1}\frac{2}{3}+\sin^{-1}\frac{1}{4}\right)$. **3**

(d)



The diagram above shows the graph of a periodic function $y = f(x)$ over the interval $-3 \le x \le 6$.

(i) State the period of $y = f(x)$. **1**

(ii) Assuming that the period of the function $y = f(x)$ continues to have the same form over the interval $-30 \le x \le 60$, calculate $f(52)$. **1**

(iii) Find $f'(x)$, when $x = 26\frac{1}{2}$. **1**

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Consider the function defined by $f(x) = x\left(\sqrt[3]{x^2 - 4}\right)$, where x is any real number, $f'(x) = \dfrac{5x^2 - 12}{3\left(x^2 - 4\right)^{\frac{2}{3}}}$ and $\left(2\sqrt{\dfrac{3}{5}}, -\dfrac{4\sqrt{3}}{5^{\frac{5}{6}}}\right)$ is one of the two stationary points on $y = f(x)$. ***You do not need to verify these facts.***

(i) Show that $f(x)$ is an odd function. **1**

(ii) Write down the coordinates of the second stationary point. **1**

(iii) Explain why there is a vertical tangent at $x = 2$. **1**

(iv) Sketch the graph of $y = f(x)$ and label the axes appropriately. **2**

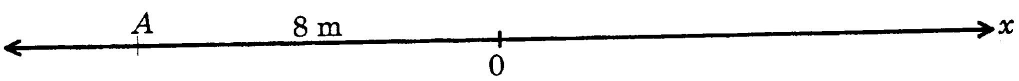
(b) The function f is given by $f(x) = \cos^{-1}\left(\dfrac{x}{3}\right)$.

(i) Find $f^{-1}(x)$. **1**

(ii) Write down the domain and range of $f^{-1}(x)$. **2**

(iii) Sketch the graph of $y = f^{-1}(x)$ and label the axes appropriately. **1**

(c)



A particle is moving in simple harmonic motion about the point O. The point A, as shown in the diagram, is 8 metres from O. When the particle passes through the point A its speed is 3 ms^{-1}. The amplitude of the motion is 10 m.

(i) Calculate the period of the motion. **2**

(ii) If x is the displacement of the particle from O, find the values of x for which the speed is zero. **1**

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The acceleration of a particle is given by $\ddot{x} = 4(1+x)$, where x is the particle's displacement from the origin. The particle is initially at the origin with a velocity of $2\,\text{m/s}$. Let $v = \dfrac{dx}{dt}$.

(i) Prove that $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \dfrac{d^2x}{dt^2}$. **2**

(ii) Find an expression for v in terms of x. **2**

(i) Show that $x = e^{2t} - 1$. Note that when $t \geq 0$, $v > 0$. **2**

(b) One hundred grams of cane sugar in water are being converted into dextrose at a rate which is proportional to the amount unconverted at any time t, that is, if M grams are converted in t minutes, then,

$$\frac{dM}{dt} = k(100 - M), \text{ where } k \text{ is a constant}$$

(i) Verify that $M = 100 + Ae^{-kt}$, where A is a constant, satisfies the given differential equation. **2**

(ii) If 40 grams are converted in the first 10 minutes, find A and k. **2**

(iii) How many grams are converted in the first 45 minutes, correct to the nearest whole gram? **2**

Question 6 commences on the next page

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve: $\dfrac{x}{x-1} \geq 5$ **3**

(b)

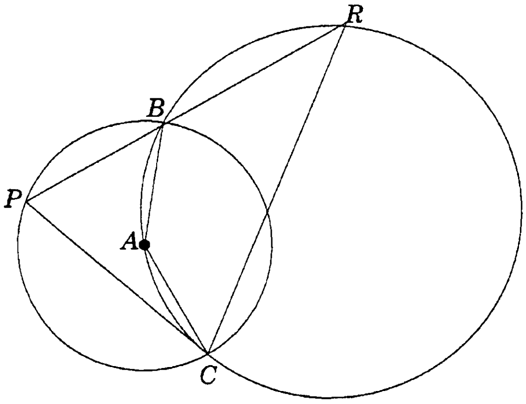


Diagram is not to scale

A is the centre of the circle BCP. The point A lies on another circle BAC. The two circles intersect in B and C as shown in the diagram. PBR is a straight line.

Copy or trace this diagram into your writing booklet.

Prove, with reasons, that $RP = RC$. **3**

(c) Two tangents from the external point $T(x_0, y_0)$ touch the parabola $x^2 = 4ay$ at $P(x_1, y_1)$ and $Q(x_2, y_2)$ respectively.

(i) Write down the Cartesian equation of the chord of contact in terms of x_0 and y_0. **1**

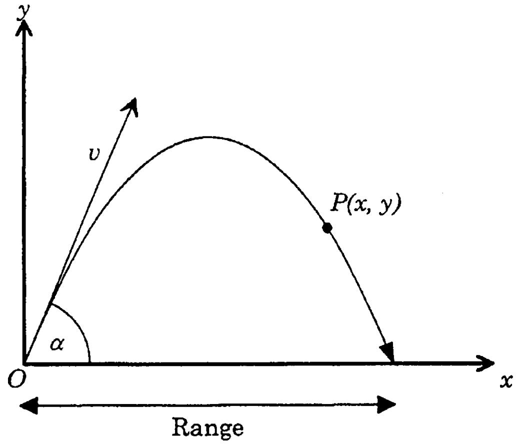
(ii) Show that the x values of the coordinates of P and Q are given by the roots of the equation $x^2 - 2x_0x + 4ay_0 = 0$. **2**

(iii) Show that the midpoint M of QP is given by $\left(x_0, \dfrac{x_0^2}{2a} - y_0\right)$. **2**

(iv) Find the Cartesian equation of the locus of M. **1**

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



A projectile is fired from level ground with an initial velocity, v metres per second, at an angle α to the horizontal. The origin, O, is taken to be at the point of projection on level ground.

(i) Starting with $\ddot{x} = 0$, $\ddot{y} = -g$ and integrating, derive the parametric equations for the position of the projectile $P(x, y)$, after t seconds. Ignore air resistance and assume the acceleration due to gravity is g m/s². **3**

(ii) Prove that the horizontal range of the projectile from the point of projection, in metres, is given by $x = \dfrac{v^2 \sin 2\alpha}{g}$. **2**

(iii) A golf ball is driven with a velocity of 50 m/s at an angle α to the horizontal towards the hole on the green 250 metres away on the same horizontal plane as the point of projection. **2**

At what angle should the golf ball be projected in order to achieve a 'hole-in-one', that is without bouncing *or* rolling first? Take $g = 9.8$ m/s² and ignore air resistance.

Question 7 continues on the next page

Question 7 continued:

(b)

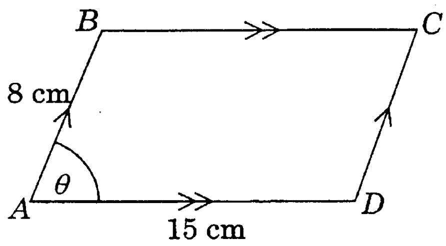


Diagram is not to scale

A parallelogram $ABCD$ has initially sides of length 8 cm and 15 cm. **3**

The angle θ at one of the vertices is decreasing at the rate of $\frac{\pi}{60}$ radians per minute.

Calculate the rate at which the area of the parallelogram is changing when $\theta = \frac{\pi}{6}$. Assume that as θ decreases, $ABCD$ remains a parallelogram.

(c) Gambler buys three tickets in a lottery for which sixty tickets are sold in all. There will be five prizes awarded. Tickets drawn will not be replaced. **2**

Find the probability that Gambler wins at least one prize.

End of Paper

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q5 ctd.

(ii) when $t=0$, $M=0$

$\therefore M = 100 + Ae^{-kt}$

$\Rightarrow 0 = 100 + Ae^{0}$

$\therefore A = -100$. ✓

$\therefore M = 100 - 100e^{-kt}$

when $t = 10$, $M = 40$

$\therefore 40 = 100 - 100e^{-10k}$

$\therefore \frac{-60}{-100} = e^{-10k}$

ie $e^{10k} = \frac{5}{3}$

$\therefore 10k = \ln(5/3)$

ie $k = \frac{1}{10}\ln(5/3)$. ✓

(iii) $M = ?$; $t = 45$

$M = 100 - 100e^{-\frac{1}{10}\ln(5/3)(45)}$ } ✓

$= 100 - 100e^{\ln(5/3 \times -4.5)}$

$= 100 - 100 \times (5/3)^{-4.5}$

$= 100 - 100 \times (3/5)^{4.5}$

$\doteqdot 90$ grams. ✓

QUESTION 6: (12 MARKS)

(a) $\frac{x}{x-1} \geq 5$

$\left(\frac{x}{x-1}\right)(x-1)^2 \geq 5(x-1)^2$

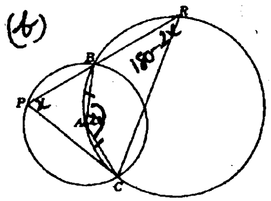
$x(x-1) - 5(x-1)^2 \geq 0$

$(x-1)[x - 5(x-1)] \geq 0$

$(x-1)(5-4x) \geq 0$

$\therefore \{x : 1 < x \leq \frac{5}{4}\}$

(b)



$AB = AC$ (equal radii)

$\angle BAC = 2x^\circ$

$\therefore \angle BPC = x^\circ$

(angle at the centre of a circle ~~on the~~ is twice the angle angle st. on the same arc). ✓

Q6 ctd.

$\angle BRC = 180 - 2x$

opp. ∠s of a cyclic quad. ABRC are supplementary.

$\therefore \angle PCR = 180 - (180 - 2x + x)$

$= 180 - (180 - x)$

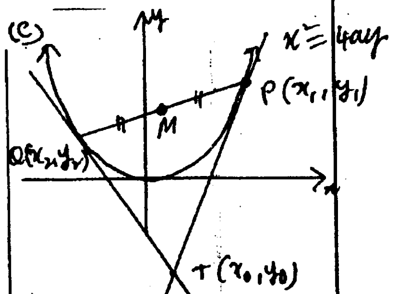
$= x$ } ✓

$\therefore \triangle PRC$ is isosceles

$\therefore RP = RC$

(sides opp equal ∠'s in a triangle are equal) ✓ reason

(c)



(i) Chord PQ:

$xx_0 = 2a(y + y_0)$. ✓

(ii) Solving $x^2 = 4ay$ ①

$xx_0 = 2a(y + y_0)$ ② } ✓ attempt to solve and lead to result.

in ① $y = \frac{x^2}{4a}$

$\therefore$ in ② $xx_0 = 2a\left(\frac{x^2}{4a} + y_0\right)$

ie $xx_0 = \frac{x^2}{2} + 2ay_0$

ie $2xx_0 = x^2 + 4ay_0$

ie $x^2 - 2xx_0 + 4ay_0 = 0$ ③

(iii) Solving ③

$x = \frac{2x_0 \pm \sqrt{(-2x_0)^2 - 4(1)(4ay_0)}}{2}$

$= x_0 \pm \sqrt{x_0^2 - 4ay_0}$ ✓

let $x_1 = x_0 + \sqrt{x_0^2 - 4ay_0}$

$x_2 = x_0 - \sqrt{x_0^2 - 4ay_0}$

$\therefore \frac{x_1 + x_2}{2} = \frac{2x_0}{2}$

$= x_0$ ④ ✓ (or)

sub ④ into $xx_0 = 2a(y + y_0)$

$\therefore \frac{x_0^2 - 2ay_0}{2a} = y$ ✓

$\therefore y = \frac{x_0^2}{2a} - y_0$

$\therefore M = \left(x_0, \frac{x_0^2}{2a} - y_0\right)$ ⑤

(iv) $x = x_0$ & $y = \frac{x_0^2}{2a} - y_0$ from ⑤ ✓

$\therefore y = \frac{x^2}{2a} - y_0$ ie $2a(y + y_0) = x^2$

Year 12- 2005 Trial HSC Mathematics EXTENSION 1 Assessment Task 4
Suggested Solutions and Marking Scheme

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

QUESTION 7: (12 MARKS)

(a)(i) $\ddot{x} = 0$ ($\dot{y} = V\sin\alpha$, $\dot{x} = V\cos\alpha$)

∴ $\dot{x} = c_1$

when $t=0$, $\dot{x} = V\cos\alpha = c_1$

∴ $\dot{x} = V\cos\alpha$

∴ $x = Vt\cos\alpha + c_2$

when $t=0$, $x=0$

∴ $c_2 = 0$

∴ $x = Vt\cos\alpha$ ① ✓

$\ddot{y} = -g$

∴ $\dot{y} = -gt + c_3$

when $t=0$, $\dot{y} = V\sin\alpha$

∴ $\dot{y} = -gt + V\sin\alpha$ ✓

$y = -\frac{gt^2}{2} + Vt\sin\alpha + c_4$

when $t=0$, $y=0$ ∴ $c_4 = 0$

∴ $y = -\frac{gt^2}{2} + Vt\sin\alpha$ ② ✓

∴ $P(x, y)$

$= \left[Vt\cos\alpha, Vt\sin\alpha - \frac{gt^2}{2}\right]$

(ii) when $y = 0$, the particle has reached the ground or is on the ground to start with.
Solving for $y=0$ in ②

$\frac{gt^2}{2} - Vt\sin\alpha = 0$

∴ $gt^2 - 2Vt\sin\alpha = 0$

∴ $t[gt - 2V\sin\alpha] = 0$

∴ either $t = 0$ or

$t = \frac{2V\sin\alpha}{g}$ ③ ✓

sub ③ into ①

$x = V\left[\frac{2V\sin\alpha}{g}\right](\cos\alpha)$

$= \frac{2V^2\sin\alpha\cos\alpha}{g}$ ✓

$x = \frac{V^2\sin 2\alpha}{g}$ as required. ④

(iii) From ④, $g = 9.8$, $x = 250$, $V = 50$.

$250 = \frac{50^2}{9.8}\sin 2\alpha$

∴ $\sin 2\alpha = 0.98$ ✓

∴ $2\alpha \doteqdot 78.52°\ldots$

$\alpha \doteqdot 39.26°\ldots$

or $2\alpha = \sin^{-1}0.98$

∴ $\alpha = \frac{1}{2}\sin^{-1}(0.98)$ (in exact form) ✓ or equivalent

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Suggested Solutions and Marking Scheme

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Q7 ctd:

(b) Area of a parallelogram

$A = ab\sin\theta$

where a, b are the adjacent sides including θ.

∴ $A = 8\times 15\sin\theta$

$A = 120\sin\theta$ ✓

Given $\frac{d\theta}{dt} = -\pi/60$ rads/min

We want:

$\frac{dA}{dt}$

$\frac{dA}{dt} = \frac{dA}{d\theta}\times\frac{d\theta}{dt}$

$= 120\cos\theta\times\left(-\frac{\pi}{60}\right)$ ✓

∴ $\frac{d}{dt}(A)$ when $\theta = \pi/6$

$= 120\times\cos\frac{\pi}{6}\times\left(-\frac{\pi}{60}\right)$

$= 120\times\frac{\sqrt{3}}{2}\times -\frac{\pi}{60}$

$= -\pi\sqrt{3}$ cm²/min ✓

∴ The area is decreasing at the rate of $\pi\sqrt{3}$ cm² per minute.

(c) P(win at least one prize)

$= 1 - P(\text{win no prize})$

$= 1 - \left[\frac{57}{60}\times\frac{56}{59}\times\frac{55}{58}\times\frac{54}{57}\times\frac{53}{56}\right]$ ✓

$= \frac{1597}{6844}$ ($\doteqdot 0.2\dot{3}$) ✓

or $1 - \left(\frac{55}{60}\times\frac{54}{59}\times\frac{53}{58}\right)$

or $= 1 - \frac{^{55}C_3}{^{60}C_3}$

or $= 1 - \frac{^{57}C_3}{^{60}C_5}$

etc....

MARKERS: Q1: DS Q2: EH Q3: LM Q4: RD Q5: AJ Q6: DS Q7: AJ

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
QUESTION 1: (12 MARKS) (a) $\frac{d}{dx}\tan^{-1}2x = \frac{2}{1+4x^2}$	✓	$\tan\theta = \left\lvert\frac{2-1}{1+2}\right\rvert$ $\tan\theta = \frac{1}{3}$ $\therefore \theta = 18.4°$	✓
(b) $k:l = 3:2$ $A(-5,12)$ $B(4,9)$ (x_1, y_1; x_2, y_2) $\therefore P(x,y) = \left[\frac{kx_2+lx_1}{k+l}, \frac{ky_2+ly_1}{k+l}\right]$ $= \left[\frac{3\times4+2\times-5}{3+2}, \frac{3\times9+2\times12}{3+2}\right]$ $= \left[\frac{2}{5}, \frac{51}{5}\right]$	✓ ✓	(e) $\int \sin^2 3x\,dx$. N.B.: $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\therefore \sin^2 3x = \frac{1}{2}(1-\cos 6x)$ $\therefore \int \sin^2 3x\,dx$ $= \frac{1}{2}\int 1-\cos 6x\;dx$ $= \frac{1}{2}\left[x - \frac{1}{6}\sin 6x\right]+c$	✓ ✓
(c) $\lim_{x\to0}\left(\frac{\tan 3x}{2x}\right)$ $= \lim_{x\to0}\left(\frac{\tan 3x}{3x}\right)\times\left(\frac{3x}{2x}\right)$ $= \frac{3}{2}\times 1$ $= 3/2$	✓	(f) let $u = x-1$ ① N.B: $\frac{dx}{du}=1$ when $x=2,\ u=1$ $x=5,\ u=4$. Also $x = u+1$ from ① $\therefore \int_2^5 \frac{x}{\sqrt{x-1}}\,dx$ $= \int_2^5 \frac{x}{\sqrt{x-1}}\frac{dx}{du}\,du$ $= \int_1^4 \frac{u+1}{\sqrt{u}}\,du$ $= \int_1^4 u^{1/2}+u^{-1/2}\,du$ $= \left[\frac{2u\sqrt{u}}{3}+2\sqrt{u}\right]_1^4$ $= \left[\frac{16}{3}+4-\frac{2}{3}-2\right]$ $= \frac{20}{3}$	✓ ✓ ✓ ✓
(d) $y_1 = x^2 \therefore y_1' = 2x$ $y_2 = x \therefore y_2' = 1$ when $x=1$, $y_1' = 2$, $y_2' = 1$ Let θ = acute angle $\therefore \tan\theta = \left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$	✓		

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Suggested Solutions and Marking Scheme

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QUESTION 2: (12 MARKS) (a)(i) $\alpha+\beta+\gamma = 5$ (ii) $\alpha\beta+\alpha\gamma+\beta\gamma = -2$ (iii) $\alpha^2+\beta^2+\gamma^2$ $= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $= 5^2 - 2\times(-2)$ $= 25+4$ $= 29$ ✓	✓ ✓ ✓ only if includes $2\sum\alpha\beta$.	(c) $V = \pi\int_0^{\frac{1}{3\sqrt2}} \frac{1}{\sqrt{9(\frac{1}{9}-x^2)}}\,dx$ $= \frac{\pi}{3}\int_0^{\frac{1}{3\sqrt2}} \frac{1}{\sqrt{(\frac{1}{3})^2-x^2}}\,dx$ $= \frac{\pi}{3}\left[\sin^{-1}3x\right]_0^{\frac{1}{3\sqrt2}}$ $= \frac{\pi}{3}\left[\sin^{-1}\frac{1}{\sqrt2} - \sin^{-1}0\right]$ $= \frac{\pi}{3}\times\frac{\pi}{4}$ $V = \frac{\pi^2}{12}$ cubic units.	✓ ✓
(b) Let $f(x) = \ln x - \sin x$ $f(2) = \ln 2 - \sin 2$ $= -0.216\ldots$ < 0 $f(2.5) = \ln 2.5 - \sin 2.5$ $= 0.3178\ldots$ > 0 Since $\lvert-0.216\ldots\rvert < \lvert0.3178\ldots\rvert$ then we conclude the root lies closer to $x=2$. or Consider $f\left(\frac{2+2.5}{2}\right) = f(2.25)$ $= 0.0328$ > 0 $\therefore$ The desired interval is $\{x: 2<x<2.25\}$ hence, the root lies closer to $x=2$.	ln radians. ✓✓ ✓ $f(2.25)$ & conclusion	(d)(i) If $5^x = e^{kx}$ then $k = \log_e 5$. (ii) $\frac{d}{dx}(5^x)$ $= \frac{d}{dx}e^{(\ln 5)x}$ $= \ln 5\, e^{(\ln 5 x)}$ $= \ln 5.(5^x)$ ✓ (or $\ln 5.\, e^{x\ln 5}$ or $\ln 5.\, e^{(\ln 5)x}$)	✓ will not accept $\frac{\ln 5^x}{x}$ but $\frac{\ln 5}{\ln e}$ is OK.
(c) $V = \pi\int_0^{\frac{1}{3\sqrt2}}\left[\frac{1}{\sqrt[4]{1-9x^2}}\right]^2 dx$ $V = \pi\int_0^{\frac{1}{3\sqrt2}}\frac{1}{\sqrt{1-9x^2}}\,dx$	✓ only if correct formula applied.		

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Suggested Solutions and Marking Scheme

Suggested Solution (s)	Comments

QUESTION 3: (12 MARKS)

(a) let $t = \tan \frac{x}{2}$

$\therefore 2\sin x + \cos x = -1$

$2\left[\frac{2t}{1+t^2}\right] + \frac{1-t^2}{1+t^2} = -1$ ✓ substitution

i.e. $4t = -2$ (N.B: t^2 term "cancels")

i.e. $t = -\frac{1}{2}$

$\therefore \tan \frac{x}{2} = -\frac{1}{2}$

$\therefore \frac{x}{2} = n\pi + \tan^{-1}\left(-\frac{1}{2}\right)$

i.e. $x = 2n\pi - 2\tan^{-1}\left(\frac{1}{2}\right)$ general solution

& $x = (2n+1)\pi$ where n is an integer.

We only want $x \in [0, 2\pi]$

$\therefore x = \pi$ or ✓

let $n=1$ in $x = 2n\pi - 2\tan^{-1}\frac{1}{2}$

$\therefore x = 2\pi - 2\tan^{-1}\frac{1}{2}$ ($\doteqdot 5.4$ radians to 2d.p) ✓

These are the only two required solutions.

(b) Let $S(n)$ be the statement:

$$\sum_{k=1}^{n} \frac{1}{(4k-3)(4k+1)} = \frac{n}{4n+1}$$

where n is a positive integer.

✓ = all stages of induction completed correctly.

Proving $S(1)$ is true:

$LHS = \frac{1}{1\times 5} = \frac{1}{5}$

$RHS = \frac{1}{4\times 1+1} = \frac{1}{5} = LHS$

$\therefore S(1)$ is true.

Assume $S(k)$ is true where $1 \le k \le n$ & k, n are positive integers.

Proving $S(k+1)$ is true:

We assume: $S(k) = \frac{k}{4k+1}$

RTP: $S(k) + \sum_{k=k+1}^{k+1} \frac{1}{(4k-3)(4k+1)} = \frac{k+1}{4k+5}$

$LHS = \frac{k}{4k+1} + \frac{1}{(4k+1)(4k+5)}$ (from assumption) ✓ or equivalent

$= \frac{4k^2+5k+1}{(4k+1)(4k+5)}$

$= \frac{(4k+1)(k+1)}{(4k+1)(4k+5)}$

$= \frac{k+1}{4k+5}$ ✓ correct simplification

$= RHS$

$\therefore S(n)$ is true for $n=1$. Whenever $S(k)$ is true the statement $S(n)$ is also true for all positive integer values of $n \ge 1$.

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Suggested Solutions and Marking Scheme

Suggested Solution (s)	Comments

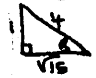
Q3 ctd.

(c) let $A = \cos^{-1}\frac{2}{3}$

$B = \sin^{-1}\frac{1}{4}$

$\therefore \cos A = \frac{2}{3}$

$\sin B = \frac{1}{4}$



$\therefore \sin(A+B)$

$= \sin A\cos B + \sin B\cos A$ ✓ or implied

$= \frac{\sqrt{5}}{3}\times\frac{\sqrt{15}}{4} + \frac{1}{4}\times\frac{2}{3}$

$= \frac{5\sqrt{3}}{12} + \frac{2}{12}$

$= \frac{5\sqrt{3}}{12} + \frac{1}{6}$ ✓

(d) (i) PERIOD = 3

(ii) N.B:

$$f(x) = \begin{cases} x & 0 \le x \le 2 \\ 6-2x & 2 \le x \le 3 \end{cases}$$

$\therefore f(52) = f(1) = 1$ ✓

(iii) $f'(26\frac{1}{2}) = f'(2\frac{1}{2}) = -2$ ✓

QUESTION 4: (12 MARKS)

(a)(i) $f(x) = x\left[\sqrt[3]{x^2-4}\right]$

$f(-x) = (-x)\left[\sqrt[3]{(-x)^2-4}\right]$

$= -x\left[\sqrt[3]{x^2-4}\right]$

$= -f(x)$ ✓

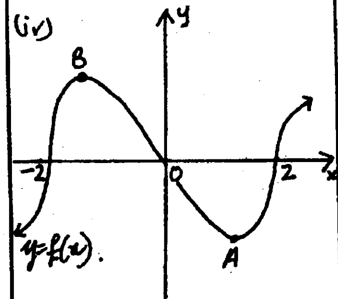
$\therefore f(x)$ is an odd f^n in x about $x=0$.

(ii) Since $f(x)$ is an odd f^n about $(0,0)$ and one of the SP's is at $A\left[2\sqrt{\frac{3}{5}}, -\frac{4\sqrt{3}}{5^{5/6}}\right]$ then the other S.P is at: $B\left[-2\sqrt{\frac{3}{5}}, \frac{4\sqrt{3}}{5^{5/6}}\right]$. ✓ must be stated.

(iii) Among the reasons, one could include: ✓

- $f'(x)$ is undefined when $x=2$
- the slope of the curve is "vertical"

(iv)



✓ intercepts
✓ vertical feature and correct shape.

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q4 ctd:

(b) $f(x) = \cos^{-1}\left(\frac{x}{3}\right)$

here: D: $\left|\frac{x}{3}\right| \le 1$

i.e. $|x| \le 3$

& R: $0 \le y \le \pi$.

(i) we seek $f^{-1}(x)$ such that: $f(f^{-1}(x)) = x$

i.e. $\cos^{-1}\left[\frac{f^{-1}(x)}{3}\right] = x$

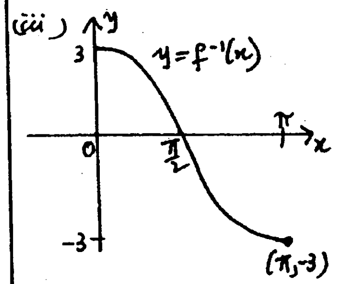
i.e. $\cos x = \frac{f^{-1}(x)}{3}$

$\therefore f^{-1}(x) = 3\cos x$. ✓

(ii) D: $0 \le x \le \pi$ ✓

R: $|y| \le 3$ ✓ or equivalent

(iii)



✓ correct shape, position, and intercepts

(c)(i) $v^2 = n^2(a^2 - x^2)$

let $v = 0$, $a = 10$, $x = -8$

$\therefore 9 = n^2(100 - 64)$ ✓

$\frac{9}{36} = n^2$

$\therefore n = \frac{3}{6} = \frac{1}{2}$ $(n > 0)$

$\therefore T = \frac{2\pi}{n} = \frac{2\pi}{\frac{1}{2}}$

$= 4\pi$ seconds. ✓

(ii) $v = 0$, $a = 10$, $n = \frac{1}{2}$

$\therefore 0 = \frac{1}{4}(100 - x^2)$

$\therefore |x| = 10$ i.e. $x = \pm 10$ m. ✓

(both must be stated).

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

QUESTION 5: (12 MARKS)

(a) $\ddot{x} = 4(1 + x)$

when $t = 0$, $x = 0$, $\dot{x} = 2$

(i) $LHS = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{1}{2} \times \frac{d}{dx}v^2$

$= \frac{1}{2} \times 2v\frac{dv}{dx}$ ✓

$= v\frac{dv}{dx}$

$= \frac{dx}{dt} \times \frac{dv}{dx}$ ✓

$= \frac{dv}{dt}$

$= \ddot{x}$

$= RHS$.

alternatively one could should show RHS = LHS.

(ii) Let $\frac{d}{dx}\frac{1}{2}v^2 = 4(1+x)$

$\therefore \int \frac{d}{dx}\frac{1}{2}v^2\,dx = 4\int 1 + x\,dx$

i.e. $\frac{1}{2}v^2 = \frac{4(1+x)^2}{2} + C_1$ ✓

when $x = 0$, $v = 2$

$\therefore 2 = \frac{4 \times 1}{2} + C_1$

$\therefore C_1 = 0$

$\therefore v^2 = 4(x+1)^2$ ✓

$\therefore v = \pm 2(x+1)$.

(iii) we are told $v > 0$

$\therefore$ let $v = 2(x+1)$

i.e. $\frac{dx}{dt} = 2(x+1)$

i.e. $\frac{dt}{dx} = \frac{1}{2(x+1)}$

$\therefore t = \frac{1}{2}\ln(x+1) + C_2$ ✓

when $x = 0$, $t = 0$

$\therefore 0 = \frac{1}{2}\ln 1 + C_2$

$\therefore C_2 = 0$

$\therefore t = \frac{1}{2}\ln(x+1)$

i.e. $e^{2t} = x + 1$ ✓

$\therefore x = e^{2t} - 1$ as required.

(b)(i) We know:

$\frac{dM}{dt} = k(100 - M)$.

Suppose $M = 100 + Ae^{-kt}$ ①

then $\frac{dM}{dt} = 0 - Ake^{-kt}$

$= -k(Ae^{-kt})$ } ✓

from ① $M - 100 = Ae^{-kt}$.

$\therefore \frac{dM}{dt} = -k(M - 100)$ ✓

i.e. $\frac{dM}{dt} = k(100 - M)$

or We know $\frac{dM}{dt} = k(100 - M)$

then $\frac{dM}{dt} = k[100 - (100 + Ae^{-kt})]$

$= k[-Ae^{-kt}]$

$= k(M - 100)$

as expected.