Number: ____________________

Teacher: ____________________

2014
Trial Higher School Certificate Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Teachers:
Mr Bradford
Mr Vuletich
Mrs Dempsey
Ms Yun

Total marks – 70

Section I: Pages 1-3

10 marks
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II: Pages 4-8

60 marks
- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Write your Board of Studies Student Number and your teacher's name on the front cover of each writing booklet

This paper MUST NOT be removed from the examination room

Number of Students in Course: 80

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 Find

$$\lim_{x\to 0} \frac{\sin 7x}{5x}$$

(A) 0

(B) $\frac{5}{7}$

(C) 1

(D) $\frac{7}{5}$

2 The point P divides the interval from $A(-2,2)$ to $B(8,\ -3)$ internally in the ratio 3:2.

What is the x-coordinate of P?

(A) 4

(B) 2

(C) 0

(D) -1

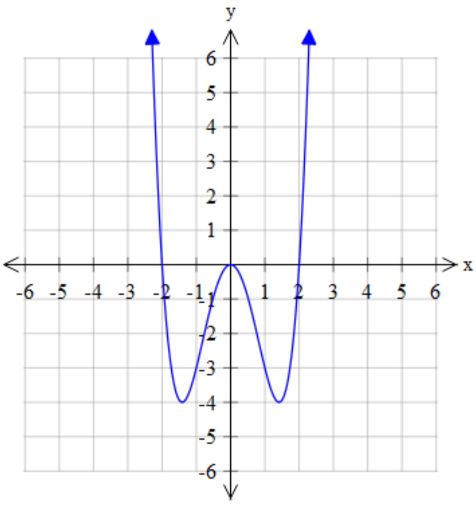
3 Which function best describes the polynomial $y = P(x)$?

(A) $y = x^2(4 - x^2)$

(B) $y = x^3(x-2)(x+2)$

(C) $y = x^2(x^2 - 4)$

(D) $y = x^3(2-x)(x+2)$



4 Which of the following is the domain of the function $y = \sin^{-1} 2x$?

(A) $-2 \le x \le 2$

(B) $0 \le x \le 2\pi$

(C) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

(D) $-\frac{1}{2} \le x \le \frac{1}{2}$

5 In the expression $(1+x)(1+x)^9$, the coefficient of x^5 is:

(A) 126

(B) 210

(C) 252

(D) 504

6 The points *A*, *B* and *C* lie on a circle centred at O. *AB* is parallel to *CD* and $\angle ABO = 42^\circ$.

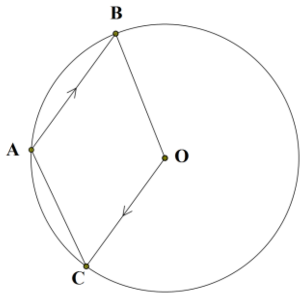
What is the size of $\angle CAB$?

(A) 42°

(B) 82°

(C) 138°

(D) 111°



7 A family of eight is seated randomly around a circular table.

What is the probability that the two youngest members of the family sit together?

(A) $\frac{6!2!}{7!}$

(B) $\frac{6!}{7!2!}$

(C) $\frac{6!2!}{8!}$

(D) $\frac{6!}{8!2!}$

8 When the polynomial *P(x)* is divided by $(x+1)(x-3)$, the remainder is $2x+7$.

What is the remainder when *P(x)* is divided by $x-3$?

(A) 1

(B) 7

(C) 9

(D) 13

9 The function $f(x)=3x-x^3$ has a relative maximum at (1, 2) and a relative minimum at $(-1,-2)$. What is the largest domain containing the origin for which *f(x)* has an inverse function $f^{-1}(x)$.

(A) $-1\leq x\leq 1$

(B) $-1<x<1$

(C) $-2\leq x\leq 2$

(D) $-2<x<2$

10 Which expression represents the ratio of the sum of *2n* terms to the sum of *n* terms of any Geometric Progression?

(A) 2 : 1

(B) $r^n:1$

(C) $(r^n+1):1$

(D) $(r^n-1):1$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Find the values of k such that $(x-2)$ is a factor of the polynomial $P(x)=x^3-2x^2+kx+k^2$. **2**

(b) Find the value of $\sum_{n=2}^{5} {}^{n}c_2$. **2**

(c) Solve the equation $\frac{4}{2x-1}<1$. **3**

(d) Find the angle that $y=\tan^{-1}2x$ makes with the <u>y-axis</u> at the origin. Give your answer to the nearest degree. **3**

(e) Use the substitution $u=\ln x$ to evaluate, **3**

$$\int_{e}^{e^2}\frac{1}{x\ln x}\,dx.$$

(f) The probability of snow falling in the Snowy Mountains on any one of the thirty-one days in August is 0.2. Find the probability that August has exactly 10 days in which snow falls. Give your answer as a percentage to the nearest whole per cent. **2**

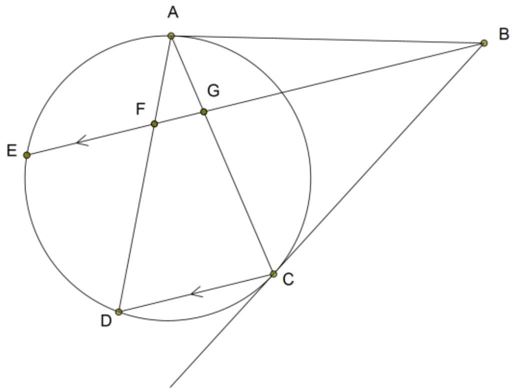
End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Show that $f(x) = 2\sin x - 10x + 5$ has a root between 0.6 and 0.7. **1**

(ii) Use one step of Newton's method and starting with *x*=0.6, find a better approximation. Answer to 2 decimal places. **2**

(b) In the diagram *EB* is parallel to *DC*. Tangents from *B* meet the circle at *A* and *C*.



Copy or trace the diagram into your writing booklet.

(i) Prove that $\angle BCA = \angle BFA$.
3

(ii) Prove *ABCF* is a cyclic quadrilateral. **1**

(c) Use mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for $n \geq 1$. **3**

(d) Consider the function $f(x) = \cos^{-1} 2x$.

(i) Sketch the function $f(x)$ for $-\frac{1}{2} \leq x \leq \frac{1}{2}$ **2**

(ii) Find the exact volume formed when $y = f(x)$ is rotated about the *y*-axis from $0 \leq y \leq \pi$. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) In how many ways can a committee of 3 men and 4 woman be selected from a group of 8 men and 10 women? **1**

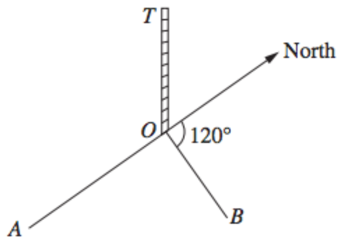
(b) It is known that two of the roots of the equation $2x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other. Find the value of *k*. **2**

(c) A salad, which is initially at a temperature of $25\,^\circ$C, is placed in a refrigerator that has a constant temperature of $3\,^\circ$C. The cooling rate of the salad is proportional to the difference between the temperature of the refrigerator and the temperature, *T* of the salad. That is, *T* satisfies the equation $\frac{dT}{dt} = -k(T-3)$ where *t* is the number of minutes after the salad is placed in the refrigerator.

(i) Show that $T = 3 + Ae^{-kt}$ satisfies this equation. **1**

(ii) The temperature of the salad is $11\,^\circ$C after 10 minutes. **3**
Find the temperature of the salad after 15 minutes.

(d) From a point *A* due south of a tower, the angle of elevation of the top of the tower *T*, is $23\,^\circ$. From another point *B*, on a bearing of $120\,^\circ$ from the tower, the angle of elevation of *T* is $32\,^\circ$. The distance *AB* is 200 metres.

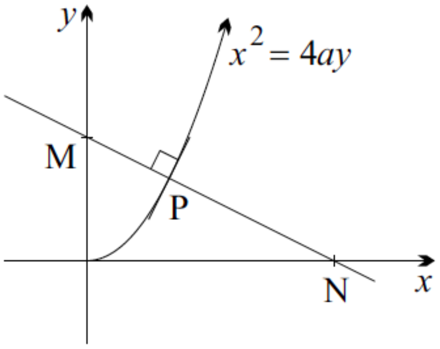


(i) Copy the diagram into your writing booklet, adding the given information to your diagram. **1**

(ii) Hence find the height of the tower. **3**

Question 13 continues on page 7

(e) A normal is drawn to the parabola $x^2 = 4ay$ at the point $P(2ap, ap^2)$, where $p > 0$. The normal intersects the x-axis at N and the y-axis at M as shown in the diagram below.



(i) Find the equation of the normal PN. **2**

(ii) It is known that the point P divides NM in the ratio 3:2. Find the value of the parameter p. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A particle moves in a straight line so that its acceleration is given by

$\frac{dv}{dt} = x - 1$, where v is its velocity and x is its displacement from the origin.

Initially, the particle is at the origin and has velocity $v = 1$.

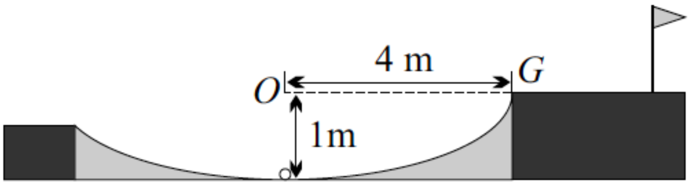
(i) Show that $v^2 = (x-1)^2$ **2**

(ii) By finding an expression for $\frac{dt}{dx}$, or otherwise, find x as a function of t. **2**

(b) The diagram below shows a golf ball in the middle of a 1m deep bunker and 4m from the edge of the green at *G*. The ball is hit with an initial speed of 12 m/s at an angle of elevation α. By taking the origin at 0, the horizontal and vertical equations of motion are:

$$x = 12t\cos\alpha \text{ and } y = -5t^2 + 12t\sin\alpha - 1.$$

(Do NOT prove these equations)



(i) Find the maximum height the ball reaches above *G* when $\alpha = 30°$. **2**

(ii) Find the range of values that α may take so that the ball lands on the green at or beyond *G*. Give your answer correct to the nearest $5°$. **3**

(c) (i) Use mathematical induction to show that $\ln(n!) > n$ for all positive integers $n \geq 6$. **3**

(ii) Hence show that $\frac{1}{n!} < \frac{1}{e^n}$ for all positive integers $n \geq 6$. **1**

(iii) Hence show that, **2**

$$\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}+\ldots<\frac{103}{60}+\frac{1}{e^5(e-1)}$$

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx \quad = \quad \frac{1}{n+1} x^{n+1}, \qquad n \neq -1; x \neq 0, \textbf{ if } n < 0$$

$$\int \frac{1}{x} \, dx \quad = \quad \ln x \quad , \qquad x > 0$$

$$\int e^{ax} \, dx \quad = \quad \frac{1}{a} e^{ax}, \qquad a \neq 0$$

$$\int \cos ax \, dx \quad = \quad \frac{1}{a} \sin ax, \qquad a \neq 0$$

$$\int \sin ax \, dx \quad = \quad -\frac{1}{a} \cos ax, \qquad a \neq 0$$

$$\int \sec^2 ax \, dx \quad = \quad \frac{1}{a} \tan ax, \qquad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \quad \frac{1}{a} \sec ax, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx \quad = \quad \frac{1}{a} \tan^{-1} \frac{x}{a}, \qquad a \neq 0$$

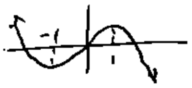
$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx \quad = \quad \sin^{-1} \frac{x}{a}, \qquad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx \quad = \quad \ln\left(x + \sqrt{x^2 - a^2}\right), \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx \quad = \quad \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

2014 Year 12 Mathematics Extension 1 Task 5 (Trial HSC) SOLUTIONS

Suggested Solution (s)	Comments
1. $\lim_{x\to 0} \frac{\sin 7x}{7x} \times \frac{7}{5} = \frac{7}{5}$ D	
2. $x = \frac{3\times 8 + 2\times -2}{5} = 4$ A	
3. $y = x^2(x^2-4)$ $= x^2(x-2)(x+2)$ C	
4. $-1 \le 2x \le 1$, $-\frac{1}{2} \le x \le \frac{1}{2}$ D	
5. $(1+x)(1+x)^9$ coeff. $x^5 = 1 \times {}^9C_5 + 1 \times {}^9C_4$ $= 126 + 126$ $= 252$ C	
6. $\angle BOC = 138^\circ$ (co-inter) Reflex $\angle BOC = 222^\circ$ $\angle CAB = 111^\circ$. D	
7. $\frac{6! \times 2!}{7!}$ A	
8. $P(x) = (x+1)(x-3)Q(x) + 2x+7$ $P(3) = 13$ D	
9.  $-1 \le x \le 1$ A	
10. $S_n = \frac{a(r^n-1)}{r-1}$, $S_{2n} = \frac{a(r^{2n}-1)}{r-1}$ $S_{2n} : S_n$ $r^{2n}-1 : r^n-1$ $(r^n-1)(r^n+1) : r^n-1$ $r^n+1 : 1$ C	

Suggested Solution (s)	Comments
QUESTION 11	
(a) $P(x) = x^3 - 2x^2 + kx + k^2$	
$P(2) = 0$, $8 - 8 + 2k + k^2 = 0$	①
$k(k+2) = 0$	
$\therefore k = 0$ or $k = -2$	①
(b) $\sum_{n=2}^{5} {}^nC_2 = {}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2$	①
$= 1 + 3 + 6 + 10$	
$= 20$	①
(c) $\frac{4}{2x-1} < 1$ $x \ne \frac{1}{2}$	
$4(2x-1) < (2x-1)^2$	①
$(2x-1)^2 - 4(2x-1) > 0$	
$(2x-1)(2x-5) > 0$	①
$\therefore x < \frac{1}{2}$ or $x > \frac{5}{2}$	①
(d) $y = \tan^{-1} 2x$	
$y' = \frac{2}{1+4x^2}$	①
$= 2$ when $x = 0$	
$\theta = \tan^{-1} 2 \Rightarrow \theta = 63^\circ 26'$	①
required angle $= 26^\circ 34'$	
$= 27^\circ$ (nearest degree)	①
(e) $\int_e^{e^2} \frac{1}{x \ln x}\,dx = \int_1^2 \frac{1}{u}\,du$	②
$u = \ln x$ $\quad = \left[\ln u\right]_1^2$	
$du = \frac{1}{x}\,dx$	
$x = e \Rightarrow u = 1$ $\quad = \ln 2$	①
$x = e^2 \Rightarrow u = 2$	
(f) $P(S) = 0{\cdot}2$, $P(\bar{S}) = 0{\cdot}8$	
P(exactly 10 days of snow)	
$= {}^{31}C_{10}(0{\cdot}8)^{21}(0{\cdot}2)^{10}$	①
$= 0{\cdot}0418894\cdots = 4\%$	①

Suggested Solution (s) | **Comments**

QUESTION 12

a)(i) $f(x) = 2\sin x - 10x + 5$

$f(0\cdot6) = 0\cdot129\ldots > 0$

$f(0\cdot7) = -0\cdot712\ldots < 0$ ①

$\therefore f(x) = 0$ has a solution between 0·6 and 0·7.

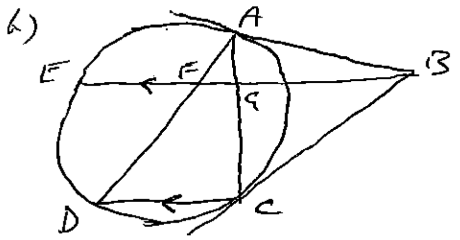
(ii) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$x_2 = 0\cdot6 - \dfrac{0\cdot129\ldots}{-8\cdot349\ldots}$ ①

$x_2 = 0\cdot6 + 0\cdot015\ldots$

$x_2 = 0\cdot62$ (2dp) ①

b)



i) Since tangents from an external point are equal, $AB = CB$. $\therefore \triangle ABC$ is isosceles. ①

(ii) $\angle BCA = \angle ADC$

(angle between chord and tangent is equal to the angle in the alternate segment). ①

$\angle BFA = \angle ADC$

(corresponding angles are equal $EB \parallel DC$). ①

$\therefore \angle BCA = \angle BFA$

(iii) Since $\angle BCA = \angle BFA$, they represent angles at the circumference standing on a chord AB (arc AB) of another circle, and ABCF is a cyclic quadrilateral. ①

c) Prove $2^{3n} - 3^n$ is divisible by 5, $n \geq 1$

When $n=1$, $2^3 - 3^1 = 8-3$
$= 5$ which is divisible by 5

Assume true when $n=k$

ie. $2^{3k} - 3^k = 5I$ for some integer I.

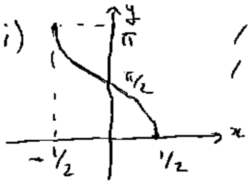
When $n=k+1$

$$\begin{aligned}2^{3(k+1)} - 3^{k+1} &= 2^{3k+3} - 3^k \times 3\\ &= 2^3 \times 2^{3k} - 3 \times 3^k\\ &= 8\left[2^{3k} - 3^k\right] + 5 \times 3^k\\ &= 8 \times 5I + 5 \times 3^k\\ &= 5\left[8I + 3^k\right]\end{aligned}$$

If true for $n=k$, then true for $n=k+1$ and since true when $n=1$, then true for all $n \geq 1$ ③

d) $f(x) = \cos^{-1} 2x$

i)



1 for shape
1 for intercepts ②

ii) $V = 2\pi \int_0^{\pi/2} x^2 \, dy$ $\quad \cos y = 2x$, $\dfrac{\cos^2 y}{4} = x^2$

$\therefore V = \dfrac{2\pi}{4} \int_0^{\pi/2} \cos^2 y \, dy$

$V = \dfrac{\pi}{2} \int_0^{\pi/2} \frac{1}{2}(\cos 2y + 1)\, dy$

$V = \dfrac{\pi}{4} \left[\dfrac{\sin 2y}{2} + y\right]_0^{\pi/2}$

Volume $= \dfrac{\pi^2}{8}$ units^3. ③

2014 Year 12 Mathematics Extension 1 Task 5 (Trial HSC) SOLUTIONS

Suggested Solution (s) | **Comments**

QUESTION 13

(a) $^{8}C_3 \times {}^{10}C_4 = 11760$ ①

(b) $T_{r+1} = {}^{n}C_r \, a^{n-r} b^r$

For $(2x+3y)^{12}$

$T_{r+1} = {}^{12}C_r (2x)^{12-r}(3y)^r$

$T_5 = {}^{12}C_4 (2x)^8 (3y)^4$ ①

$= {}^{12}C_4 . 2^8 . 3^4 . x^8 . y^4$

Coefficient of x^8y^4

$= {}^{12}C_4 . 2^8 . 3^4 = 10\,264\,320$ ①

(c) i) $T = 3 + Ae^{-kt}$

$\therefore T - 3 = Ae^{-kt}$ —①

$\frac{dT}{dt} = -kAe^{-kt}$

$= -k(Ae^{-kt})$

$= -k(T-3)$ from ①

$\therefore T = 3 + Ae^{-kt}$ satisfies. ①

(ii) When t=0, T=25

$\therefore 25 = 3 + Ae^{-kt}$

$A = 22$ ①

when $t = 10$, $T = 11$

$\therefore 11 = 3 + 22e^{-10k}$

$e^{-10k} = \frac{4}{11}$

$k = -\frac{1}{10}\ln\left(\frac{4}{11}\right)$ ①

$= 0{\cdot}10116\ldots$

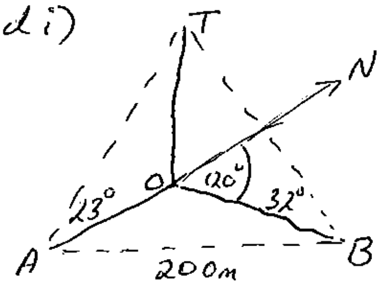
When $t = 15$

$T = 3 + 22e^{-15k}$

$T = 7{\cdot}8241\ldots$

$\therefore$ Temperature is approx. 8°c. ①

d i) ①



(ii) $\angle AOB = 60°$

$OA = \frac{OT}{\tan 23°}$, $OB = \frac{OT}{\tan 32°}$ ①

By cosine rule,

$200^2 = \frac{OT^2}{\tan^2 23°} + \frac{OT^2}{\tan^2 32°}$

$- 2\left(\frac{OT}{\tan 23°}\right)\left(\frac{OT}{\tan 32°}\right)\cos 60°$ ①

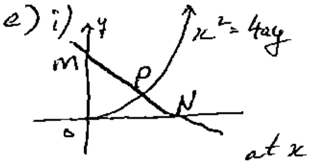
$200^2 = OT^2\left[\frac{1}{\tan^2 23°} + \frac{1}{\tan^2 32°} - \frac{1}{\tan 23° \tan 32°}\right]$

$40000 = OT^2 \times 4{\cdot}3409\ldots$

$OT^2 = 9214{\cdot}5535\ldots$

$\therefore OT = 96$ m (nearest m) ①

e) i)



$y = \frac{x^2}{4a}$

$y' = \frac{x}{2a}$

at $x = 2ap$, $y' = p$

$\therefore$ gradient of normal $= -\frac{1}{p}$ ①

Equ. normal, $y - ap^2 = -\frac{1}{p}(x - 2ap)$

$x + py = 2ap + ap^3$ ①

(ii) x-coordinate of M is 0. ①

x-coordinate of N is $2ap + ap^3$

Ratio of differences is

$\frac{ap^3}{2ap} = \frac{3}{2}$

$p^2 = 3$

$p = \sqrt{3}$ $(p>0)$ ①

2014 Year 12 Mathematics Extension 1 Task 5 (Trial HSC) SOLUTIONS

Suggested Solution (s) | **Comments**

Question 14

(a)(i) $\frac{dv}{dt} = x-1$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x-1$

$\frac{1}{2}v^2 = \frac{1}{2}x^2 - x + c$

when $x=0$, $v=1$

$\therefore \frac{1}{2} = 0-0+c$

$c = \frac{1}{2}$ ①

$\therefore \frac{1}{2}v^2 = \frac{1}{2}x^2 - x + \frac{1}{2}$

$v^2 = x^2 - 2x + 1$

$v^2 = (x-1)^2$ ①

(ii) $\left(\frac{dx}{dt}\right)^2 = (x-1)^2$

$\frac{dx}{dt} = \pm(x-1)$

at $x=0$, $\frac{dx}{dt} = 1$

$\therefore \frac{dx}{dt} = -(x-1)$

$= 1-x$ ①

$\therefore \frac{dt}{dx} = \frac{1}{1-x}$

$t = \int \frac{1}{1-x}\,dx$

$t = -\ln(1-x) + c$

$t=0, x=0 \Rightarrow c=0$

$\therefore t = -\ln(1-x)$

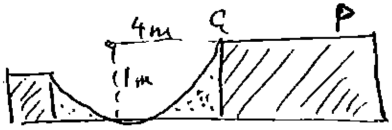
$-t = \ln(1-x)$

$e^{-t} = 1-x$

$\therefore x = 1-e^{-t}$ ①

Suggested Solution (s) | **Comments**

d) i)



when $\alpha = 30°$, $y = -5t^2+6t-1$

Axis of symmetry

$t = \frac{-6}{-10}$

$t = 3/5$ ①

when $t = 3/5$, $y = -5(3/5)^2 + 6(3/5) - 1$

$y = 4/5$ ①

Max height above green when $\alpha = 30°$ is $4/5$ m.

ii) To reach G, $x=4$.

$\therefore 4 = 12t\cos\alpha \Rightarrow t = \frac{1}{3\cos\alpha}$

$\therefore y = -5t^2 + 12t\sin\alpha - 1$

ie $0 = -5\left(\frac{1}{3\cos\alpha}\right)^2 + 12\left(\frac{1}{3\cos\alpha}\right)\sin\alpha - 1$ ①

$\therefore 5\left(\frac{1}{9\cos^2\alpha}\right) - 12\left(\frac{\sin\alpha}{3\cos\alpha}\right) + 1 = 0$

$\frac{5}{9}\sec^2\alpha - 4\tan\alpha + 1 = 0$

$5\sec^2\alpha - 36\tan\alpha + 9 = 0$

$5\tan^2\alpha - 36\tan\alpha + 14 = 0$ ①

$\therefore \tan\alpha = \frac{36-\sqrt{1016}}{10}$ or $\frac{36+\sqrt{1016}}{10}$

$\alpha \doteqdot 22.4°$ or $81.6°$

In order to land at or beyond G, $25° \leq \alpha \leq 80°$ ①

accept $20° \leq \alpha \leq 80°$ even though at the lower limit the ball does not reach the green.

PTO for C

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 14 cont'd. c) i) To prove $\ln(n!) > n$ for $n \geq 6$ When $n=6$, $\ln(6!) = 6{\cdot}58 > 6$ $\therefore$ True when $n=6$ Assume true when $n=k$ ie $\ln(k!) > k$, $k \geq 6$ When $n=k+1$ $\ln\left[(k+1)!\right] = \ln\left[(k+1) \times k!\right]$ $= \ln(k+1) + \ln k!$ $> \ln e + \ln k! \quad (k+1 > e)$ $> 1 + k \quad (\ln(k!) > k)$ $> k+1$ If $\ln(n!) > n$ is true, then $\ln(n+1)! > n+1$ and since $\ln(n+1)! > n+1$ when $n=6$ then $\ln(n!) > n$ for all $n \geq 6$ ii) $\ln(n!) > n$, $n \geq 6$ $\therefore n! > e^n$, $n \geq 6$ $\therefore \frac{1}{n!} < \frac{1}{e^n}$, $n \geq 6$.	③ ①	iii) $\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\ldots$ $< \frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{e^6}+\frac{1}{e^7}+\ldots$ $< 1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{e^6}\left[1+\frac{1}{e}+\frac{1}{e^2}+\ldots\right]$ $< \frac{206}{120}+\frac{1}{e^6}\left[1+\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}+\ldots\right]$ $< \frac{103}{60}+\frac{1}{e^6}\left[\frac{1}{1-\frac{1}{e}}\right]$ $< \frac{103}{60}+\frac{1}{e^5(e-1)}$	$S_n = \frac{a}{1-r}$ $-1 < 1/e < 1$ ②