Northern Beaches Secondary College
Manly Selective Campus

2007
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators and templates may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1 - 7
- All questions are of equal value

Marks

Question 1 Use a <u>SEPARATE</u> writing booklet. (12)

a) In how many ways can a committee of three girls and two boys be chosen from eight girls and four boys? **1**

b) Find the Cartesian equation of the curve defined by the parametric equations $x = \sin\theta$ and $y = \cos^2\theta - 3$. **2**

c) Solve the inequality $\dfrac{x^2}{x-2} \geq -1$ **3**

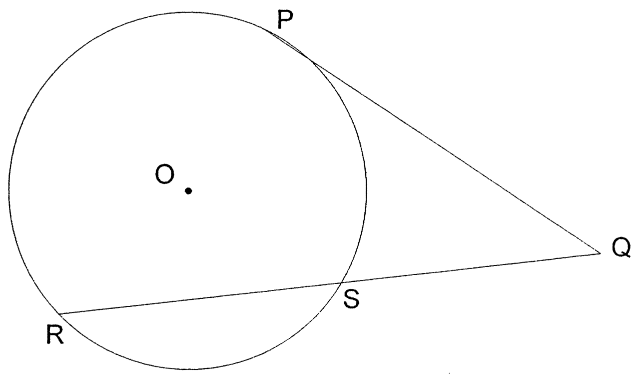
d) Point A has coordinates (7, 4) and P has coordinates (4, -2).
Point P divides the interval joining AB externally in the ratio of 3 : 2.
Determine the coordinates of point B. **3**

e) Use the substitution $t = u^2 - 1$ to evaluate $\displaystyle\int_0^1 \frac{t}{\sqrt{1+t}}\,dt$ **3**

Marks

Question 2 Use a SEPARATE writing booklet. *(12)*

a) In the circle centre O, the tangent PQ is 4 cm. The secant RQ is x cm and the chord RS is y cm.



i) Show that $y = x - \frac{16}{x}$ 1

ii) Show that as x increases, so does y. 2

b) $P\ (2ap, ap^2)$, $Q\ (2aq, aq^2)$ and $R\ (2ar, ar^2)$, where $p < q < r$, are three points on the parabola $x^2 = 4ay$. The tangent to the parabola at point Q has a gradient of q. 3

If the chord PR is parallel to the tangent at Q, show that p, q and r are consecutive terms in an arithmetic sequence.

c) Find the exact value of $\int_0^1 \frac{1}{\sqrt{4 - 3x^2}}\, dx$ 3

d) The equation $\ln x = \sin x$ has a first approximation for its solution at x =2.5. Use Newton's method once to find a better approximation. 3

Marks

Question 3 Use a SEPARATE writing booklet. (12)

a) Evaluate $\lim_{x \to 0} \frac{\sin 4x}{3x}$. 1

b) Evaluate $\int_0^{\frac{\pi}{4}} \sin x \cos^2 x \, dx$ and express your answer in the simplest surd form. 3

c) Find $\int \cos^2 9x \, dx$ 2

d) A spherical balloon is expanding so that its volume is increasing at the rate of $24 \, cm^3/s$. Determine the rate at which its surface area is increasing when the radius is 8 cm. 3

e) A particle is moving with acceleration given by $\ddot{x} = -2\sin x \; cm/s^2$. 3

Initially the particle is at the origin and its velocity, $v = 4$ cm/s.
Show that $v = \; + 2\sqrt{\cos x + 3}$

Marks

Question 4 Use a <u>SEPARATE</u> writing booklet. (12)

a) i) Use the expansion of the equation 2

$(1+x)^{n+1} = (1+x)(1+x)^n$ to show that :

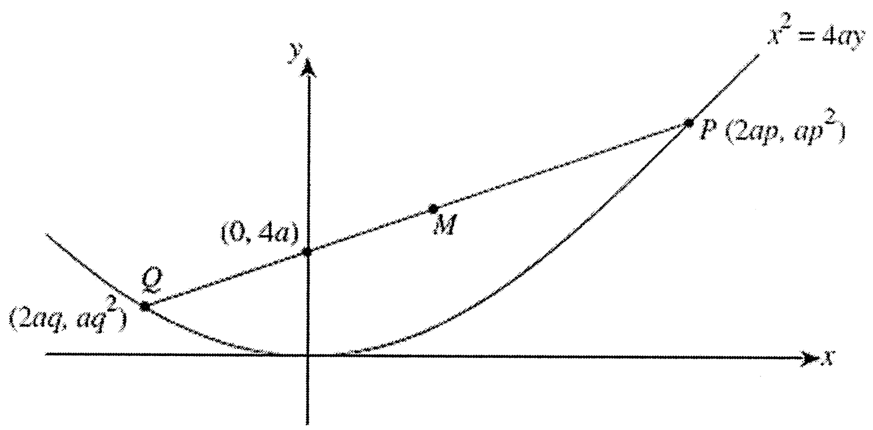
$$\binom{n+1}{2} = \binom{n}{1} + \binom{n}{2}$$

(ii) By differentiation of $(1+x)^{2n}$ show that

$$\binom{2n}{1} + 2\binom{2n}{2} + 3\binom{2n}{3} + \dots\dots + 2n\binom{2n}{2n} = n.4^n$$ 2

b) Show that $\tan 2x + \tan x = \dfrac{\sin 3x}{\cos 2x \cos x}$ 2

c) The diagram shows the graph of the parabola $x^2 = 4ay$.
The points $P\ (2ap, ap^2)$ and $Q\ (2aq, aq^2)$ lie on the parabola.



(i) Determine the coordinates of M, the midpoint of the chord PQ. 1

(ii) The chord PQ passes through the point $(0, 4a)$. Show that $pq = -4$ 3

(iii) Hence determine the locus of M as P and Q move along the parabola. 2

Marks

Question 5 Use a SEPARATE writing booklet. (12)

a) Use mathematical induction to prove that, for $n \geq 1$ **4**

$$1 \times 2 + 3 \times 4 + 5 \times 6 + \ \ldots \ + (2n-1) \times 2n = \tfrac{1}{3}n(n+1)(4n-1)$$

b) Consider the function $y = 2\sin^{-1} 3x$

(i) State its domain and range. **2**

(ii) Sketch the function. **1**

(iii) Find the area, in the first quadrant, between the curve, the y-axis and $y = 0$ and $y = \pi$. **2**

c) A bottle of medicine which is initially at a temperature of 10^0C is placed in a room which has a constant temperature of 25^0C. The medicine warms at a rate proportional to the difference between the temperature of the room and the temperature (T) of the medicine.
That is, T satisfies the equation

$$\frac{dT}{dt} = -k(T-25) \ .$$

(i) Show that $T = 25 + Ae^{-kt}$ satisfies this equation. **1**

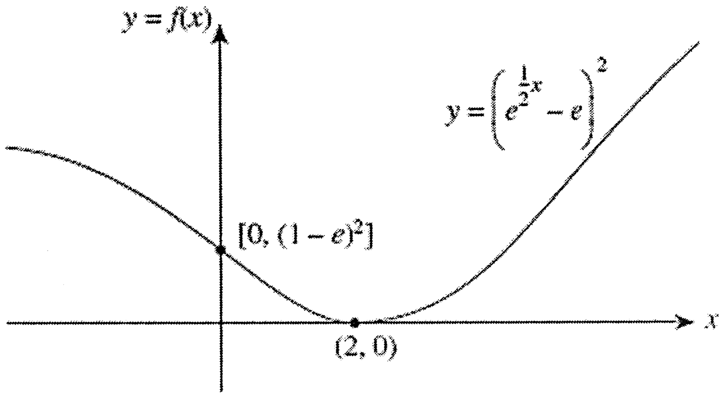
(ii) If the temperature after ten minutes is 16^0C, find the value of k. **2**

Marks

Question 6 Use a SEPARATE writing booklet. (12)

a) In Aunt Emily's pantry, the probability that any one item will be past the use-by date is 0·3. Determine the probability that of 12 items in Aunt Emily's pantry, no more than two will be past the use-by date. Express your answer as a decimal correct to two decimal places. 2

b) The diagram shows the graph of $f(x) = \left(e^{\frac{x}{2}} - e\right)^2$.



(i) Explain mathematically why the inverse of $f(x) = \left(e^{\frac{x}{2}} - e\right)^2$ is not a function. 2

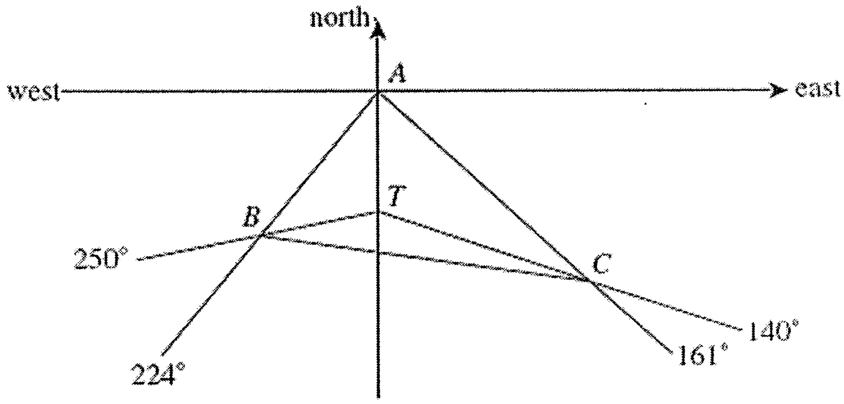
(ii) What is the largest domain that includes $x = e$ for which $f(x) = \left(e^{\frac{x}{2}} - e\right)^2$ has an inverse function, $f^{-1}(x)$? 1

(iii) Determine the equation of the inverse function, $f^{-1}(x)$ 2

Marks

Question 6 *(continued)*

c) From point A, the bearings of B and C are 224° and 161° respectively. From point T, 30 km due south of A, the bearings to B and C are 250° and 140° respectively.



(i) Show that the distance from B to C in kilometres is given by 3

$$(BC)^2 = 900\left[\left(\frac{\sin 44}{\sin 26}\right)^2 + \left(\frac{\sin 19}{\sin 21}\right)^2 - \frac{2\sin 44 \sin 19 \cos 110}{\sin 26 \sin 21}\right]$$

(ii) Hence, or otherwise, determine the time it will take to sail from B to C at an average speed of 10 km/h. 2

Marks

Question 7 Use a SEPARATE writing booklet. (12)

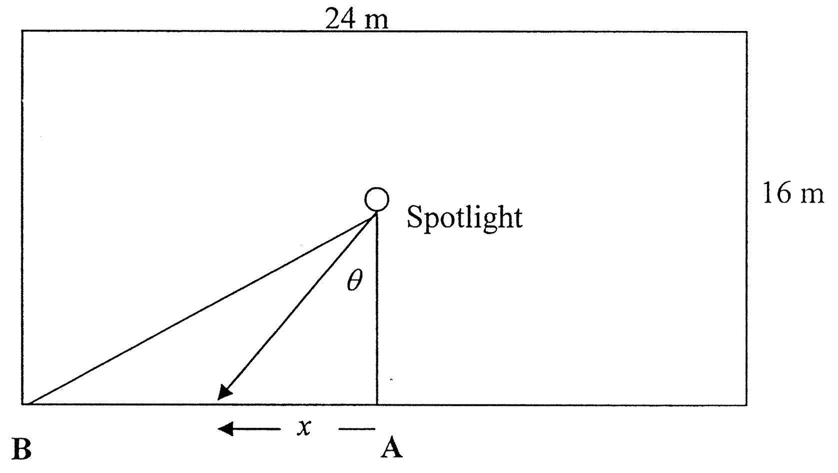
(a) Consider the curve $f(x) = x^3 - 3Bx^2 + 24$ where B is an integer with a constant value.

(i) Show that the curve has stationary points at $x = 0$ and $x = 2B$. Determine the nature of the stationary points. 3

(ii) The polynomial $y = f(x)$ has one root and only one root - at $x = \alpha$. Show, by graphical means or otherwise, that it is impossible for α to have a positive value. 2

(iii) Determine the value(s) B can take when α is the only root and $\alpha < 0$. 2

(b) A spotlight is in the centre of a rectangular nightclub which measures 24 m by 16 m. It is spinning at a rate of 20 rev/min. Its beam throws a spot which moves along the walls as it spins.



(i) Write the rate of rotation $\frac{d\theta}{dt}$ in radians/sec. 1

(ii) The spot moves along the wall from A to B at a velocity of $\frac{dx}{dt}$. 2

Show that $\frac{dx}{dt} = \frac{16\pi}{3}\left(1 + \frac{x^2}{64}\right)$

(iii) What is the difference in the velocities at which the spot appears to be moving at the points A, nearest to the light and B, furthest from the light? 2

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq 1;\ x \neq 0,\ \textit{if}\ n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

Where $\ln x = \log_e x,\ x > 0$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Question 1

(a)	${}^8C_3 \times {}^4C_2 = 336$	1 mark: correct answer
(b)	$x = \sin\theta$ *i.e.* $x^2 = \sin^2\theta$ $y = \cos^2\theta - 3$ *i.e.* $y + 3 = \cos^2\theta$ *but* $\sin^2\theta + \cos^2\theta = 1$ $\therefore x^2 + y + 3 = 1$ *i.e.* $y = -x^2 - 2$	2 marks: correct working and answer 1 mark: correct method with an error
(c)	*Consider* $\frac{x^2}{x-2} = -1$ $x \neq 2, \quad x^2 = -x + 2$ $x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ *Critical points are* $-2, 1$ *and* 2 *Test* $x = 3$, *true* *test* $x = 1\frac{1}{2}$, *false* *test* $x = 0$, *true* *test* $x = -3$, *false* $\therefore$ *solution is* $x > 2$ *or* $-2 \le x \le 1$	3 marks: correct soln by a valid method 2 marks: correct method but with an error 1 mark: stating x cannot equal 2
(d)	$A(7,4),\ P(4,-2), B(x,y) \qquad m:n = 3:-2$ $(4,-2) = \left(\frac{3x - 2.7}{3-2}, \frac{3y - 2.4}{3-2}\right)$ $\therefore 3x - 14 = 4 \qquad 3y - 8 = -2$ $3x = 18 \qquad 3y = 6$ $x = 6 \qquad y = 2 \qquad \therefore P = (6,2)$	3 marks: correct soln by a valid method 2 marks: correct method with an error 1 mark: correct method with multiple errors, or formula but wrong substitution

(e)	$t = u^2 - 1$ $dt = 2udu$ $t = 1 \quad u = \sqrt{2}$ $t = 0 \quad u = 1$ $\int_0^1 \frac{tdt}{\sqrt{1+t}} = \int_1^{\sqrt{2}} (u^2 - 1)\frac{2udu}{u}$ $= 2\int_1^{\sqrt{2}} (u^2 - 1)du$ $= 2\left[\frac{u^3}{3} - u\right]_1^{\sqrt{2}}$ $= 2\left[\left(\frac{2\sqrt{2}}{3} - \sqrt{2}\right) - \left(\frac{1}{3} - 1\right)\right]$ $= \frac{-2\sqrt{2}}{3}$	3 marks: correctly substituting and finding and evaluating integral 2 marks :correct method, mistake in evaluation, or incorrect limits 1 mark: showing correct substitution

Question 2

(a) (i)	$QP^2 = QS \times QR$ $4^2 = (x-y) \times x$ $\frac{16}{x} = x - y$ $y = x - \frac{16}{x}$	*1 mark – correct proof*
(ii)	EITHER $\frac{dy}{dx} = 1 + \frac{16}{x^2}$ $\therefore$ as $x \to \infty$ $\frac{dy}{dx} \to \infty$ Hence y is an increasing function in x OR $y = x - \frac{16}{x}$ as x $\to \infty$ $\frac{16}{x} \to 0$ $\therefore y \to x$ and as $x \to \infty$ $y \to \infty$	*2 marks – correct demonstration* *1 mark – demonstration approached correctly*
(b)	Gradient of PR $= \frac{ap^2 - ar^2}{2ap - 2ar}$ $= \frac{p+r}{2}$ Gradient at Q is q (by definition) $\therefore \frac{p+r}{2} = q$ – so q is the arithmetic mean of p and r OR let q = p + d and r = p + 2d $\therefore \frac{p+r}{2} = \frac{p+p+2d}{2}$ $= p + d = q$ $\therefore$ p, q and r are in arithmetic progression	*3 marks - correct demonstration* *2 marks – gradients equated but subsequent error* *1 mark – gradients established correctly*

Question 2 (continued)

(c)	$\int_0^1 \frac{1}{\sqrt{4-3x^2}}\,dx = \int_0^1 \frac{1}{\sqrt{3} \times \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 - x^2}}$ $= \frac{1}{\sqrt{3}}\left[\sin^{-1}\left(\frac{\sqrt{3}\,x}{2}\right)\right]_0^1$ $= \frac{1}{\sqrt{3}}\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}(0)\right)$ $= \frac{1}{\sqrt{3}} \times \frac{\pi}{3} = \frac{\pi}{3\sqrt{3}}$	*3 marks - correct answer* *2 marks – correct integration but substitution error OR 1/a coefficient used by mistake and substitution correct* *1 mark – fraction transformed appropriately*
(d)	Equation is $f(x) = lnx - sinx$ $f'(x) = \frac{1}{x} - cosx$ Remember as x= 2.5, this measure is in radians $f(2.5) = ln\ 2.5 - sin\ 2.5$ (radians) $= 0.3178$ $f'(2.5) = 1.2011$ $x_2 = 2.5 - \frac{0.3178}{1.2011} = 2.235$ NOT 3.9568 if x taken as degrees	*3 marks - correct answer* *2 marks – correct differentiation and substitution but 2.5 used as degrees* *1 mark – correct differentiation but subsequent error*
	Comments: (a ii) and (b) Marks were given only when statements were precise. Too often assumptions were substituted into a proof and even actual numbers were substituted to establish a trend – against all what has been said. (c) The coefficient for the square root must be explicitly considered by many as most students had no idea of what it was – or even if it existed!!!	

Question 3

(a)	$\frac{1}{3}\lim_{x\to 0}\frac{\sin 4x}{x}$ $=\frac{4}{3}\lim_{x\to 0}\frac{\sin 4x}{4x}$ $=\frac{4}{3}\times 1$ $=\frac{4}{3}$	1 mark - Gives the correct answer, with no wrong method displayed
(b)	$\int_0^{\frac{\pi}{4}}\sin x\cos^2 x\,dx=-\left[\frac{1}{3}\cos^3 x\right]_0^{\frac{\pi}{4}}$ $=-\frac{1}{3}\left[\cos^3\frac{\pi}{4}-\cos^3 0\right]$ $=\frac{2\sqrt{2}-1}{6\sqrt{2}}$ or $\frac{4-\sqrt{2}}{12}$	2 marks - Gives the correct answer• 1 mark - Gives the correct integral
(c)	$\int\cos^2 9x\,dx=\int\frac{1}{2}(1+\cos 18x)\,dx$ $=\frac{1}{2}\left(x+\frac{1}{18}\sin 18x\right)+c$ $=\frac{x}{2}+\frac{1}{36}\sin 18x+c$	2 marks – correct answer 1 mark – correct change of $\cos^2 9x$ *to* $\frac{1}{2}(1+\cos 18x)$
(d) .	$\frac{dV}{dt}=24\ \text{cm}^3/\text{s},\ r=8$ $\frac{dSA}{dt}=\frac{dSA}{dr}\times\frac{dr}{dv}\times\frac{dv}{dt}$ $=8\pi r\times\frac{1}{4\pi r^2}\times 24$ $=\frac{48}{r}$ $=6\ \text{cm}^2/\text{s}$	3 marks - Gives the correct answer 2 marks - Determines $\frac{dSA}{dt}=\frac{dSA}{dr}\times\frac{dr}{dv}\times\frac{dv}{dt}$ or equivalent. Uses one piece of information correctly . 1

Question 3 (continued)

(e)	$\frac{d}{dx}\left(\frac{1}{2}v^2\right)=-2\sin x$ $\frac{1}{2}v^2=2\cos x+C$ given $t=0, x=0, v=4$ $8=2\cos 0+C$ $C=6$ $\frac{1}{2}v^2=2\cos x+6$ $v^2=4\cos x+12$ $v=\pm 2\sqrt{\cos x+3}$ but $v>0$ when $x=0$, so + is required $\therefore v=+2\sqrt{\cos x+3}$	3 marks - Gives a correct demonstration 2 marks - Gives an essentially correct demonstration that omits checking ± or similar . . . . 2 • Uses the . . . $\frac{d\left(\frac{1}{2}v^2\right)}{dx}$ 1

Question 4

(a) (i)	$(1+x)^{n+1} = \binom{n+1}{0}x^0 + \binom{n+1}{1}x^1 + \binom{n+1}{2}x^2 \ldots$ $(1+x)(1+x)^n = (1+x)\left(\binom{n}{0}x^0 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \ldots\right)$ Equating coefficients of x^2 $\binom{n+1}{2} = \binom{n}{2} + 1 \times \binom{n}{1}$	*2 marks – correct proof* *1 mark – correct expansions*
(ii)	$(1+x)^{2n} = \binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \binom{2n}{3}x^3 + \ldots + \binom{2n}{2n}x^{2n}$ Differentiating both sides $2n(1+x)^{2n-1} = \binom{2n}{1} + 2\binom{2n}{2}x + 3\binom{2n}{3}x^2 + \ldots + 2n\binom{2n}{2n}x^{2n}$ Let x =1 $2n \times 2^{2n-1} = \binom{2n}{1} + 2\binom{2n}{2} + 3\binom{2n}{3} + \ldots + 2n\binom{2n}{2n}$ *But* $2n \times 2^{2n-1} = n \times 2^{2n} = n \times 4^n$ $\therefore n \times 4^n = \binom{2n}{1} + 2\binom{2n}{2} + 3\binom{2n}{3} + \ldots + 2n\binom{2n}{2n}$	*2 marks – correct proof* *1 mark – correct expansions*
(b)	RHS $= \dfrac{\sin 3x}{\cos 2x \cos x}$ $= \dfrac{\sin 2x \cos x + \cos 2x \sin x}{\cos 2x \cos x}$ $= \tan 2x + \tan x =$ LHS	*2 marks – correct proof* *1 mark – correct expansions*

Question 4 (continued)

(c)(i)	Midpoint is $X = \dfrac{2ap + 2aq}{2} = a(p+q)$ $Y = \dfrac{ap^2 + aq^2}{2} = \dfrac{a(p^2+q^2)}{2}$	*1 mark – both coordinates correct*
(ii)	Eqn of PQ is $\dfrac{y - ap^2}{aq^2 - ap^2} = \dfrac{x - 2ap}{2aq - 2ap}$ If passing through (0,4a) $\dfrac{a(4-p^2)}{a(q-p)(q+p)} = \dfrac{-2ap}{2a(q-p)}$ $4 - p^2 = -(pq + p^2)$ $pq = -4$	*3 marks – correct proof* *2 marks – substitution correct but subsequent error* *1 mark – equation formed*
(iii)	$p + q = \dfrac{X}{a}$ $\dfrac{2Y}{a} = p^2 + q^2 = (p+q)^2 - 2pq$ $= \dfrac{X^2}{a^2} - 2(-4)$ $2aY = X^2 + 8a^2$	*2 marks – correct solution* *1 mark – combined equation formed*

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2007 HSC Mathematics Extension 1 – Trial - solutions

Question 5

(a)	*Prove that* $1\times 2 + 3\times 4 + ... + (2n-1)\times 2n = \frac{1}{3}n(n+1)(4n-1)$ *Step 1: Let* $n=1$ *LHS* $= 1\times 2$ *RHS* $= \frac{1}{3}1\times 2\times 3$ $\therefore$ *true for* $n = 1$ *Step 2: Assume true for n=k* *i.e.* $1\times 2 + 3\times 4 + ... + (2k-1)\times 2k = \frac{1}{3}k(k+1)(4k-1)$ *Try to show true for n=k+1,* *i.e. that* $1\times 2 + 3\times 4 + ... + (2k-1)\,2k + (2k+1)\,(2k+3) =$ $\frac{1}{3}(k+1)\,(k+2)\,(4k+3)$ *LHS* $= 1\times 2 + 3\times 4 + ...(2k-1)\,2k + (2k+1)(2k+2)$ $= \frac{1}{3}k(k+1)(4k-1) + (2k+1)(2k+2)$ *from assumption*	4 marks: correct soln 3 marks: correct method but with one error in algebra 2 marks: n=1 correct and correct n=k+1 line, multiple errors in algebra 1 mark: correctly showing for n=1
	$= (k+1)\left[\frac{1}{3}k(4k-1) + 2(2k+1)\right]$ $= \frac{k+1}{3}\left[4k^2 - k + 12k + 6\right]$ $= \frac{k+1}{3}\left[4k^2 + 11k + 6\right]$ $= \frac{k+1}{3}(k+2)(4k+3)$ $=$ *RHS* $\therefore$ *if true for n=k then true for n=k+1* *Step 3: But true for n=1,* $\therefore$ *true for n=1+1=2, and similarly for* *n=3,4,5,....and all posive integral n*	

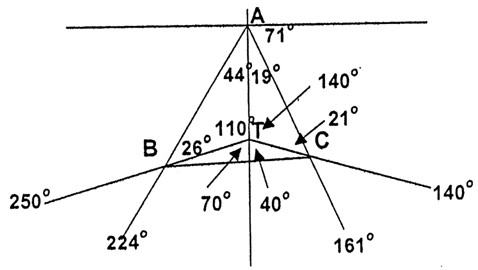
(b)(i)	$y = 2\sin^{-1}3x$ *for domain* $-1 \le 3x \le 1$ *i.e.* $-\frac{1}{3} \le x \le \frac{1}{3}$ *for range* $-\frac{\pi}{2} \le \sin^{-1}3x \le \frac{\pi}{2}$ *i.e.* $-\pi \le y \le \pi$	1 mark: correct domain 1 mark: correct range 1 mark: correct diagram
(iii)	*Area* $= \int_0^{\pi} x\,dy$ $y = 2\sin^{-1}3x$ $x = \frac{1}{3}\sin\left(\frac{y}{2}\right)$ $= \int_0^{\pi} \frac{1}{3}\sin\left(\frac{y}{2}\right)dy$ $= \left[-\frac{1}{3}\frac{\cos\left(\frac{y}{2}\right)}{\frac{1}{2}}\right]_0^{\pi}$ $= \left(-\frac{2}{3}\cos\left(\frac{\pi}{2}\right)\right) - \left(-\frac{2}{3}\cos 0\right)$ $= 0 - \left(-\frac{2}{3}\right)$ $= \frac{2}{3}u^2$	2 marks: correct answer by appropriate working 1 mark: correct method but with one mistake

(c)(i)	$T = 25 + Ae^{-kt}$ $\frac{dT}{dt} = -kAe^{-kt}$ *but* $Ae^{-kt} = T - 25$ $\therefore \frac{dT}{dt} = -k(T-25)$ *as required*	1 mark: correctly showing
(ii)	*when t=0, T=10,* $\therefore$ $10 = 25 + Ae^{0}$ i.e. $A = -15$ *when t=10, T=16,*$\therefore$ $16 = 25 - 15e^{-10k}$ $15e^{-10k} = 9$ $e^{-10k} = \frac{3}{5}$ $-10k = \ln\left(\frac{3}{5}\right)$	2 marks: correctly finding value of k 1 mark: correct value of A

Question 6

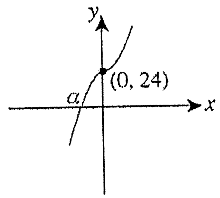
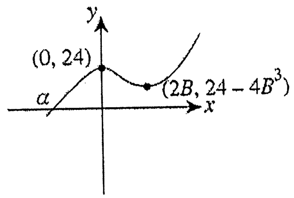
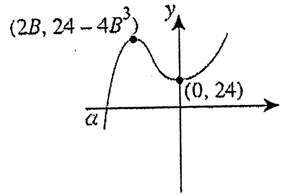
(a)	Pr(use-by) = 0.3 Pr(no more than 2) = Pr(0 items)+Pr (1 item)+pr(2 items) $= {}^{12}C_0\,(0.3)^0(0.7)^{12} + {}^{12}C_1(0.3)^1(0.7)^{11} + {}^{12}C_2\,(0.3)^2(0.7)^{10}$ = 0.25	*2 marks - correct answer* *1 mark – correct statement with binomial probability but arithmetic error*
(b)	(i) The inverse of f(x) is not a function because there is a turning point in the domain. Hence for any given value on the range, there are two possible x values. An inverse of f(x) ($f^{-1}(x)$)therefore has two possible y values for a given x value which does not conform to the definition of a function.	*2 marks - complete answer* *1 mark – partially correct statement*
	(ii) For the function to be an inverse, the function on either side of the stationary point must be defined. Hence the largest domain is $x \geq 2$ or $x \leq 2$ NOTE: the answer x>2 is not acceptable.	1 mark – correct domain
(iii)	$y = \left(e^{\frac{1}{2}x} - e\right)^2$ $x = \left(e^{\frac{1}{2}y} - e\right)^2$ $\sqrt{x} = e^{\frac{1}{2}y} - e$ $e^{\frac{1}{2}y} = e + \sqrt{x}$ $y = 2\ln(e + \sqrt{x})$	*2 marks –inverse correctly determined* *1 mark – interchange of x and y and some progress made to determine inverse.*
	NOTES: In part (i) ,a common error resulted from misreading the question – the emphasis should have been on the INVERSE not being a function but many answers test the original equation and showed that was (or was not) a function. In addition, there is no such thing as the "horizontal line test" and so it cannot be quoted as evidence.	

Question 6 (continued)

(c) (i)	In these questions, always begin by analysing the diagram to identify where the angles are which are listed in the "to prove" statement.  First use the sine rule to $\Delta\ ABT$ to find BT $BT = 30 \times \frac{\sin 44}{\sin 26}$ Then examine $\Delta\ ATC$ to find TC $TC = 30 \times \frac{\sin 19}{\sin 21}$ Now using ΔBTC with the cos rule focussing on angBTC = 70+40=110 $BC^2 = BT^2 + TC^2 - 2 \times BT \times TC \times \cos BTC$ $= \left(30^2 \times \frac{\sin 44}{\sin 26}\right)^2 + 30^2 \times \left(\frac{\sin 19}{\sin 21}\right)^2$ $- 2 \times \left(\frac{\sin 44}{\sin 26}\right) \times \left(\frac{\sin 19}{\sin 21}\right) \times \cos 110)$	*3 marks- correct proof for BC^2* *2 marks – expressions determined for BT and CT but subsequent error or lack of justification.* *1 mark – correct statement for either BT or TC or significant attempt(with justification) in either triangle to develop the sine rule relationship.*
(ii)	BC = 62.36 Hence at 10km/hr time = 6.236 hours = 6 hours 14 minutes	*2 marks - correct answer* *1 mark – correct answer for BC reported*
	NOTE: In (c) (i) it is essential that all angles used are shown and proved to have the values attributed. This can be done even through a diagram.	

Question 7

7 (a) - i	(i) $f'(x) = 3x^2 - 6Bx = 0$ $3x(x - 2B) = 0$ $\therefore x = 0$ or $x = 2B$ Therefore, the stationary points are $(0, 24)$ and $(2B, 24 - 4B^3)$ $f''(x) = 6x - 6B$ When $B > 0$ $f''(0) < 0$ $(0, 24)$ maximum $f''(2B) > 0$ $(2B, 24 - 4B^3)$ minimum When $B < 0$ $f''(0) > 0$ $(0, 24)$ minimum $f''(2B) < 0$ $(2B, 24 - 4B^3)$ maximum When $B = 0$ $(0, 24)$ will be a horizontal inflexion	3 marks – full answer. 2 marks – for considering B either neg. or pos. 1 mark – showing stationary points but not taking into account different possibilities for B

7(a) - ii	There are three possible graphs for $y = f(x)$ with exactly one root. $B = 0$  $B > 0$  $B < 0$  In each case, $\alpha < 0$. Therefore, α cannot be positive.	2 marks – complete answer 1 mark – explaining one case only of three possible cases.

7 (a) - iii	When $B = 0$, there is a valid solution (see graph). When $B > 0$, $0 < 24 - 4B^3 < 24$ $-24 < -4B^3 < 0$ $\therefore 0 < 4B^3 < 24$ $0 < B^3 < 6$ $0 < B < \sqrt[3]{6}$ As B is an integer, B could be 1. When B < 0, obviously from the graph all values will satisfy. algebraically, $24 - 4B^3 > 24$ $\therefore -4B^3 > 0$ $B < 0$ Thus, as B is an integer, B could have the values $\{1, 0, -1, -2 \ldots\}$.	2 marks – correct answer. 1 mark – attempt to use 24 - 4B³ in a correct manner.
(b) - i	20 rev/min $= 20 \times 2\pi$ rad/min $= \frac{40\pi}{60} = \frac{2\pi}{3}$ rad/sec	1 mark – correct answer
(b) - ii	$\tan\theta = \frac{x}{8}$ $x = 8\tan\theta$ $\frac{dx}{d\theta} = 8\sec^2\theta$ $\frac{dx}{dt} = \frac{dx}{d\theta}.\frac{d\theta}{dt}$ $= 8\sec^2\theta.\frac{2\pi}{3}$ $= \frac{16\pi}{3}\sec^2\theta$ $= \frac{16\pi}{3}(1+\tan^2\theta)$ $= \frac{16\pi}{3}(1+\tan^2\theta)$ $= \frac{16\pi}{3}\left(1+\left(\frac{x}{8}\right)^2\right)$ $= \frac{16\pi}{3}\left(1+\frac{x^2}{64}\right)$	2 marks for full solution 1 mark if done in terms of θ or otherwise incomplete

(b) - iii	At A, $x = 0$ $\frac{dx}{dt} = \frac{16\pi}{3}\left(1+\left(\frac{0}{8}\right)^2\right)$ $= \frac{16\pi}{3}$ At B, $x = 12$ $\frac{dx}{dt} = \frac{16\pi}{3}\left(1+\left(\frac{12}{8}\right)^2\right)$ $= \frac{16\pi}{3}\left(\frac{13}{4}\right)$ $= \frac{52\pi}{3}$ Difference $= \frac{52\pi - 16\pi}{3}$ $= 12\pi$ m/s	2 marks – correct answer 1 mark if only one found correctly or if subtraction incorrect.