Northern Beaches Secondary College
Manly Selective Campus

2010
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators and templates may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1- 7
- All questions are of equal value

Marks

Question 1 *(Answer in a separate booklet)* *(12)*

a) Solve for x in the following. $\dfrac{2x}{x-2} \le 5$ *(2)*

b) Determine the size of the acute angle formed by the intersection of the lines $2x + y = 3$ and $3x - 5y = 10$. *(2)*

c) How many ways could the letters in the word LEVEL be arranged if the two letter E's are together? *(2)*

d) Show that $(2x + 1)$ is a factor of $P(x) = 2x^3 - x^2 - 13x - 6$. *(1)*

Hence find a second expression which is a factor of $P(x)$ *(1)*

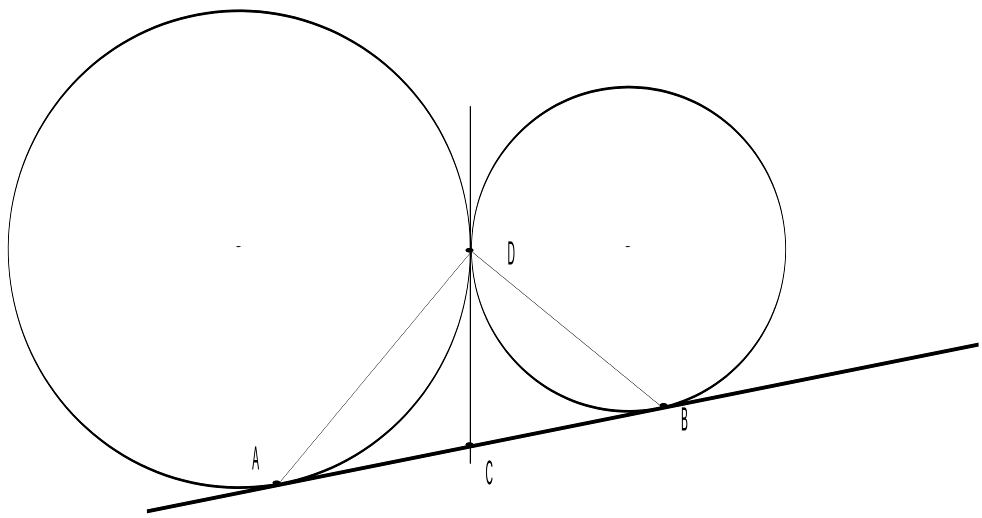
e) Find $\int \sin^2 3x \, . \, dx$ *(2)*

f) Determine the exact value of $sin 165^{o}$. *(2)*

Marks

Question 2 *(Answer in a separate booklet)* *(12)*

a)



Two circles have a common tangent CD and meet externally at D.
AB is another common tangent with A and B being points of tangency.
AB and CD intersect at C.

(i) Give reasons to explain why AC = BC *(1)*

(ii) Prove that <ADB =90^{o} *(3)*

b) One quarter of all jellybeans in a mixture of jellybeans are black. A random sample of ten jellybeans is taken. Find the probability that the sample contains

(i) no black jellybeans *(1)*

(ii) two black jellybeans *(2)*

c) Prove by mathematical induction for all positive integers n
that $13 \times 6^n - 3$ is divisible by 5 *(3)*

d) Find the constant term in the expansion of

$$\left(x - \frac{1}{2x^2}\right)^{15}$$ *(2)*

Marks

Question 3 *(Answer in a separate booklet)* *(12)*

a) From the top of a lighthouse 55 metres high, a boat is observed due south at an angle of depression of $18^{o}50'$. The boat is sailing due east and after 5 minutes, the angle of depression is found to be $12^{o}12'$.

Calculate the speed of the boat in km/hr to the nearest km. *(4)*

b) The equation $x^3 - 3x^2 + 9 = 0$ has a single root. In an effort to determine its value, Dani takes $x = 3$ as her first approximation for that root and uses Newton's method twice.

(i) Interpret the value you have calculated based on Dani's choice for the value of the root. *(2)*

(ii) What would be a better starting approximation? Give reasons for your answer. *(2)*

c) (i) Show that $\dfrac{u}{u+1} = 1 - \dfrac{1}{u+1}$ *(1)*

(ii) Hence find $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$ using the substitution $x = u^2$ ($u \geq 0$). *(3)*

Marks

Question 4 *(Answer in a separate booklet)* *(12)*

(a) Four girls and two boys are to be seated ***around*** a table.

(i) How many seating arrangements are possible without restriction? *(1)*

(ii) Determine the probability that if the seating is arranged randomly that the two boys do **NOT** sit together. *(3)*

(b) A monic polynomial, when divided by $x^2 - 4$ leaves a remainder of $7x - 4$ and, when divided by x, there is a remainder of -8.

Write out the polynomial in the form $P(x) = ax^3 + bx^2 + cx + d$

(4)

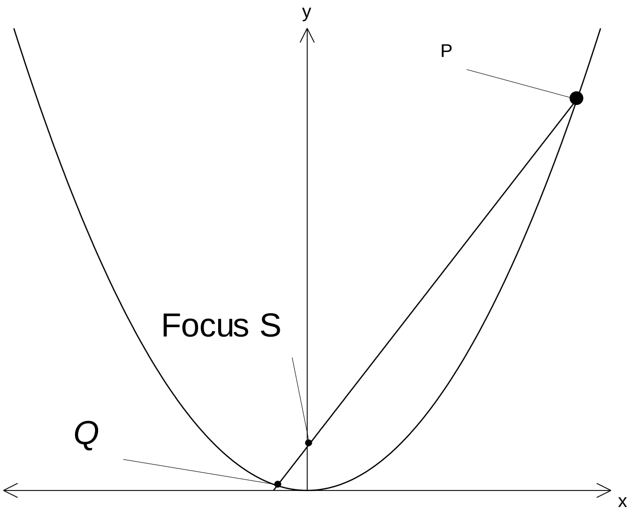
(c) (i) Show that $\cos\left(\frac{\pi}{2} + A\right) = -\sin A$ for all A *(1)*

(ii) Taking $A = 5\theta$, write down the value of B such that $-\sin 5\theta = \cos B$.
Hence find the least value of θ, where $0 \le \theta \le 2\pi$,
such that $\sin 5\theta + \cos 8\theta = 0$ *(3)*

Marks

Question 5 *(Answer in a separate booklet)* *(12)*

(a)



PQ is a chord on the parabola $x^2 = 4ay$ drawn above. If the parametric coordinates of P and Q are respectively $(2ap, ap^2)$ and $(2aq, aq^2)$ then show that:

(i) the equation of the chord PQ is $y = \dfrac{p+q}{2}x - apq$ *(3)*

(ii) if PQ is a focal chord then $pq = -1$ *(1)*

(iii) the length of the focal chord PQ is $a\left(p + \dfrac{1}{p}\right)^2$ *(2)*

(b) (i) Simplify $\cos^{-1}\left(\cos\left(\dfrac{4\pi}{3}\right)\right)$ *(1)*

(ii) Find $\dfrac{d}{dx}\left(\tan^{-1}\left(\dfrac{x}{3}\right)\right)$ in simplest form *(2)*

(iii) Evaluate $\displaystyle\int_{0.25}^{0.5} \frac{dx}{\sqrt{1-4x^2}}$ *(3)*

Marks

Question 6 *(Answer in a separate booklet)* *(12)*

(a) Assume that the rate at which a body warms in air is proportional to the difference between its temperature T and the constant room temperature A of the surrounding air. The rate can be expressed by the differential equation $\frac{dT}{dt} = k(T - A)$ where t is the time in minutes and k is a constant.

(i) Show that $T = A + C.e^{kt}$, where C is a constant, is a solution of the differential equation. *(1)*

(ii) A cooled body warms from 5^0C to 10^oC in 20 minutes. The air temperature around the body is 25^oC.

Using this information, show that the value for $k = 0.05 \times \ln 0.75$ *(2)*

(iii) Determine the temperature of object of the body after a further 40 minutes have elapsed. Give your answer to the nearest degree. *(2)*

(iv) By referring to the equation for T, explain the behaviour of T as t becomes larger. *(1)*

(b) Show that $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ *(1)*

(c) The acceleration of a particle moving in a straight line is given by $\frac{d^2x}{dt^2} = 2x - 3$ where x is the displacement, in metres, from the origin O and t is the time in seconds. Initially the particle is at rest $x = 4$.

(i) If the velocity of the particle is $v\ ms^{-1}$, determine and expression for v^2 as a function of x. *(2)*

(ii) Show that the particle does not pass through the origin. *(1)*

(iii) Determine the position of the particle when $v = 10$. Justify your answer. *(2)*

Marks

Question 7 *(Answer in a separate booklet)* *(12)*

a) If $(1 + x)^7 = 1 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_7x^7$ and

$(1 - x)^{10} = 1 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 \ \ldots + b_{10}\, x^{10}$

Find:

(i) the value of $a_4b_7 - b_4a_7$ *(2)*

(ii) the greatest value of a_nb_n where $1 \leq n \leq 7$. *(2)*

b) A particle moving in simple harmonic motion has a period of $\frac{\pi}{3}$ sec and a maximum velocity of 2 metres/sec. Find the amplitude of the motion and its displacement after $\frac{\pi}{6}$ seconds. *(2)*

c) The displacement x metres of a particle moving in simple harmonic motion is given by $x = 4 \sin \pi$ t where t is the time expressed in seconds.

(i) What is the period of oscillation? *(1)*

(ii) What is the speed of the particle as it moves through the centre of motion? *(2)*

(iii) Show that the acceleration of the particle is proportional to the displacement from the equilibrium position (centre of motion). *(1)*

(iv) What is the velocity of the particle at the position x = 3? *(2)*

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

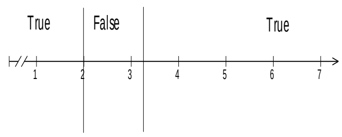
$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Manly Selective Campus
2010 HSC Mathematics Extension 1 – Trial HSC - Solutions

Question 1

(a)	$\dfrac{2x}{x-2} \le 5 \qquad \therefore x \ne 2$ $\dfrac{2x}{x-2} \le 5$ $2x \le 5x - 10$ $10 \le 3x$ $\dfrac{10}{3} \le x$  $\therefore \quad x < 2 \ \ or \ \ \dfrac{10}{3} \le x$	2 marks – correct answer 1 mark – incorrect inequality for 2
(b)	$2x + y = 3 \qquad$ m=-2 $3x - 5y = 10 \qquad$ m= $\dfrac{3}{5}$ $\tan\theta = \left\lvert \dfrac{m_1 - m_2}{1 + m_1 \times m_2} \right\rvert$ $= \left\lvert \dfrac{-2 - \frac{3}{5}}{1 + \frac{3}{5} \times -2} \right\rvert$ $= \left\lvert \dfrac{-13}{5} \times \dfrac{-5}{1} \right\rvert$ $\theta = \tan^{-1} 13$ $= 85^\circ 36'$	2 marks – correct answer 1 mark – correct substitution into correct formula
(c)	LEVEL No. of arrangements = $\dfrac{4!}{2!} = 12$	2 marks – correct answer 1 mark – incorrect denominator
(d) - i	P(x)=$2x^3 - x^2 - 13x - 6 \qquad x= -\dfrac{1}{2}$ $P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 - 13\left(-\frac{1}{2}\right) - 6$ $= 0$	1 marks - Correct demonstration.

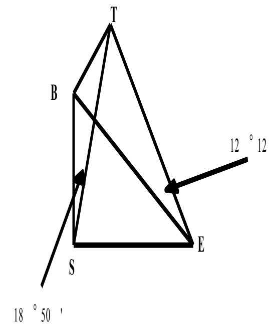
Question 1 (continued)

(d)ii	Factors of 6 = ±(1,2,3,6) P(1) = -18 P(2)=-20 P(3)=0 P(-2) = 0 Therefore (x-3) and (x + 2) are both factors	1 mark – correct answer
(e)	$\int \sin^2 3x\ .\ dx$ $= \frac{1}{2}\int 1 - \cos 6x\ .\ dx$ $= \frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + C$	2 marks – correct answer 1 mark – correct substitution.
(f)	$\sin 165^\circ = \sin(120 + 45)$ $= \sin 120 . \cos 45 + \sin 45 . \cos 120$ $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times -\frac{1}{2}$ $= \frac{\sqrt{3} - 1}{2\sqrt{2}}$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$	2 marks – correct answer 1 mark – correct expansion for compound angle.

Question 2

Q2(a) (i)	AC=DC= BC (tangents drawn from an external point are equal)	1- Correct reason
(ii)	As CA=CB=CD then C is the centre of a circle with A,B and D on its circumference. A,C and B are collinear thus <ACB =180^o AB is a diameter <ADB = $\frac{1}{2} \times \ < ACB \ = \ 90^\circ$ (Angle in a semi-circle is a right angle)	3- Correct reasons given for all steps. 2- All but one reason given 1- Determining that C is centre of circle
(b) (i)	$\left(\frac{1}{4} + \frac{3}{4}\right)^{10}$ P(no black) = $\left(\frac{3}{4}\right)^{10} \cong 0.0563$	1- correct answer
(b)(ii)	$\begin{bmatrix} 10 \\ 2 \end{bmatrix} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^8 \cong 0.2816$	2-Correct term of binomial and answer 1-Correct term
(c)	* Test true for n = 1 *LHS* = 13x6 -3 = 75 *RHS* *75* = 5x15 Thus true for n=1 * Assume true for $n = k$ Thus $13 \times 6^k - 3 = 5p$ (p is a positive integer) * Prove true for $n=k+1$ That is *LHS* = $13 \times 6^{k+1} - 3 = 5q = RHS$ Proof: *LHS*= $6 \times 13 \times 6^k - 3 = 6(5p + 3) - 3 = 30p + 15$ $= \quad 5(6p + 3) = 5q$ * Tested true for $n=1$, proved true for $n=k+1$, thus true for all n	3- Complete proof with , testing, assumption, proof and conclusion 2- setting out correct with assumption used at proof stage but incomplete thereafter 1- Test and assumption correct
(d)	*General term is:* $\begin{bmatrix} 15 \\ k \end{bmatrix} x^{15-k} \left(\frac{-1}{2x^2}\right)^k = \begin{bmatrix} 15 \\ k \end{bmatrix} \left(-\frac{1}{2}\right)^k x^{15-3k}$ $\therefore 15 - 3k = 0 \quad \therefore k = 5$ Term is: $\begin{bmatrix} 15 \\ 5 \end{bmatrix} \left(-\frac{1}{2}\right)^5 = -93\frac{27}{32}$	2- Correct term identified and value found 1- strong evidence of correct term being identified

Question 3

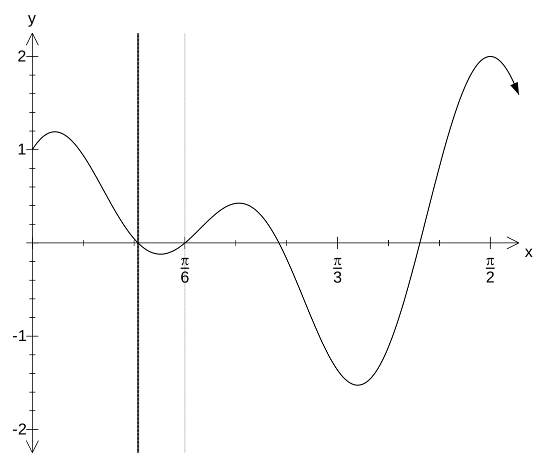
(a)	$\text{BS} = \dfrac{55}{\tan 18^\circ 50'}$ and $\text{BE} = \dfrac{55}{\tan 12^\circ 12'}$ $\therefore \quad \text{SE} = \sqrt{\dfrac{55^2}{\tan^2 12^\circ 12'} - \dfrac{55^2}{\tan^2 18^\circ 50'}}$ $= 196.75 \text{ m}$ $\therefore$ in one hour, distance = 2361 m $\therefore$ speed = 2 km/hr  *Note: Confusion over where an angle of depression is located. Clear need to draw a 3D diagram properly so you know where the right angle is and you don't think 2D (and hence just subtract).*	*4 marks – correct speed* *3 marks – correct distance in 1 hour* *2 marks – correct distance between positions* *1 mark – correct distances substituted into Pythagoras*
(b) (i)	$f(x) = x^3 - 3x^2 + 9$ $f'(x) = 3x^2 - 6x$ $f(3) = 9$ and $f'(3) = 9$ $\therefore x_1 = 3 - \dfrac{9}{9} = 2$ Dani used the method twice ... so $f(2) = 5$ and $f'(2) = 0$ $\therefore x_2 = 2 - \dfrac{5}{0}$ which is indeterminate so x = 2 must be a stationary point so Dani cannot obtain a better approximation with this starting point Indeed the derivative shows SP at x = 0 and x = 2.	*2 marks – can't proceed because gradient= 0 and so tangent being horizontal cannot cut through the x-axis.* *1 mark – at x = 2 there is a stationary point.* *NOTE: Surely by now the number $\frac{5}{0}$ looks different and is ≠ 0!!* *It is easy to identify stationary points from the derivative and hence sketch a rough curve to assist this part and the next.*
(ii)	A better starting point tries to avoid the SP. If Dani chose x = 4, f(4) = 25 which is a big value and so a long way from the root. Go the other way. Clearly f(0) = 9. So try x = -1. f(-1) = 5. That is probably OK. It is a reasonably small value for the ordinate. *NOTE: Be careful of making statements in a rush and not writing down what you really mean. One statement, repeated several times in different forms, was " the first estimate should be a negative number greater than 3".*	*2 marks – estimate of new root with appropriate reason.* *1 mark – a better estimate cited.*
(c) (i)	$\text{RHS} = 1 - \dfrac{1}{u+1}$ $= \dfrac{u + 1 - 1}{u + 1}$ $= \dfrac{u}{u+1} = \text{LHS}$	*1 mark – correct proof with correct setting out* *NOTE: Proof or "Show" answer should start with LHS or RHS*

(ii)	$x = u^2$ $dx = 2u\,du$ $\therefore \int \frac{2u\,du}{1+u} = 2\int \left[1 - \frac{1}{u+1}\right] du$ $= 2[u - \ln(u+1)] + c$ $= 2[\sqrt{x} - \ln(\sqrt{x} + 1)] + c$	*3 marks – correct integral in terms of x.* *2 marks – correct integral in terms of u* *1 mark – correct substitution*

Question 4

(a) (i)	$5! = 120$	*1 mark – correct answer*
(ii)	Number = Total – No. (Boys sit together) $n = 5! - 4! \times 2!$ $= 72$ $\text{Prob} = \frac{72}{120} = \frac{3}{5}$	*3 marks - Boys seated together calculated, then boys NOT together calculated then probability found.* *2 marks - Boys seated together calculated, then boys NOT together calculated* *1 marks - An reasonable attempt at the calculation*
(b)	$P(x) = ax^3 + bx^2 + cx + d$ Polynomial is monic so $a = 1$ If divided by x $P(0) = d = -8$ $P(x) = x^3 + bx^2 + cx - 8$ $P(2) = 8 + 4b + 2c - 8 = 14 - 4$ $\therefore 4b + 2c = 10$ $P(-2) = -8 + 4b - 2c - 8 = -14 - 4$ $4b - 2c = -2$ $8b = 8 \therefore b = 1$ $c = 3$ $\therefore x^3 + x^2 + 3x - 8$	*4 marks → 1 mark for each coefficient*

Question 4 (c) (continued)

$LHS = \cos\left(\frac{\pi}{2} + A\right)$ $= \cos\frac{\pi}{2}.\ \cos A - \sin\frac{\pi}{2}\sin A$ $= 0 - 1 \times \sin A$ $= -\sin A$ $= RHS$	*1 mark – correct solution fully demonstrated.*
$B = \frac{\pi}{2} + 50$ $-\sin 5\theta = \cos\left(\frac{\pi}{2} + 5\theta\right) = \cos 8\theta$ *In 1st and 4th quad.* $\frac{\pi}{2} + 5\theta = 8\theta$ $\theta = \frac{\pi}{6}$ *In 2nd and 3rd quads,* $\cos(-\theta) = \cos\theta$ *so reverse one side of the equation* $\frac{\pi}{2} + 5\theta = -8\ \theta$ $13\ \theta = -\frac{\pi}{2} = \frac{3\pi}{2}$ *(as no soln in the given range for -)* $13\theta = \frac{3\pi}{2}$ $\theta = \frac{3\pi}{26}$ *Of these 2 solns,* $\frac{3\pi}{26} < \frac{\pi}{6}$	*3 marks - correct answer identified* *2 marks - correct answer for* $\frac{\pi}{6}$ *1 mark – correct expression for B* *A graph helps to see the context of this question (not expected to be drawn – just for background):* 

Question 5

5(a) (i)	m(gradient of chord)$= \dfrac{ap^2 - aq^2}{2aq - 2ap} = \dfrac{a(p-q)(p+q)}{2a(p-q)}$ $= \dfrac{p+q}{2}$ equation of chord: $y - ap^2 = \left(\dfrac{p+q}{2}\right)(x - 2ap)$ $y = \left(\dfrac{p+q}{2}\right)x - \left(\dfrac{p+q}{2}\right)2ap^2 + ap^2$ $y = \left(\dfrac{p+q}{2}\right)x - ap^2 - apq + ap^2$ $y = \left(\dfrac{p+q}{2}\right)x - apq$	3- Correct equation with correct gradient found 2- Correct gradient from correct factoring found substituted into equation format 1- Correct gradient from correct factoring found
(ii)	If focal chord: $a = -apq$ $-1 = pq$	1- Correctly demonstrated
(iii)	Use lengths to directrix: $SP=PD_1=ap^2+a=a(p^2+1)$ $SQ=QD_2=aq^2+a=a(q^2+1)$ $\therefore \quad \overrightarrow{PQ} = a(p^2+1+q^2+1) \quad$ but $q = -\dfrac{1}{p}$ $= a\left(p^2+2+\dfrac{1}{p^2}\right) = a\left(p+\dfrac{1}{p}\right)^2$	2- correct length found from appropriate calculation 1-Correct initial statement of length with $q = -\dfrac{1}{p}$ used at some stage
(b) (i)	$\cos^{-1}\left(-\dfrac{1}{2}\right) = \dfrac{2\pi}{3}$	1- Correct value
(ii)	$\dfrac{1}{1+\left(\dfrac{x}{3}\right)^2} \times \dfrac{1}{3} = \dfrac{1}{\dfrac{9+x^2}{9}} \times \dfrac{1}{3} = \dfrac{3}{9+x^2}$	2- Correct derivative process then simplified 1- Correct derivative but not fully simplified
(iii)	$\dfrac{1}{2}\displaystyle\int_{0.25}^{0.5} \dfrac{2xdx}{\sqrt{1-(2x)^2}} = \left[\dfrac{1}{2}\sin^{-1}(2x)\right]_{0.25}^{0.5}$ $= \left[\dfrac{1}{2} \times \dfrac{\pi}{2} - \dfrac{1}{2} \times \dfrac{\pi}{6}\right]$ $= \dfrac{\pi}{6}$	3- Correct integral found then correct exact value 2- Correct integral found 2 0.5236 2- $\sin^{-1}(2x)$ then $\dfrac{\pi}{3}$ 1- $\sin^{-1}(2x)$ sighted
Comments:(a)(i)(ii) done well by most students(iii) Students attempted to use distance formula which proved most difficult. (b) (i) Too many said the easy wrong answer $\dfrac{4\pi}{3}$ (ii) (iii) generally answered well		

Question 6

6(a)(i)	$T = A + Ce^{kt}$ $\frac{dT}{dt} = k.\, Ce^{kt}$ $= kA + k.\, Ce^{kt} - kA$ $= k(A + Ce^{kt}) - kA$ $= kT - kA$ $= k(T - A)$ $= RHS$	1 mark – correctly demonstrated.
(ii)	$T = 5 \;\; t = 0 \qquad A = 25$ $T = A + Ce^{kt}$ $5 = 25 + C.\, e^0$ $-20 = C$ $T = 10 \qquad t = 20 \qquad A = 25$ $10 = 25 - 20.\, e^{k \times 20}$ $\frac{15}{20} = e^{20k}$ $\frac{1}{20} ln\left(\frac{3}{4}\right) = k$	2 marks – correct explanation 1 mark – correct value for C
(iii)	$T = x \;\; t = 60 \qquad A = 25$ $T = A + Ce^{kt}$ $x = 25 - 20.\, e^{60 \times \frac{1}{20} \ln\frac{3}{4}}$ $x = 16{\cdot}56$ Therefore temperature of particle is 17^0 – nearest degree.	2 marks – correct answer 1 mark – a correct answer using incorrect time.

iv	*As* $t \to \infty$ *then* $e^{kt} \to e^{-\infty} \to 0$ $\therefore \lim_{t \to \infty} T$ $= \lim_{t \to \infty} 25 - e^{kt}$ $= 25 - 0$ $= 25$ Therefore T approaches 25^0C	1 mark – fully and correctly demonstrated with indication that $e^{kt} \to e^{-\infty} \to 0$
6b-(i)	$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $RHS = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dv}\left(\frac{1}{2}v^2\right) \times \frac{dv}{dx}$ $= v \times \frac{dv}{dx}$ $= \frac{dx}{dt} \times \frac{dv}{dx}$ $= \frac{dv}{dt}$ $= \ddot{x}$ $= RHS$	1 mark – correct explanation with all steps fully demonstrated.
6c (i)	$\frac{d^2x}{dt^2} = 2x - 3$ $\therefore \quad \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x - 3$ $\frac{1}{2}v^2 = x^2 - 3x + \frac{C}{2}$ $v^2 = 2x^2 - 6x + C$ *When v=0 , x=4* $0 = 2 \times (4)^2 - 6 \times 4 + C$ $C = -8$ $v^2 = 2x^2 - 6x - 8$	2 marks – correct answer fully explained. 1 mark - incorrect value for C.
c (ii)	Origin $x = 0 \quad v^2 = -8$ As this is not possible therefore particle does not pass through the origin.	1 mark – correct explanation.

c(iii)	$when\ v = 10$ $100 = 2x^2 - 6x - 8$ $0 = x^2 - 3x - 54$ $0 = (x-9)(x+6)$ $x = 9\ \ or\ \ -6$ $x = 9$ as particle starts at 4 and does not pass through the origin.	2 marks – correct answer with correct explanation. 1 mark – failure to explain correctly the exclusion of $x = -6$.

Comments:

6a-iv – Brevity is not an asset in Proof questions – too many students failed to adequately show $e^{kt} \rightarrow e^{-\infty} \rightarrow 0$ *as part of the required answer.*

6b - A number of students introducing steps without previous justification and/or introducing assumptions/ statements in incorrect manner.

6 c-i *A number of students left out* $\frac{1}{2}$ *in* $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x}$

Question 7

(a) (i)	$a_4 b_7 - b_4 a_7 = {}^7C_4(-{}^{10}C_7) - {}^{10}C_4\,{}^7C_7$ $= -4410$	*2 marks – correct value determined* *1 mark – correct identification of coefficients*
(ii)	*The n subscripts are the same in both terms.* $\dfrac{{}^7C_n\,{}^{10}C_n}{7C_{n-1}\,10C_{n-1}} > 1$ After expanding and cancelling factorials we get $\dfrac{8-k}{k} \times \dfrac{11-k}{k} > 1$ $88 - 19k + k^2 > k^2$ $\dfrac{88}{19} > k$ $\therefore \quad k = 4$ $\therefore \quad {}^7C_4\,{}^{10}C_4 = 7350$	*2 marks – correct greatest value* *1 mark – correct approach to determining k*
(b)	Period $= \dfrac{\pi}{3}$ $\therefore \dfrac{2\pi}{n} = \dfrac{\pi}{3} \quad \therefore n = 6$ $v^2 = n^2(a^2 - x^2)$ $4 = 36\,(a^2 - 0)$ Amplitude $= a = \dfrac{1}{3}$ $\therefore x = \dfrac{1}{3}\cos(6t)$ at $t = \dfrac{\pi}{6}$ $x = -\dfrac{1}{3}$ which is $\frac{1}{2}$ a wavelength away from the starting point If using a sin curve the particle is back ot the origin so again half a wavelength different	*2 marks – correct amplitude and displacement* *1 mark – correct amplitude*

Question 7 (continued)

(c) (i)	$n = \pi$ period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{\pi} = 2$ seconds	*1 mark – correct period*
(ii)	*The particle starts at the origin at t = 0 so it returns after half the period – i.e. at t = 1.* $x = 4 \sin \pi t$ $\dot{x} = 4\pi \cos \pi t$ at $t = 1$ $\dot{x} = 4\pi \cos \pi = -4\pi$	*2 marks – correct velocity determined* *1 mark – correct approach but minor arithmetic error*
(iii)	$\dot{x} = 4\pi \cos \pi t$ (from (ii)) $\ddot{x} = -4\pi^2 \sin \pi t$ $= -\pi^2 x$ $\therefore$ acceleration is negatively proportional to the displacement from origin *NOTE: You must set out questions of SHOW, PROVE etc fully (as noted previously). If you do not, you will be heavily penalized. So do not short cut!! Ever!!*	*1 mark – correct demonstration*
(iv)	$v^2 = n^2 (a^2 - x^2)$ $= \pi^2 (4^2 - 3^2)$ $v = \sqrt{7}\ \pi$	*2 marks – correct answer* *1 mark –correct approach using v^2*