Northern Beaches Secondary College
Manly Selective Campus

2011
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators and templates may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1- 7
- All questions are of equal value

Marks

Question 1 *(Answer in a separate booklet)* *(12)*

(a) Solve for x in the following: $\dfrac{2x-1}{x} > 4$. *(2)*

(b) Find the general solution for θ in the equation sin θ = -1. *(2)*

(c) What is the remainder when $P(x) = x^3 - 4x + 5$ is divided by $x + 3$? *(2)*

(d) Find $\displaystyle\int 3\sin^2 x\,dx$ *(2)*

(e) In an exam paper, the first question has six parts and each part tests a different topic. Given the inequality question must be first, in how many ways can the remaining parts be ordered if they are not in alphabetical order? *(2)*

(f) What is the acute angle between two lines L_1 and L_2 if L_1 has a gradient of $\dfrac{1}{3}$ and L_2 is parallel to the y axis? *(2)*

Marks

Question 2 *(Answer in a separate booklet)* *(12)*

(a) Find the inverse function for $y = \dfrac{1}{x-2}$. *(2)*

(b) For the binomial expansion of $(2x-3)^{12}$ find the term containing x^6. *(2)*

(c) Given $v = \dfrac{1}{x}$ and that $t = 0$ when $x = 1$, find an equation for x as a function of time.
You may assume that x is always positive. *(2)*

(d) Prove by mathematical induction that for positive integers $n \geq 2$, *(3)*

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n-1}{n!} = 1 - \frac{1}{n!} .$$

(e) (i) Show that $\dfrac{d}{dx}(10^x) = \ln 10 \times 10^x$ *(1)*

(ii) Taking $x_1 = 0{\cdot}5$ as the first approximation, use one application of Newton's method to find a closer approximation to the root of the equation $10^x - 3 = 0$.
Give your answer correct to 2 decimal places. *(2)*

Marks

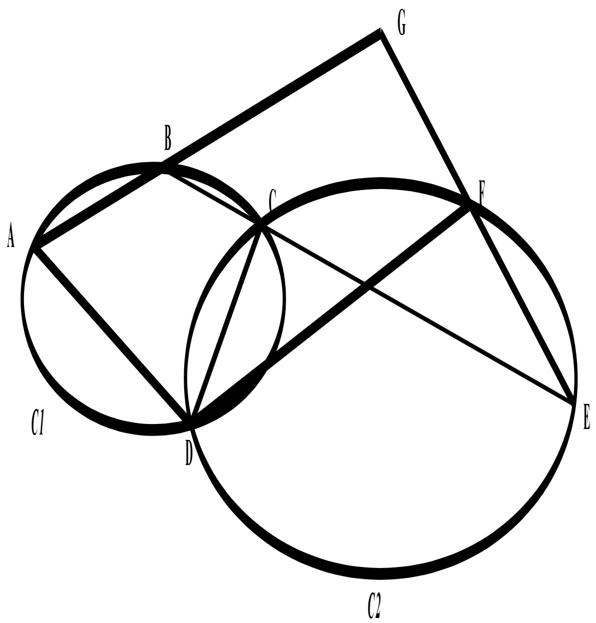
Question 3 *(Answer in a separate booklet)* *(12)*

(a) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 5x}{x}$ *(1)*

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin 2x \cos 2x \, dx$ *(2)*

(c) Use the substitution $u = 3x - 1$ to find $\displaystyle\int \frac{x}{3x-1}\, dx$ *(3)*

(d)



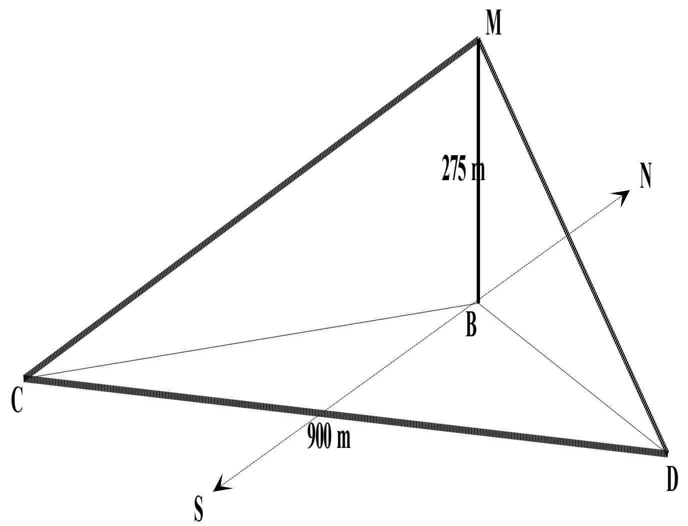
Two circles *C1* and *C2* intersect at C and D. BC produced meets circle *C2* at E. AB produced meets EF produced at G.

(i) Copy or trace the diagram onto your answer booklet.
(ii) Prove ADFG is a cyclic quadrilateral. *(3)*

Marks

***Question 3** (continued)*

(e) June is on the top of a 275 metre cliff (*M*) in the Megalong Valley. At the base of the cliff directly below her, June sees *B*, the base camp for an orienteering exercise involving Team X and Team Y. She sees Team X out on exercises at checkpoint C and determines their angle of depression to be 55°. From the base camp B, Team X is on a bearing of 214° and Team Y, at checkpoint D, is on a bearing of 137° and the two teams are 900 metres apart.



(i) Copy or trace this diagram into your writing booklet.

(ii) Show that CB = $275 \cot 55^\circ$. *(1)*

(iii) Show that $\sin \angle BDC = \dfrac{275 \cot 55^\circ \sin 77^\circ}{900}$.

DO NOT calculate the actual value for $\angle BDC$. *(2)*

Marks

Question 4 *(Answer in a separate booklet)* *(12)*

(a) Sand falling from a funnel forms a conical pile whose height is one and a half times its radius.

(i) Show that the volume of the conical pile is $V = \dfrac{\pi r^3}{2}$ *(1)*

(ii) If the sand is falling at the rate of $\dfrac{\pi}{10}$ metres^3 / minute, find the rate at which the radius is increasing when the pile is 3 metres high. *(2)*

(b) Find the probability that a random arrangement of the letters of the word

SELECTION

has the vowels and consonants in alternating positions. *(2)*

(c) For the function $f(x) = \dfrac{2x}{(x-1)^2}$

(i) State the equation of the vertical asymptote. *(1)*

(ii) Evaluate $\lim\limits_{x \to \infty} f(x)$. *(1)*

(iii) Sketch the graph of $f(x)$. You DO NOT need to find the co-ordinates of any turning points. *(2)*

(d) The rate at which a body cools in air is proportional to the difference between the constant air temperature, C, and its own temperature, T. This can be expressed by the differential equation $\dfrac{dT}{dt} = -k(T-25)$, where t is time in hours and k is a constant.

You are given that $T = 25 + Ae^{-kt}$ is a solution to the differential equation where A is a constant.

A heated piece of metal cools from $90^\circ C$ to $70^\circ C$ in one hour.
Find the temperature of the metal after another 2 hours (answer to the nearest degree). *(3)*

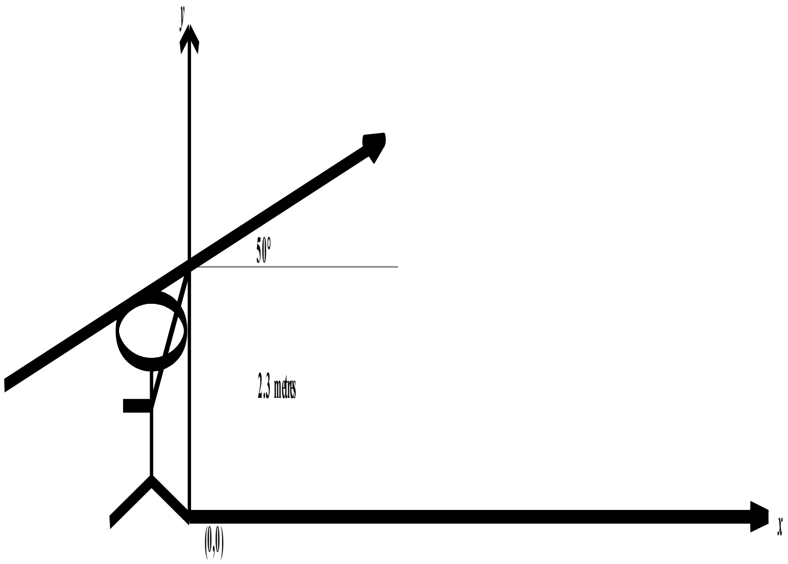
Marks

Question 5 *(Answer in a separate booklet)* *(12)*

(a) At a school athletics carnival, a competitor is trying to break the school's 17 year old boys' javelin throwing record of 47.2 metres.

The competitor projects the javelin at an initial velocity of 20 metres per second from a height of 2.3 metres with an angle of projection of 50°.

Take the acceleration due to gravity as 10m/s^2 and assume that air resistance is ignored. Define the origin as being 2.3 metres vertically below the point of projection, as in the diagram.



(i) Use integration to show that the equations of motion are *(2)*

$$x = 20t\cos 50^\circ \qquad y = 2.3 + 20t\sin 50^\circ - 5t^2$$

(ii) Show that, correct to 1 decimal place, the time of flight is 3.2 seconds. *(2)*

(iii) At what angle and with what speed does the tip of the javelin hit the ground? *(3)*

(iv) Prove that the competitor does not break the school record with this javelin throw. *(1)*

(b) Solve the equation $8x^3 - 14x^2 - 7x + 6 = 0$ given that two of the roots are reciprocals of each other. *(4)*

Marks

Question 6 *(Answer in a separate booklet)* *(12)*

(a) (i) Show that $\frac{d}{dx}(\text{cosec}x) = -\cot x \,\text{cosec}x$ *(2)*

(ii) Show that $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$ *(2)*

(iii) Prove that $\text{cosec}^2 x + \text{cosec}x\cot x = \frac{1}{1-\cos x}$ *(2)*

(iv) Hence find $\int \frac{1}{1-\cos x}\,dx$ *(2)*

(b) The function $f(x) = \sec x$ for $0 \le x < \frac{\pi}{2}$ and is not defined for any other values of x.

(i) Write down the domain of the inverse function. *(1)*

(ii) Show that $f^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$. *(1)*

(iii) Find $\frac{d}{dx}(f^{-1}(x))$. *(2)*

Marks

***Question** 7 (Answer in a separate booklet)* *(12)*

(a) The acceleration of a particle when it is x metres from the origin is given by

$$a = -e^{-x}.$$

Given that $v = 2$ when $x = 0$, find v when $x = 2$. *(4)*

(b) Using the expansion of $(1 + x)^n$, show that *(4)*

$$\frac{2^{n+2} - 3 - n}{(n+1)(n+2)} = \frac{\binom{n}{0}}{1 \times 2} + \frac{\binom{n}{1}}{2 \times 3} + \frac{\binom{n}{2}}{3 \times 4} + \ldots + \frac{\binom{n}{n}}{(n+1)(n+2)}$$

(c) A chord PQ is drawn for a parabola between P $(4p, 2p^2)$ and Q $(4q, 2q^2)$.
P and Q vary such that, for all other values of the parameters, the chord is always parallel to that drawn first.

(i) Find the locus of the midpoint M of PQ in terms of the gradient of PQ. *(3)*

(ii) Sketch a locus of M. *(1)*

End of Examination

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

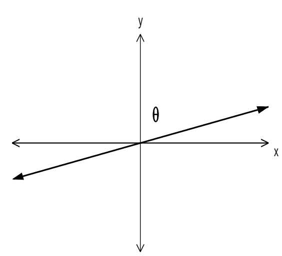
$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Manly Selective Campus
2011 HSC Mathematics Extension 1 – Trial HSC - Solutions

Question 1

(a)	$\frac{x^2(2x-1)}{x} > 4x^2$ $2x^2 - x = 4x^2$ (using equality) $2x^2 + x = 0$ $x = 0 \text{ or } -\frac{1}{2}$ $\text{Testing } x = -\frac{1}{4}$ shows $-\frac{1}{2} < x < 0$	*2 marks – correct result proved* *1 mark – correct limits*
(b)	$\sin\theta = -1$ $\theta = -\frac{\pi}{2}$ $\theta = 2k\pi - \frac{\pi}{2} \text{ or } \pi n + (-1)^n\left(-\frac{\pi}{2}\right)$ etc	*2 marks – correct statement (any version)* *1 mark – basic angle correct*
(c)	$P(x) = x^3 - 4x + 5$ $P(-3) = -27 + 12 + 5$ $= -10$ So remainder is -10	*2 marks – correct result proved* *1 mark – correct limits*
(d)	$\cos 2x = 1 - 2\sin^2 x$ $\therefore \sin^2 x = \frac{1}{2}(1 - \cos 2x)$ (doing this in full to be careful in Q1!!!) $\therefore \quad I = \int 3\sin^2 x\, dx$ $= \frac{3}{2}\int 1 - \cos 2x\, dx$ $= \frac{3}{2}\left[x - \frac{\sin 2x}{2}\right] + C$	*2 marks – correct integral* *1 mark – correct substitution*
(e)	*1*(5!) – 1 = 119*	*2 marks – correct result* *1 mark – correction for alphabetical order not included*
(f)	*Draw a picture!!!* *We want θ.* *So find the complement* *As* $\tan(90^o - \theta) = \frac{1}{3}$ *Hence* $\theta = 71^o\ 34'$ 	*2 marks – correct integral* *1 mark – correct substitution*

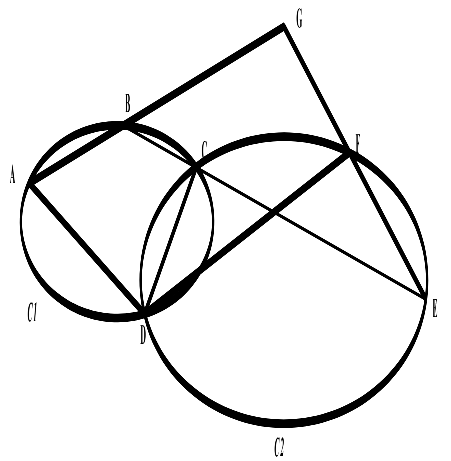
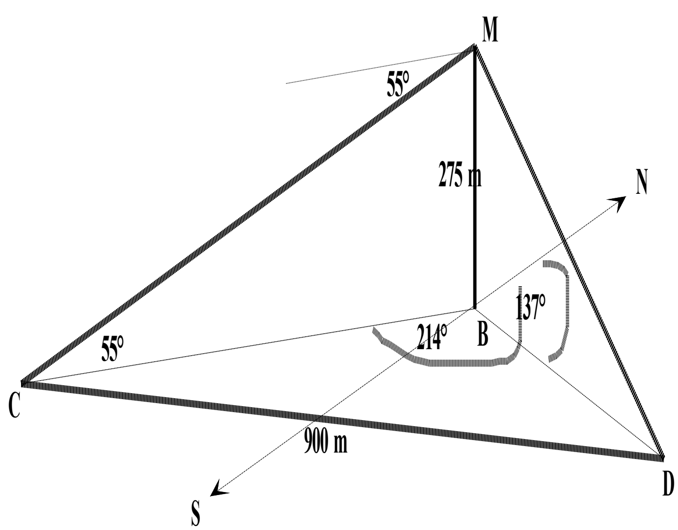
Question 2

(i)	$f(x)\colon\ y=\dfrac{1}{x-2}$ $f^{-1}(x)\colon\ x=\dfrac{1}{y-2}$ $y-2=\dfrac{1}{x}$ $y=2+\dfrac{1}{x}$	***2 marks for correct inverse function.*** ***1 mark for swapping x and y.***
(b)	$T_k={}^{12}\mathbf{C}_k(2x)^{12-k}(-3)^k$ *Let 12-k=6* $k=6$ $T_6={}^{12}\mathbf{C}_6\,(2x)^6(-3)^6$ $T_6=43110144x^6$	***2 marks for correct term even if not evaluated.*** ***1 mark for k=6 or only giving co-efficient.***
(c)	$v=\dfrac{1}{x}$ $\dfrac{dx}{dt}=\dfrac{1}{x}$ $\dfrac{dt}{dx}=x$ $t=\dfrac{x^2}{2}+c$ when $x=1,\ t=0$ $0=\dfrac{1}{2}+c$ $c=-\dfrac{1}{2}$ $t=\dfrac{x^2}{2}-\dfrac{1}{2}$ $t+\dfrac{1}{2}=\dfrac{x^2}{2}$ $x^2=2t+1$ $x=\sqrt{2t+1}$ for $x>0$	***2 marks for correct demonstration.*** ***1 mark for*** $t=\dfrac{x^2}{2}+c$

(d)	For $n = 2$, $LHS = \frac{2-1}{2!} = \frac{1}{2!}$ $RHS = 1 - \frac{1}{2!} = \frac{2!-1}{2!} = \frac{2-1}{2!} = \frac{1}{2!} = LHS$ So true for $n = 2$. For n= k, assume that $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{k-1}{k!} = 1 - \frac{1}{k!}$ For n=k+1 R.T.P. $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{k-1}{k!} + \frac{k}{(k+1)!} = 1 - \frac{1}{(k+1)!}$ $LHS = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{k-1}{k!} + \frac{k}{(k+1)!}$ $= 1 - \frac{1}{k!} + \frac{k}{(k+1)!}$ $= 1 - \frac{k+1}{(k+1)!} + \frac{k}{(k+1)!}$ $= 1 - \frac{1}{(k+1)!}$ $= RHS$ So the result is proven by the principle of mathematical induction.	***3 marks for correct proof.*** ***2 marks for correct substitution of assumption and for showing true for n=2 OR for having step 1 wrong but proving correctly for n=k+1.*** ***1 mark for showing true for n=2 or correct use of assumption.*** ***Comment: many algebraic errors occurred in Step 3 if minus was in front of finding common denominator***
(ei)	$\frac{d}{dx}(10^x)$ $= \frac{d}{dx}\left(e^{\ln(10)\ \times x}\right)$ $= \ln(10) \times e^{\ln(10)\ \times x}$ $= \ln(10) \times 10^x$	***1 marks for correct demonstration.*** ***Comment: This was generally very poorly done. Students need to revise 2U work***
(eii)	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $= 0{\cdot}5 - \frac{\sqrt{10} - 3}{\ln(10) \times \sqrt{10}}$ $= 0{\cdot}4777134395$ $= 0{\cdot}48\ (2d.p.)$	***2 marks for correct solution*** ***1 mark for correct substitution into correct rule OR for correct result but error in derivative.***

Question 3

(a)	$\lim_{x \to 0} \frac{\sin 5x}{5x} \text{ x } 5 = 1\text{x } 5 = 5$	*1 mark – correct limit derived using* $\lim_{x \to 0} \frac{\sin 5x}{5x} = 1$
(b)	$\sin 2x \cos 2x = \frac{1}{2} \sin 4x$ $\therefore \quad I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \sin 4x \, dx = -\frac{1}{2}\left[\frac{\cos 4x}{4}\right]_0^{\frac{\pi}{4}}$ $= \quad -\frac{1}{8}(-1 - 1) = \frac{1}{4}$	*2 marks – correct solution* *1 mark – correct integration, error in substitution*
(c)	$u = 3x - 1 \qquad \therefore x = \frac{u+1}{3} \qquad \frac{du}{dx} = 3 \qquad \therefore dx = \frac{1}{3} du$ $I = \int \frac{u+1}{3.\ 3u} du$ $I = \frac{1}{9} \int 1 + \frac{1}{u} du$ $I = \frac{1}{9}(u + \ln u) + c$ $I = \frac{1}{9}(3x - 1 + \ln(3x - 1)) + c \qquad x > \frac{1}{3}$	*3 marks for correct solution* *2 marks for correct integration but failure to substitute back at end* *1 mark for error in substitution that simplifies the integration as long as integration is correct and substitution occurs*

(d)	 $\angle DCE = \angle DFE$ (angles in the same segment) $\angle BAD = \angle DCE$(exterior angle of cyclic quadrilateral ABCD = opposite interior angle) $\therefore \angle BAD = \angle DFE$ i.e. exterior angle of quadrilateral ADFG = opposite interior angle $\therefore$ *ADFG* is a cyclic quadrilateral	*3 marks – correct proof* *2 marks - 1 error in proof statement i.e. incorrect angle or inadequate or incorrect statement of reason* *1 mark- 1 correct and relevant statement 7 reason*
(e) (i)	 $\cot 55^\circ = \dfrac{\text{CB}}{275}$ $\therefore \quad \text{CB} = 275 \cot 55^\circ$	1 mark – correct and clear derivation

(e) (ii)	$\angle CBD = 214^\circ - 137^\circ = 77^\circ$ $\dfrac{\sin \angle BDC}{\text{CB}} = \dfrac{\sin 77^\circ}{900}$ $\sin \angle BDC = \dfrac{275 \cot 55^\circ \sin 77^\circ}{900}$	2 marks – correct derivation 1 mark – correct substitution into sine rule

Question 4

(ai)	$V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi r^2\left(\frac{3r}{2}\right)$ $V = \frac{\pi r^3}{2}$	*1 mark – correct and clear derivation*
(aii)	$h = \frac{3}{2}r$ When h=3 metres, $3 = \frac{3}{2} \times r$ $r = 2$ $\frac{dV}{dr} = \frac{3\pi r^2}{2}$, when r=2, $\frac{dV}{dr} = \frac{3\pi(2)^2}{2} = 6\pi$ $\frac{dr}{dt} = \frac{dr}{dv} \times \frac{dv}{dt}$ $= \frac{1}{6\pi} \times \frac{\pi}{10}$ $= \frac{1}{60}\, m/min$	*2 marks – correct solution* *1 mark – 1 error in working*
(b)	There are 5 consonants and 4 vowels so to arrange them alternately a consonant must come first. C VCVCVCVC. Five consonants can be arranged in $5! = 120$ ways. Four consonants can be arranged in $\frac{4!}{2!} = 12$ ways. The sample space is $\frac{9!}{2!} = 181440$ So required probability is $\frac{120 \times 12}{181440} = \frac{1}{126}$	*2 marks – correct probability* *1 mark – correct number of arrangements*
(ci)	$x = 1$	*1 mark*
(cii)	$\lim_{x \to \infty} \frac{2x}{x^2 - 2x + 1}$ $= \lim_{x \to \infty} \frac{\frac{2}{x}}{1 - \frac{2}{x} + \frac{1}{x^2}}$ $= \frac{0}{1 - 0 - 0}$ $= 0$	*1 mark*

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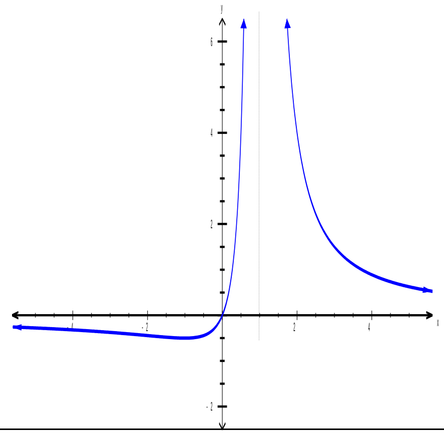
<tr><td>(ciii)</td><td></td><td>2 marks – correct graph

1 mark :
1 section of graph correctly drawn AND either correct x & y intercepts OR graph tends to zero from below as x tends to neg infinity OR correct from error in (i) or (ii)</td></tr>
<tr><td>(d)</td><td>$T = 25 + Ae^{-kt}$
when t=0, T=90
$90 = 25 + A$
$A = 65$
$T = 25 + 65e^{-kt}$
when t=1, T=70
$70 = 25 + 65e^{-k}$
$\frac{45}{65} = e^{-k}$
$\ln\left(\frac{45}{65}\right) = -k$
$k = -\ln\left(\frac{45}{65}\right)$
When t=3(after another 2 hours)
$T = 25 + 65e^{3\ln\left(\frac{45}{65}\right)}$
$T = 46{\cdot}56804734$
$T = 47^{\circ}$(nearest degree)</td><td>3 marks – correct solution

2 marks – correct A and k OR 1 only 1 error in working

1 mark – Correct value for A</td></tr>
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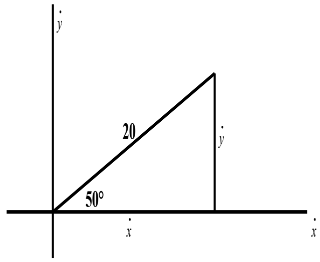
<tr><td>Questi on 5(ai)</td><td>


Initially, $\dot{x} = 20\cos 50^\circ$ $\dot{y} = 20\sin 50^\circ$
$\ddot{x} = 0$
$\dot{x} = c$
$t = 0,\ \dot{x} = 20\cos 50^\circ \quad \therefore c = 20\cos 50^\circ$
$\therefore \dot{x} = 20\cos 50^\circ$
$x = 20t\cos 50^\circ + c_1$
$t = 0,\ x = 0 \qquad \therefore c_1 = 0$
$\therefore\ x = 20t\cos 50^\circ$
$\ddot{y} = -10$
$\dot{y} = -10t + c_2$
$t = 0,\ \dot{y} = 20\sin 50^\circ \ \therefore c_2 = 20\sin 50^\circ$
$\dot{y} = -10t + 20\sin 50^\circ$
$y = -5t^2 + 20t\sin 50^\circ + c_3$
$t = 0,\ y = 2.3 \qquad \therefore c_3 = 2.3$
$y = 2.3 + 20t\sin 50^\circ - 5t^2$
</td><td>2 marks – correct derivation

1 mark – failure to justify values of constants of integration and initial velocity conditions</td></tr>
<tr><td>(aii)</td><td>
Time of flight, $y = 0$
$0 = 2.3 + 20t\sin 50^\circ - 5t^2$
$t = \dfrac{-20\sin 50^\circ \pm \sqrt{(20\sin 50^\circ)^2 + 4(5)(2.3)}}{-5 \times 2}$
$t = 3.2075 \text{ or } -0.14 \quad \text{but } t \geq 0$
$\therefore\ t = 3.2s$
</td><td>2 marks – correct proof

1 mark – error in solving y=0</td></tr>
</table>

(aiii)	$t = 3.2$, Speed $= \sqrt{\dot{x}^2 + \dot{y}^2}$ $\therefore$ Speed $= \sqrt{(20\cos 50^\circ)^2 + (20\sin 50^\circ - 32)^2}$ $= 21.056 \approx 21ms^{-1}$ $\tan\theta = \frac{\dot{y}}{\dot{x}} = \frac{20\sin 50^\circ - 32}{20\cos 50^\circ}$ Note - this is neg value $\theta = 127^\circ 37'$	3 marks correct solution (including accepting 52° and correct substitution for speed, even with calculator error 2 marks – Correct horizontal and vertical velocity values and either correct speed or correct angle of impact 1 mark - Correct horizontal and vertical velocity values
(aiv)	Range = 20 x 3.2 cos 50 = 41.138 m which is less than the record. Does not break record	1 mark
(b)	Roots are $\alpha, \frac{1}{\alpha}, \beta$ product of roots $= -\frac{d}{a} = -\frac{6}{8} = -\frac{3}{4} = \beta$ Sum of roots $= -\frac{b}{a} = \frac{14}{8} = \frac{7}{4}$ $\therefore \quad \frac{7}{4} = \alpha + \frac{1}{\alpha} - \frac{3}{4}$ $0 = \alpha + \frac{1}{\alpha} - \frac{5}{2}$ $0 = 2a^2 - 5\alpha + 2$ $0 = (2\alpha - 1)(\alpha - 2)$ $\alpha = \frac{1}{2}, 2$ $\therefore \quad x = \frac{1}{2}, 2, -\frac{3}{4}$	4 marks – correct solutions 3 marks – 1 error and all else correct 2 marks – finding x = -3/4 and correct approach to finding other roots but error in algebra 1 mark – finding x = *-3/4*

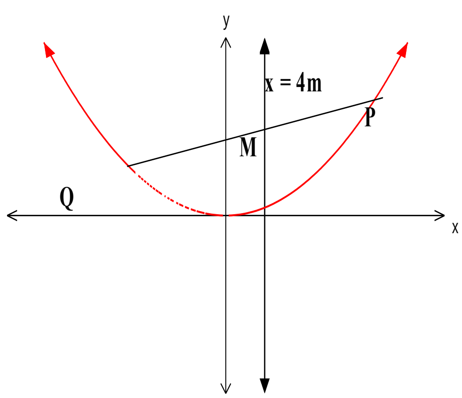
Question 6

(a)(i)	$\frac{d}{dx}(\text{cosec } x) = \frac{d}{dx}((\sin x)^{-1})$ $= -1(\sin x)^{-2}(\cos x)$ $= \frac{-\cos x}{(\sin x)^2}$ $= -\cot x \text{ cosec } x$	*2 marks – correct demonstration* *1 mark – correct first step in derivative*
(ii)	$\frac{d}{dx}(\cot x) = \frac{d}{dx}(\tan x)^{-1}$ $= \frac{-\sec^2 x}{(\tan x)^2}$ $= -\frac{1}{\cos^2 x} \times \frac{\cos^2 x}{\sin^2 x}$ $= -\text{cosec}^2 x$	*2 marks – correct demonstration* *1 mark – correct first step in derivative*
(iii)	$\text{cosec}^2 x + \text{cosec } x \cot x = \frac{1}{\sin^2 x} + \frac{1}{\sin x} \times \frac{\cos x}{\sin x}$ $= \frac{1 + \cos x}{\sin^2 x}$ $= \frac{1 + \cos x}{(1 - \cos x)(1 + \cos x)}$ $= \frac{1}{1 - \cos x}$	*2 marks – correct result proved* *1 mark – correct approach with progress*
(iv)	$\int \frac{dx}{1 - \cos x} = \int (\text{cosec}^2 x + \text{cosec } x \cot x)\, dx$ $= [-\cot x - \text{cosec } x] + c$ $= -(\cot x + \text{cosec} x) + c$	*2 marks – correct integral* *1 mark – correct substitution*
(b)(i)	$x \geq 1$	*1 mark – correct domain*
(ii)	$y = \sec x$ $x = \frac{1}{\cos y}$ $\therefore \cos y = \frac{1}{x}$ $y = \cos^{-1}\left(\frac{1}{x}\right)$	*1 mark – correct demonstration*
(iii)	$\frac{d}{dx}(\cos^{-1}(x^{-1})) = -\frac{1}{\sqrt{1 - \left(\frac{1}{x}\right)^2}} \times (-x^{-2})$ $= \frac{1}{x^2} \times \frac{x}{\sqrt{x^2 - 1}}$ $= \frac{1}{x\sqrt{x^2 - 1}}$	*2 marks – correct derivative* *1 mark – correct incorporation of $(-x^{-2})$ in first step* *COMMON ERRORS: the derivative of x^{-1} is ln x!!!* *AND forgetting to multiply by the derivative (truly amazing for good students).*

Question 7

(a)(i)	Accel $= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $\therefore \quad \frac{1}{2}v^2 = \int -e^{-x}\,dx$ $= e^{-x} + c$ $2^2 = 2 \times e^{-0} + c$ $c = 2$ $\therefore \quad v^2 = 2e^{-x} + 2$ When $x = 2, \quad v^2 = \frac{2}{e^2} + 2$ $v = \sqrt{\frac{2}{e^2} + 2}$	_4 marks – correct equation for v_ _3 marks – subsequent error_ _2 marks – correct equation for v^2_ _1 mark – use of basic statement for acceleration_
(b)	$(1 + x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n$ integrating $\frac{(1+x)^{n+1}}{n+1} = \binom{n}{0}x + \frac{\binom{n}{1}x^2}{2} + \frac{\binom{n}{2}x^3}{3} \ldots + \frac{\binom{n}{n}x^{n+1}}{n+1} + c$ when $x = 0$ $c = \frac{1}{n+1}$ integrating again $\frac{(1+x)^{n+2}}{(n+1)(n+2)} = \frac{\binom{n}{0}x^2}{1 \times 2} + \frac{\binom{n}{1}x^3}{2 \times 3} + \frac{\binom{n}{2}x^4}{3 \times 4} + \ldots + \frac{\binom{n}{n}x^{n+2}}{(n+1)(n+2)} + \frac{x}{n+1} + c$ when $x = 0, \quad c = \frac{1}{(n+1)(n+2)}$ Let x = 1 $\frac{2^{n+2}}{(n+1)(n+2)} = \frac{\binom{n}{0}}{1 \times 2} + \frac{\binom{n}{1}}{2 \times 3} + \frac{\binom{n}{2}}{3 \times 4} + \ldots \frac{\binom{n}{n}}{(n+1)(n+2)} + \frac{1}{n+1} + \frac{1}{(n+1)(n+2)}$ $\frac{2^{n+2} - n - 3}{(n+1)(n+2)} = \frac{\binom{n}{0}}{1 \times 2} + \frac{\binom{n}{1}}{2 \times 3} + \frac{\binom{n}{2}}{3 \times 4} + \ldots \frac{\binom{n}{n}}{(n+1)(n+2)}$	_4 marks – correct proof_ _3 marks – correct statement including the 2^nd constant_ _2 marks – correct 1^st integral including the constant_ _1 mark – correct expansion and use of integration_

***Question** 7 (continued)*

(c)(i)	Gradient of PQ $= \dfrac{2p^2 - 2q^2}{4p - 4q}$ $= \dfrac{2(p+q)(p-q)}{4(p-q)}$ $= \dfrac{p+q}{2} = m$ Midpoint of PQ: $x = 2(p+q)$ $y = (p^2 + q^2)$ $\therefore$ locus is x = 4m	*3 marks – correct locus equation* *2 marks – correct gradient and midpoint determined in simplified form* *1 mark – either gradient or midpoint determined correctly* *NOTE: remember that with all locus questions, the aim is to eliminate the parameters from one equation at least.*
(ii)		*1 mark – correct diagram*