Northern Beaches Secondary College

Manly Selective Campus

2014 HSC Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using blue or black pen.
- Board-approved calculators and templates may be used.
- All necessary working should be shown in every question.
- Multiple choice questions are to be completed on the special answer page.

Total marks – 70

- Attempt Questions 1-14

Multiple choice section

Answer each of the following ten (10) questions on the special answer sheet provided.

Q1. $\dfrac{2\tan\theta}{1+\tan^2\theta}$ is equivalent to?

A) $\cos 2\theta$

B) $\sin 2\theta$

C) $\tan 2\theta$

D) $\cot 2\theta$

Q2. The remainder theorem when $P(x) = x^3 - 2x^2 - 4x + 7$ is divided by $2x + 3$ is:

A) 4

B) $\dfrac{41}{8}$

C) $-\dfrac{1}{8}$

D) -26

Q3. The point $P(2,2)$ divides the interval joining A (-2, -4) and B (x_2, y_2) in the ratio 2:1. The coordinates of B are?

A) (4, 5)

B) (6, 8)

C) (0, -1)

D) (10, 14)

Q4. The term independent of x in the expansion $\left(x + \dfrac{3}{x}\right)^4$ is?

A) 3

B) 6

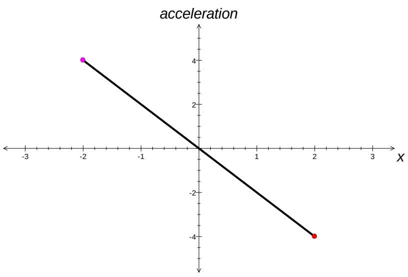
C) 18

D) 54

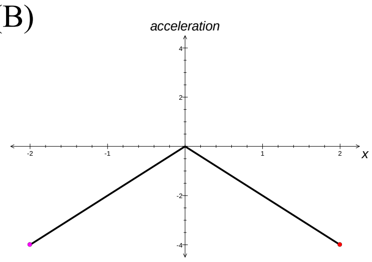
Q5. A particle is moving along a straight line. The displacement of the particle from a fixed point O is given by x. The graphs below show acceleration against displacement.

Which of the graphs below best represents a particle moving in simple harmonic motion?

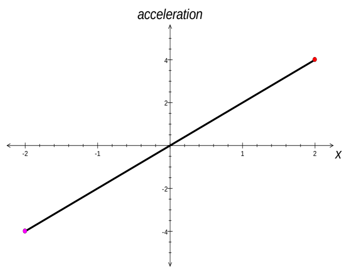
(A)



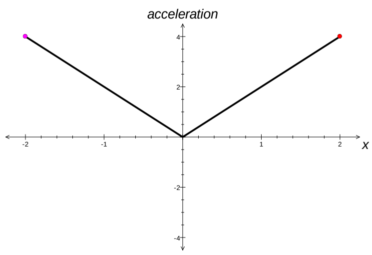
(B)



(C)



(D)



Q6. The definite integral $\int_e^{e^2} \frac{2}{x(\log_e x)^2}\,dx$ is evaluated using the substitution $u = \log_e x$.

The value of the integral is?

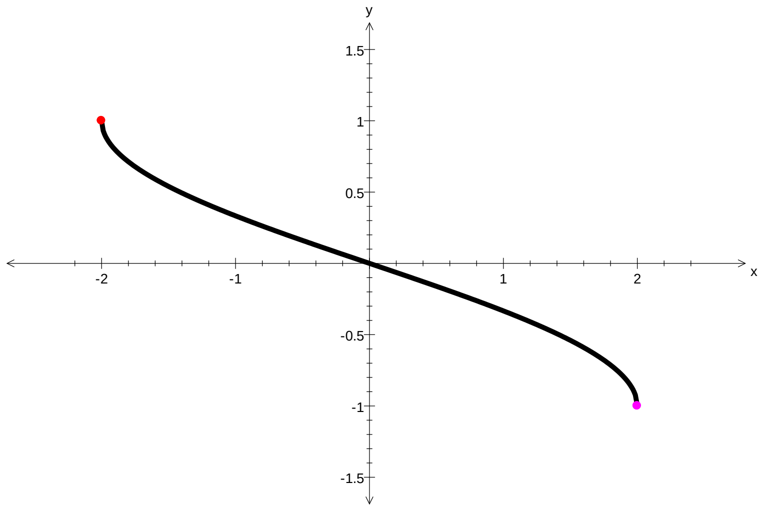
A) $2\left(\frac{1}{e} - \frac{1}{e^2}\right)$

B) $2\left(\frac{1}{e^2} - \frac{1}{e}\right)$

C) -1

D) 1

Q7. The function shown in the diagram below has the equation $y = A\sin^{-1} Bx$.
Which of the following is true?



(A) $A = 1,\ B = \dfrac{1}{2}$

(B) $A = -1,\ B = 2$

(C) $A = \dfrac{-2}{\pi},\ B = \dfrac{1}{2}$

(D) $A = \dfrac{2}{\pi},\ B = 2$

Q8. Three English books, four Mathematics books and five Science books are randomly placed along a bookshelf. What is the probability that the Mathematics books are all next to each other?

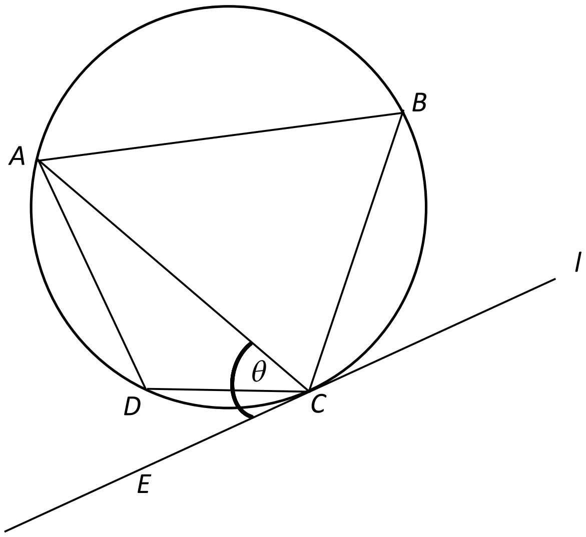
A) $\dfrac{1}{3!5!}$

B) $\dfrac{4!9!}{12!}$

C) $\dfrac{4!3!5!}{12!}$

D) $\dfrac{4!}{9!}$

Q9. *A*, *B*, *C* and *D* are points on a circle. The line *l* is tangent to the circle at *C*. $\angle ACE = \theta$.



What is $\angle ADC$ in terms of θ?

A) $90° - \theta$

B) $180° - \theta$

C) $180° - 2\theta$

D) θ

Q10. What is the indefinite integral for $\int\left(\cos^2 x + \sec^2 x\right)dx$?

A) $\frac{1}{2}x + \frac{1}{4}\sin 2x + \frac{1}{2}\tan x + c$

B) $\frac{1}{2}x - \frac{1}{4}\sin 2x + \frac{1}{2}\tan x + c$

C) $\frac{1}{2}x + \frac{1}{4}\sin 2x + \tan x + c$

D) $\frac{1}{2}x - \frac{1}{4}\sin 2x + \tan x + c$

End of Multiple Choice

Free response questions – answer each question in a separate Booklet

Question 11: ***Start a new Booklet*** **15 Marks**

a) Solve $\left(\frac{x+3}{x^2-1}\right) \leq 0$ **2**

b) A container ship brings a total of 1200 cars into Australia. Of these cars, three hundred have defective brakes.

A total of five hundred cars are unloaded at Sydney.

Find an expression for the probability that one hundred of the cars unloaded at Sydney have defective brakes. **2**

(NOTE: You don't need to simplify or evaluate the expression).

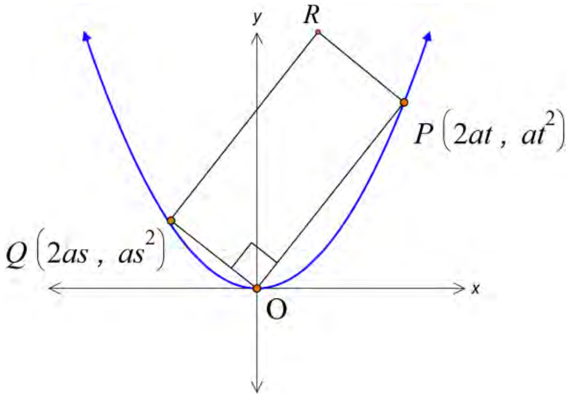
c) How many times must a die be rolled so that the probability of at least one six exceeds 0.5? **2**

d) Line A is defined by the equation $y = 2x + 1$.
Line B is defined by the equation $y = mx + b$

If the acute angle between the two is 45 degrees, what are the possible values of m? **2**

Question 11: continued on next page

Question11: continued.



e) In the diagram above the point R is the fourth vertex of the rectangle *POQR*.

The points *P* and *Q* move such that $\angle POQ = 90^\circ$.

(i) Show that $st = -4$ ***1***

(ii) Find the locus of the point *R* ***2***

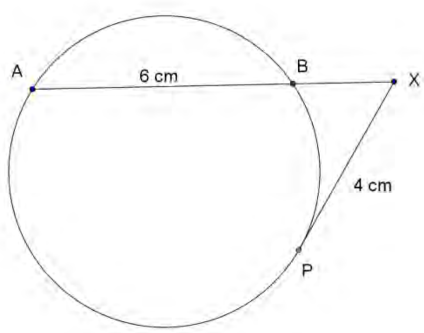
f) Find the general solution of the equation: ***2***

$$\sin\theta\ \cos\theta = \frac{1}{2}$$

g) The diagram shown below shows a circle and *AB* is a chord of length 6 *cm*.

The tangent to the circle at *P* meets *AB* produced at *X*. $PX = 4$ *cm*.

Find the length of *BX*. ***2***



Question 12: ***Start a new Booklet*** **15marks**

a) (i) Show that a zero of the function

$$f(x) = \log_e x - \frac{1}{x}$$

lies between $x = 1$ and $x = 2$. **1**

(ii) Use one application of Newton's Method with an initial approximation of $x = 1 \cdot 5$ to obtain an improved estimate to the solution of the equation:

$$\log_e x - \frac{1}{x} = 0.$$

(State your answer to one decimal place.) **2**

b) The rate of change of the temperature T of a cool item placed in a hot environment is determined by the equation.

$$\frac{dT}{dt} = k(S - T)$$

where k is a constant and T is the temperature of the object, and S is the temperature of the environment.

(i) Show that $T = S - Ae^{-kt}$ is a solution to the differential equation:

$$\frac{dT}{dt} = k(S - T).$$ **1**

Jamie is cooking a large roast in an oven set to 160°C. The roast will be cooked when the thermometer shows that the temperature of the centre of the roast is 150°C. When Jamie started cooking, the temperature of the centre of the roast was 4°C and 30 minutes later it was 60°C.

(ii) How long will it take for the roast to be cooked? **3**

Question 12 continued on next page

Question 12 continued

c) (i) Show that $\dfrac{1}{(n+1)!} - \dfrac{n+1}{(n+2)!} = \dfrac{1}{(n+2)!}$ ***1***

(ii) Use mathematical induction to show that, for all integers $n \geq 1$,

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots\ldots\ldots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}.$$ ***3***

d) Given the function $f(x) = \dfrac{x^2 - 1}{x^3 - 8}$

(i) State the vertical asymptote. ***1***

(ii) Sketch the graph of the function. Clearly show on your diagram the x – intercepts. ***3***
(Your diagram should be one third of a page in size)

Question 13:* *Start a new Booklet **15 marks**

a) Find $\int e^{x+e^x}\,dx$ using the substitution $u = e^x$ **3**

b) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line. The velocity is given by

$$\dot{x} = -\frac{1}{8}x^3$$

The acceleration of the particle is given by $\ddot{x}$. The particle is initially 2 metres to the right of O.

(i) Show that $\ddot{x} = \frac{3}{64}x^5$ **2**

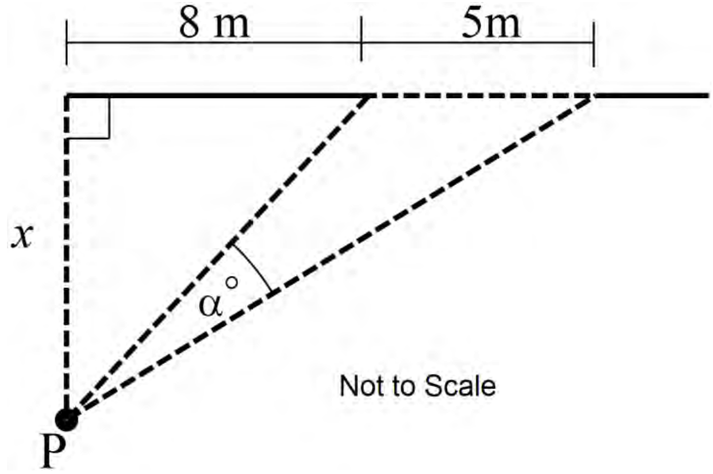
(ii) Find an expression for x in terms of t **3**

c) The letters of the word **INTEGRAL** are arranged in a row. Calculate the probability that there are three letters between the letters "**N**' and "**T**". **3**

Question 13 continued on next page

Question 13 continued

d) During the medieval wars, the enemy wanted to attack a fortress with a 5 metre opening along a front wall. The strategy was to stand at the point P, on a line 8 metres from the opening and perpendicular to the wall, as per the diagram. The archer stands x metres away from the wall, thus giving an angle of vision, α, through which to fire arrows from a cross-bow.



(i) Show that the angle of vision α is given by

$$\alpha = \tan^{-1}\left(\frac{13}{x}\right) - \tan^{-1}\left(\frac{8}{x}\right)$$ **1**

(ii) Determine the distance x which gives the maximum angle of vision α. **3**

Question 14:* *Start a new Booklet **15 marks**

a) Let α, β , γ be the roots of $P(x) = 2x^3 - 5x^2 + 3x - 5$.

Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ ***3***

b) Let $(3 + 2x)^{20} = \sum_{r=0}^{20} a_r x^r$

(i) Write down an expression for a_r ***1***

(ii) Show that $\frac{a_{r+1}}{a_r} = \frac{40 - 2r}{3r + 3}$ ***2***

(iii) Find the value of r which produces the greatest coefficient in the expansion of $(3 + 2x)^{20}$ ***1***

c) A particle is moving in a straight line in simple harmonic motion. At time t it has displacement x metres from a fixed point O on the line where

$$x = (\cos t + \sin t)^2$$

At time t, the velocity of the particle is $\dot{x}\ ms^{-1}$ and the acceleration is $\ddot{x}\ ms^{-2}$

(i) Show that $\ddot{x} = -4(x - 1)$ ***2***

(ii) Find the extreme positions of the particle during its motion. ***2***

Question 14 continued on next page.

Question 14continued

d) (i) Show that:

$$(1-x)^n\left(1+\frac{1}{x}\right)^n = \left(\frac{1-x^2}{x}\right)^n$$ ***1***

(ii) By considering the expansion of $(1-x)^n\left(1+\frac{1}{x}\right)^n$ or otherwise, express

$$\binom{n}{2}\binom{n}{0} - \binom{n}{3}\binom{n}{1} + \ \dots \ + (-1)^n\binom{n}{n}\binom{n}{n-2}$$

in simplest form. ***3***

END OF EXAMINATION PAPER

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Q1	Let $t = \tan\theta$ then $\dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{2t}{1+t^2}$ $= \sin 2\theta$	B
Q2	$x = -\dfrac{3}{2}$ $\left(-\dfrac{3}{2}\right)^3 - 2\times\left(-\dfrac{3}{2}\right)^2 - 4\times\left(-\dfrac{3}{2}\right) + 7 = \dfrac{41}{8}$	B
Q3	$\dfrac{1(-2)+2x}{2+1} = 2 \Rightarrow x = 4$ $\dfrac{1(-4)+2y}{2+1} = 2 \Rightarrow y = 5$	A
Q4	$T_{k+1} = {}^4C_k x^{4-k}\left(\dfrac{3}{x}\right)^k$ $= {}^4C_k x^{4-k}\left(3^k x^{-k}\right)$ $= {}^4C_k 3^k x^{4-2k}$ independent of $x \Rightarrow 4-2k=0$ $k=2$ the term is ${}^4C_2 \times 3^2 = 54$	D
Q5	For simple harmonic motion $\ddot{x} = -n^2 x$	A

Q6	$\int_e^{e^2} \frac{2}{x(\log_e x)^2} dx = 2\int_e^{e^2} \frac{1}{(\log_e x)^2} \times \frac{1}{x} dx$ $= 2\int_1^2 \frac{1}{u^2} du$ $= 2\int_1^2 u^{-2} du$ $= 2\left[\frac{u^{-1}}{-1}\right]_1^2$ $= -2\left[\frac{1}{u}\right]_1^2$ $= -2\left(\frac{1}{2} - 1\right)$ $= 1$	D
Q7	$A = \frac{-2}{\pi}$ Domain is $D: -1 \le Bx \le 1$ $-\frac{1}{B} \le x \le \frac{1}{B}$ $\therefore \frac{1}{B} = 2$ $B = \frac{1}{2}$	C
Q8	$\frac{4!9!}{12!}$	B
Q9	$180° - \theta$	B
Q10	$\frac{1}{2}x + \frac{1}{4}\sin 2x + \tan x + c$	C

Q11

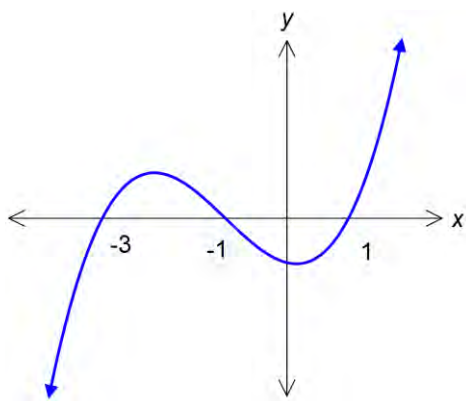
<table>
<tr><td>Q11 -a</td><td>

$$\left(\frac{x+3}{x^2-1}\right) \le 0 \qquad nb\ x \ne \pm 1$$

$$\left(\frac{x+3}{x^2-1}\right)\left(x^2-1\right)^2 \le 0 \times \left(x^2-1\right)^2$$

$$(x+3)\left(x^2-1\right) \le 0$$

$$(x+3)(x-1)(x+1) \le 0$$



therefore $x \le -3 \ or \ -1 < x < 1$

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2 marks: correct solution

1 mark:

$x \ne 1 \ or \ -1 \ and \ (x+3)\left(x^2-1\right) \le 0$

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<tr><td>Q11 -b</td><td>

$$P_{\delta fective} = p = \frac{1}{4}$$

$$P_{non\text{-}defective} = q = \frac{3}{4}$$

$$(p+q)^{500}$$

$$100\ defective = {}^{500}\mathbf{C}_{400}\ p^{100}\ q^{400}$$

$$= {}^{500}\mathbf{C}_{400}\left(\frac{1}{4}\right)^{100}\left(\frac{3}{4}\right)^{400}$$

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2 marks: correct solution

1 mark:

$p = \frac{1}{4},\ q = \frac{3}{4}\ \ and\ \ considering\ (p+q)^{500}$

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</table>

Q11-c	$p_6 = \frac{1}{6} \qquad q_{not\,6} = \frac{5}{6}$ $\therefore \quad p_{no\ 6's} = \left(\frac{5}{6}\right)^n$ $\therefore\ 1 - \left(\frac{5}{6}\right)^n > 0{\cdot}5$ $0{\cdot}5 > \left(\frac{5}{6}\right)^n$ $\ln(0{\cdot}5) > n\ln\left(\frac{5}{6}\right)$ $\frac{\ln(0{\cdot}5)}{\ln\left(\frac{5}{6}\right)} < n$ $3{\cdot}8 < x$ $\therefore \quad n = 4$ Therefore 4 throws of dice required.	2 marks: correct solution 1 mark: $1 - \left(\frac{5}{6}\right)^n > 0.5$ Or bald correct answer Note: many students failed to recognise that $\ln\left(\frac{5}{6}\right)$ is negative requiring inequality sign to be reversed
Q11-d	$y = 2x + 1 \qquad m_1 = 2 \qquad m_2 = m$ $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $1 = \left\lvert \frac{2 - m}{1 + 2m} \right\rvert$ $-1 = \frac{2 - m}{1 + 2m} \qquad 1 = \frac{2 - m}{1 + 2m}$ $-2m - 1 = 2 - m \qquad 1 + 2m = 2 - m$ $m = -3 \qquad m = \frac{1}{3}$	2 marks: correct solution 1 mark: correct formula

Q11-e-i	$st = -4$ $m_{OQ} = \frac{as^2}{2as} = \frac{s}{2}$ $m_{OP} = \frac{at^2}{2at} = \frac{t}{2}$ $m_{OQ} \times m_{OP} = -1$ $\frac{s}{2} \times \frac{t}{2} = -1$ $\therefore \quad st = -4$	1 mark: correct demonstration
Q11-e-ii	$Q(2as\,, as^2)\ P(2at,\ at^2)$ *As shape is a rectangle* $R = (2as + 2at, as^2 + at^2)$ $x = 2a(s + t) \qquad y = a(s^2 + t^2)$ $\frac{x}{2a} = (s + t)$ $(s + t)^2 = s^2 + 2st + t^2$ $(s + t)^2 - 2st = s^2 + t^2$ $\frac{x^2}{4a^2} - 2(-4) = s^2 + t^2$ $\therefore \quad y = a\left(\frac{x^2}{4a^2} + 8\right)$ $y = \frac{x^2}{4a} + 8a$ $4ay = x^2 + 32a^2$ $x^2 = 4a(y - 8a)$	2 marks: correct demonstration 1 mark: applying $(s + t)^2 = s^2 + 2st + t^2$ Note: too many students were unable to write down coords of R, but wasted time deriving them
Q11-f	$\sin\theta \cos\theta = \frac{1}{2}$ $\therefore 2\sin\theta\cos\theta = 1$ $\sin 2\theta = 1$ $2\theta = \frac{\pi}{2} + 2n\pi$ $\theta = \frac{\pi}{4} + n\pi$	2 marks :correct solution 1 mark: correct base value

Q11-g	$XB.AX = XP^2$ $(6+x)x = 16$ $x^2 + 6x - 16 = 0$ $(x+8)(x-2) = 0$ $x = -8 \text{ or } 2$ $\therefore \quad x = 2 \text{ distance}$	2 marks: correct solution 1 mark: correct equation $(6+x)\,x = 16$

Q12

Q12-a-i	$f(x) = \log_e x - \frac{1}{x}$ $f(1) = \log_e 1 - \frac{1}{1} = 0 - 1 = neg$ $f(2) = \log_e 2 - \frac{1}{2} = 0 \cdot 19 = pos$ Therefore at least one root must lie between 1 and 2 as graph moves from negative to positive.	1 mark – correct explanation based on correct values calculated.
Q12-a-i	$f(x) = \log_e x - \frac{1}{x}$ $f'(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$ $a_2 = a_1 - \frac{f(x)}{f'(x)}$ $= 1 \cdot 5 - \frac{\log_e 1 \cdot 5 - \frac{1}{1 \cdot 5}}{\frac{1 \cdot 5 + 1}{(1 \cdot 5)^2}}$ $= 1 \cdot 735$	2 marks – correct solution 1 mark – correct substitution into correct formula
Q12b-i	*Version 1* $T = S - Ae^{-kt}$ $\frac{dT}{dt} = kAe^{-kt}$ $= kS - kS + kAe^{-kt}$ $= -k\left(-S + S - Ae^{-kt}\right)$ $= -k(-S + T)$ $= k(S - T)$ *Verision 2* $T = S - Ae^{-kt}$ $\therefore Ae^{-kt} = S - T \quad ①$ $\frac{dT}{dt} = kAe^{-kt}$ $= k(S - T) \quad \text{from } ①$	1 mark – fully explained in explanation of substitution.

q12b-ii	$T = S - Ae^{-kt}$ $at\ t = 0 \qquad T = 4$ $4 = 160 - Ae^{0}$ $A = 156$ $at\ t = 30 \qquad T = 60$ $60 = 160 - 156e^{-30k}$ ① $e^{-30k} = \frac{100}{156}$ $-30k = \ln\frac{100}{156}$ $k = \ln\frac{100}{156} \div -30 = 0{\cdot}014823$ $150 = 160 - 156e^{-kt}$ $e^{-kt} = \frac{10}{156}$ $t = \ln\frac{10}{156} \div -k$ $= \ln\frac{10}{156} \div \left(-\ln\frac{100}{156} \div -30\right)$ $= 185min\ 20\sec$	3 marks – correct solution 2 marks – correct value for k 1 mark – correct to line (1)
Q12-c	$\frac{1}{(n+1)!} - \frac{n+1}{(n+2)!}$ $= \frac{1}{(n+1)!} - \frac{n+1}{(n+2)(n+1)!}$ $= \frac{n+2}{(n+2)(n+1)!} - \frac{n+1}{(n+2)!}$ $= \frac{(n+2)-(n+1)}{(n+2)!}$ $= \frac{1}{(n+2)!}$	1 mark – correct solution

<table>
<tr><td>Q12c-ii</td><td>

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ \dots\ \frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}$$

$For\ n=1$

$$LHS=\frac{1}{(1+1)!}=\frac{1}{2}$$

$$RHS=1-\frac{1}{(1+1)!}=\frac{1}{2}$$

</td><td rowspan="2">3 marks – correct solution clearly showing use of assumption and or substitution or matching algebra

2 marks – solution partially showing use of assumption and or substitution or matching algebra

1 mark – correct for $n=1$</td></tr>
<tr><td>Q12c-ii</td><td>

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ \dots\ \frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}$$

Assume true for $n=k$

$$ie.\ \frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ \dots\ \frac{k}{(k+1)!}=1-\frac{1}{(k+1)!}$$

RTP true for $n=k+1$

$$LHS=\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ \dots\ \frac{k}{(k+1)!}+\frac{k+1}{((k+1)+1)!}$$

$$=1-\frac{1}{(k+1)!}+\frac{k+1}{(k+2)!}$$

$$=1-\left\{\frac{1}{(k+1)!}-\frac{k+1}{(k+2)!}\right\}$$

$$=1-\frac{1}{(k+2)!}$$

$$=RHS$$

Therefore proved true for all n by process of mathematical induction.

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<tr><td>Q12d-i</td><td>

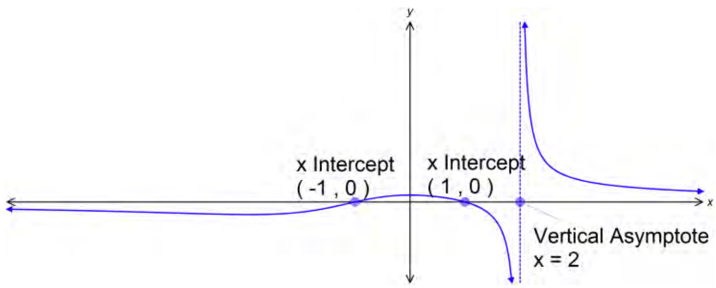
$$f(x)=\frac{x^2-1}{x^3-8}$$

$$=\frac{x^2-1}{(x-2)\left(x^2+x+4\right)}$$

$\therefore\quad x=2$ *is vertical asymptote*

</td><td>1 mark – correct solution

$x\neq 2$ not accepted.</td></tr>
<tr><td>Q12d-ii</td><td>



</td><td>3 marks

- correct shape
- correct asymptotes
- correct x intercepts</td></tr>
</table>

Q13

Q13-a	$\int e^{x+e^x}\,dx$ $u = e^x \qquad du = e^x\,dx$ $= \int e^x \times e^{e^x}\,dx$ $= \int e^u\,du$ $= e^u + C$ $= e^{e^x} + C$	3 marks – correct solution 2 marks – for correct expression $\int e^u\,du = e^u + C$ 1 mark for $e^{x+e^x} = e^x.e^{e^x}$
Q13b-i	$\dot{x} = -\frac{1}{8}x^3 = v$ $\therefore \frac{1}{2}v^2 = -\frac{1}{128}x^6$ $\ddot{x} = \frac{d\left(\frac{1}{2}v^2\right)}{dx} = \frac{6}{128}x^5$ $= \frac{3}{64}x^5$	2 marks for correct solution. 1 mark for $\frac{1}{2}v^2 = \frac{1}{128}x^6$

<table>
<tr><td>Q13b-ii</td><td>

$\dot{x} = -\frac{1}{8}x^3$

$\frac{dx}{dt} = -\frac{1}{8}x^3$

$\therefore \frac{dt}{dx} = -\frac{8}{x^3} = -8x^{-3}$

$t = \int -8x^{-3}dx = \frac{-8x^{-2}}{-2}$

$t = \frac{4}{x^2} + C$

$at\ t = 0 \qquad x = 2$

$0 = \frac{4}{4} + C$

$\therefore C = -1$

$t = \frac{4}{x^2} - 1$

$t + 1 = \frac{4}{x^2}$

$x^2 = \frac{4}{t+1}$

$x = \sqrt{\frac{4}{t+1}}$

Note; Positive answer only to agree original conditions</td><td>3 marks for correct solution

2 marks
- $t = \frac{4}{x^2} + C\ and\ C = -1$
- $t = \frac{4}{x^2} + C$ and correct primitive from incorrect value for C

1 mark - $t = \frac{4}{x^2}$</td></tr>
<tr><td>Q13c</td><td>

INTEGRAL

N				T			
	N				T		
		N				T	
			N				T

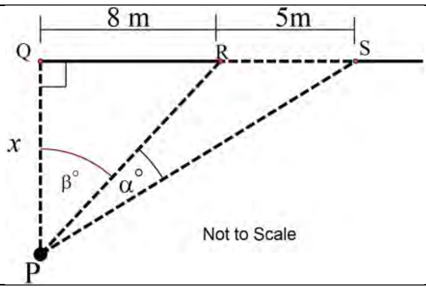
$Total\ Number\ of\ arrangements = 8!$

$Number\ of\ restricted\ arrangements = 4 \times 2! \times 6!$

$Prob = \frac{4 \times 2! \times 6!}{8!} = \frac{1}{7}$</td><td>3 marks – correct solution

2 marks – for *n(S)=8!* and any two correct of 4 or 2! or 6!

1 mark
- 8!
-any two of 4 or 2! or 6!</td></tr>
</table>



Q13-d	$\alpha = \tan^{-1}\left(\frac{13}{x}\right) - \tan^{-1}\left(\frac{8}{x}\right)$ $\alpha = \angle QPS - \angle QPR$ $\tan(\angle QPR) = \frac{8}{x} \qquad \tan(\angle QPS) = \frac{13}{x}$ $\angle QPR = \tan^{-1}\frac{8}{x} \quad \angle QPS = \tan^{-1}\frac{13}{x}$ $\therefore \quad \alpha = \tan^{-1}\left(\frac{13}{x}\right) - \tan^{-1}\left(\frac{8}{x}\right)$	1 mark for correct solution
Q13-d-ii	$\alpha = \tan^{-1}\left(\frac{13}{x}\right) - \tan^{-1}\left(\frac{8}{x}\right)$ $\frac{d\alpha}{dx} = -\frac{13}{x^2}\left(\frac{1}{1+\left(\frac{13}{x}\right)^2}\right) + \frac{8}{x^2}\left(\frac{1}{1+\left(\frac{8}{x}\right)^2}\right)$ $\therefore at \frac{d\alpha}{dx} = 0$ $\frac{13}{x^2}\left(\frac{1}{1+\left(\frac{13}{x}\right)^2}\right) = \frac{8}{x^2}\left(\frac{1}{1+\left(\frac{8}{x}\right)^2}\right)$ $\frac{8x^2}{x^2+64} = \frac{13x^2}{x^2+169}$ $8(x^2+169) = 13(x^2+64)$ $x^2 = \frac{520}{5}$ $x = \sqrt{104}$ Test for maximum $x = 10 \quad \frac{d\alpha}{dx} = -\frac{13}{100}\left(\frac{1}{1+\left(\frac{13}{10}\right)^2}\right) + \frac{8}{100}\left(\frac{1}{1+\left(\frac{8}{10}\right)^2}\right) = 0 \cdot 0004$ $x = 10 \cdot 2 \quad \frac{d\alpha}{dx} = -0 \cdot 000004$ Therefore change in gradient positive, zero to negative therefore maximum.	3 marks – correct solution 2 marks - $x = \sqrt{104}$ 1 mark - correct for $\frac{d\alpha}{dx}$ - value of x correctly obtained from incorrect $\frac{d\alpha}{dx}$

Q14-a	$\alpha+\beta+\gamma = -\frac{b}{a} = \frac{5}{2}$ $\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a} = \frac{3}{2}$ $\alpha\beta\gamma = -\frac{d}{a} = \frac{5}{2}$ $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2} = \frac{\beta^2\gamma^2+\alpha^2\gamma^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$ $(\alpha\beta+\alpha\gamma+\beta\gamma)^2$ $= \alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2+2\left(\alpha^2\beta\gamma+\alpha\beta^2\gamma+\alpha\beta\gamma^2\right)$ $= \alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2+2\alpha\beta\gamma(\alpha+\beta+\gamma)$ $\therefore \quad \left(\frac{3}{2}\right)^2 = \alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2+2\times\frac{5}{2}\times\frac{5}{2}$ $\frac{9}{4}-\frac{25}{2} = \alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2$ $\frac{\beta^2\gamma^2+\alpha^2\gamma^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2} = \left(-\frac{41}{4}\right) \div \frac{25}{4} = -\frac{41}{25}$	3 marks: correct solution 2 marks : partial correct with a least 2 values of ratios of coefficients correct with correct required expansion 1 mark: at least two ratios correct only Or 1 mark: correct required expansion only
	$(3+2x)^{20}$ $\therefore x^r\ term = {}^{20}\mathbf{C}_r 3^{20-r}.(2x)^r$ $a_r = {}^{20}\mathbf{C}_r 3^{20-r}.(2)^r$	1 mark: correct solution

	$a_r = {}^{20}\mathbf{C}_r 3^{20-r}.(2)^r$ $a_{r+1} = {}^{20}\mathbf{C}_{r+1} 3^{19-r}.(2)^{r+1}$ $\frac{a_{r+1}}{a_r} = \frac{{}^{20}\mathbf{C}_{r+1} 3^{19-r}.(2)^{r+1}}{{}^{20}\mathbf{C}_r 3^{20-r}.(2)^r}$ $= \frac{20! \times 3^{19-r}.(2)^{r+1}}{(19-r)!(r+1)!} \times \frac{(20-r)!r!}{20! \times 3^{20-r}.(2)^r}$ $= \frac{2(20-r)}{3(r+1)}$ $= \frac{40-2r}{3r+3}$	2 marks: correct solution 1 mark: partial correct with both terms correct
	$\frac{40-2r}{3r+3} \le 1$ $40 - 2r \le 3r + 3$ $37 \le 5r$ $7{\cdot}4 \le r$ $\therefore \quad r = 8$ $r = 8 \qquad {}^{20}\mathbf{C}_8 3^{12} \times 2^8 = 1{\cdot}714 \times 10^{13}$ $r = 7 \qquad {}^{20}\mathbf{C}_7 3^{13} \times 2^7 = 1{\cdot}582 \times 10^{13}$ Test $r = 9 \qquad {}^{20}\mathbf{C}_9 3^{11} \times 2^9 = 1{\cdot}523 \times 10^{13}$	1 mark: correct solution
14c-i	$x = (\cos t + \sin t)^2 = \cos^2 t + 2\sin t\cos t + \sin^2 t$ $\therefore \quad x = 1 + \sin 2t$ $\dot{x} = 2\cos 2t$ $\ddot{x} = -4\sin 2t$ $= -4(1 + \sin 2t - 1)$ $= -4(x - 1)$	2 marks: corrcet solution 1 mark: correct expression for velocity
14c-ii	$x = 1 + \sin 2t$ $-1 \le \sin 2t \le 1$ $0 \le 1 + \sin 2t \le 2$ $0 \le x \le 2$ therefore extremes of position are $x = 0$ and $x = 2$	2 marks: correct solution 1 mark: only one correct extreme

14 d -i	$(1-x)^n\left(1+\frac{1}{x}\right)^n = \left[(1-x)\left(1+\frac{1}{x}\right)\right]^n$ $= \left[1+\frac{1}{x}-x-1\right]^n$ $= \left(\frac{1}{x}-x\right)^n$ $= \left[\frac{1-x^2}{x}\right]^n$	1 mark: correct solution

14d-ii	Determine the coefficient of x^2 on both sides of the equation. $$RHS = (1-x)^n\left(1+\frac{1}{x}\right)^n$$ $$= \left[\sum_{k=0}^{n}(-1)^k\, {}^n\mathbf{C}_k x^k\right]\left[\sum_{k=0}^{n} {}^n\mathbf{C}_k x^{-k}\right]$$ $\therefore$ *coefficient of* x^2 $$\binom{n}{2}\binom{n}{0}-\binom{n}{3}\binom{n}{1}+\ldots\ldots+(-1)^n\binom{n}{n}\binom{n}{n-2}$$ $$LHS = \left(\frac{1-x^2}{x}\right)^n = \left(\frac{1}{x}-x\right)^n$$ $$= \sum_{k=0}^{n}(-1)^k\binom{n}{k}x^{-(n-k)}x^k$$ $$= \sum_{k=0}^{n}(-1)^k\binom{n}{k}x^{2k-n}$$ $\therefore$ *coefficient of* x^2 *at* $$2k-n = 2$$ $$k = 1+\frac{n}{2}$$ If n is odd, then k is not an integer therefore cannot exist. therefore $$\binom{n}{2}\binom{n}{0}-\binom{n}{3}\binom{n}{1}+\ \ldots\ +(-1)^n\binom{n}{n}\binom{n}{n-2}=0$$ If n is even then $(-1)^{1+\frac{n}{2}}\binom{n}{1+\frac{n}{2}}$ is coefficient of x^2 therefore $$\binom{n}{2}\binom{n}{0}-\binom{n}{3}\binom{n}{1}+\ \ldots\ +(-1)^n\binom{n}{n}\binom{n}{n-2}=(-1)^{1+\frac{n}{2}}\binom{n}{1+\frac{n}{2}}$$	3 marks : correct solution 2 marks: partial correct with significant progress to equating coefficients of squared term 1 mark: one expression for the squared term correct