NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

HIGHER SCHOOL CERTIFICATE

Trial Examination

2015

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Write your Student Number at the top of each page
- Answer Section I- Multiple Choice on Answer Sheet provided
- Answer Section II – Free Response in a separate booklet for each question.
- Board approved calculators and templates may be used.

Section I Multiple Choice

- 10 marks
- Attempt all questions

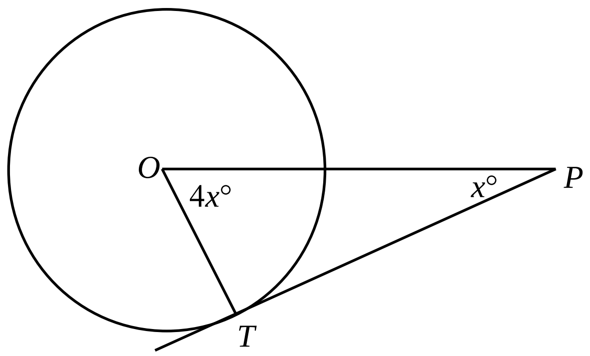
Section II – Free Response

- 60 marks
- Each question is of equal value
- All necessary working should be shown in every question.

Weighting: 40%

Multiple Choice: Answer questions on provided answer sheet.

Q1. The diagram shows a circle with centre O. The line PT is tangent to the circle at the point T. $\angle TOP = 4x°$ and $\angle TPO = x°$.



What is the value of x?

(A) 9

(B) 18

(C) 36

(D) 72

Q2. Which of the following is a simplified expression for $\dfrac{\sin 2x}{1-\cos 2x}$

(A) $\sin x$

(B) $\cos x$

(C) $\tan x$

(D) $\cot x$

Q3. The point P divides the interval AB in the ratio 3:7. In what external ratio does the A divide the interval PB

(A) 3:10

(B) 3:4

(C) 7:3

(D) 10:3

Q4. What is the obtuse angle between lines $y = -x$ and $\sqrt{3}\,y = x$?

(A) 15°

(B) 75°

(C) 105°

(D) 165°

Q5. What is the value of $\int_1^2 \frac{1}{\sqrt{4-x^2}}\,dx$?

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Q6. In how many ways can 5 people be selected from a group of 6 and then arranged in a line so that the two oldest people in the selected group are at either end of the line? (NB. No two people are the same age.)

(A) 720

(B) 144

(C) 72

(D) 36

Q7. The remainder of the division $\frac{x^5+1}{x^2-1}$ is equal to

(A) 1

(B) 2

(C) $x+2$

(D) $x+1$

Q8. The power of x in the 7[th] term of the expansion of $\left(\frac{4x}{5}-\frac{8}{5x}\right)^9$ is

(A) 3

(B) -3

(C) 5

(D) -5

Q9. The velocity of a particle is given by the equation $v=\sqrt{4+4x}$. If the particle is initially located at the origin, what displacement at $t = 3$?

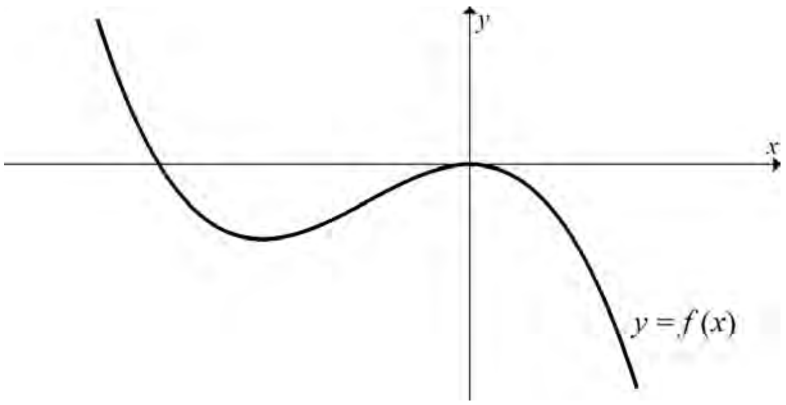
(A) 3

(B) 8

(C) 15

(D) 16

Q10. The diagram show the graph of a cubic function $y=f(x)$.



Which is a possible equation of this function?

(A) $f(x)= -x(x-2)(x+2)$

(B) $f(x)=x^2(x-2)$

(C) $f(x)= -x^2(x+2)$

(D) $f(x)= -x^2(x-2)$

End of Multiple Choice

Question 11: Start A New Booklet 15 Marks

(a) Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\sin^2 x + \cos^2 x \ dx$. **(3)**

(b) (i) Verify that $(\alpha\beta+\alpha\gamma+\beta\gamma)^2 = \alpha^2\beta^2+\alpha^2\gamma^2+\beta^2\gamma^2 + 2\alpha\beta\gamma(\alpha+\beta+\gamma)$. **(1)**

(ii) Hence, or otherwise, if α, β and γ are the roots of $x^3 - 3x^2 - x + 3 = 0$, evaluate $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ **(3)**

(c) (i) Determine the vertical asymptotes for $y = \frac{x+1}{x^2 - 16}$ **(2)**

(ii) Hence sketch the curve $y = \frac{x+1}{x^2 - 16}$ **(2)**

(d) Find the general solution of the equation

$$\cot\theta \sec^2\theta - \cot\theta - \sqrt{3}\sec^2\theta + \sqrt{3} = 0$$ **(4)**

Question 12 Start A New Booklet 15 Marks

(a) Use the substitution $x = 2\sin\theta$ to show $\frac{\pi}{3}+\frac{\sqrt{3}}{2}=\int_0^1 \sqrt{4-x^2}\ dx$ **(3)**

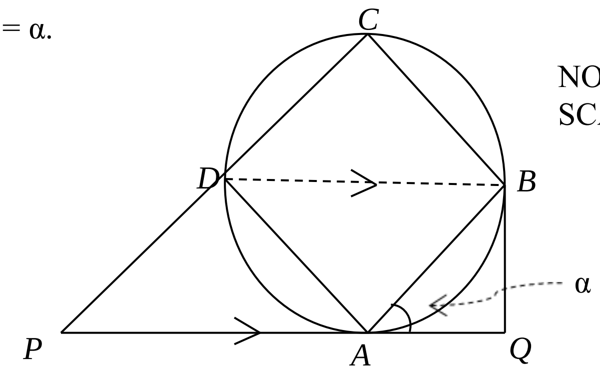
(b) A particle moves with acceleration $\ddot{x} = -\frac{1}{4\sqrt{x}}$. Initially, the particle is one metre to the right of the origin and its velocity is 4*m/s*.

Find the displacement of the particle when it is at rest. **(3)**

(c) *ABCD* is a cyclic quadrilateral. The tangents from *Q* touch the circle at *A* and *B*.

The diagonal *DB* is parallel to the tangent *AQ*, and *QA* produced intersects *CD* produced at *P*.

Let $< QAB = \alpha$.



(i) Prove that ΔBAD is isosceles, giving reasons. **(2)**

(ii) Find $< DCB$ in terms of α, stating reasons. **(1)**

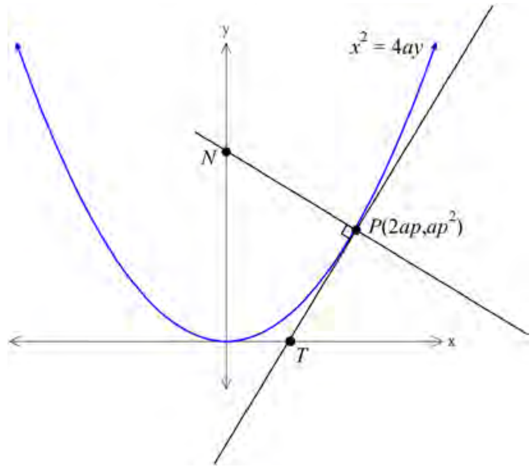
(iii) Show that P,C, B and Q are concyclic points **(2)**

(d) (i) Show that $x = 1.8$ is a reasonable approximation for the x value of the point of intersection of $y = 2\sin x$ and $y = x$ in the domain $0.78 < x < 2.35$. **(1)**

(ii) Use Newton's method with one application to find a better approximation to the x value of this point on intersection. **(3)**

Question 13 Start A New Booklet 15 marks

(a) The diagram shows the parabola $x^2 = 4ay$



The tangent to the parabola at $P(2ap, ap^2)$ cuts the x-axis at T and the normal at P cuts the y-axis at N.

The equation to the tangent is given by $y = px - ap^2$

(i) Show the coordinates of N are $\left(0, a(p^2 + 2)\right)$ **(2)**

(ii) Let M be the midpoint of NT. Find the Cartesian equation of the locus. (3)

(b) Use mathematical induction to prove that $(n+1)^2 + n - 1$ is divisible by 2 for all integers $n \geq 1$. **(3)**

(c) A school band is to be formed with a brass section containing 8 students and a percussion section containing 4 students.

(i) In how many ways can the band be formed if 12 students audition for the brass section and 10 students audition for the percussion section? **(1)**

(ii) In how many ways can the band be formed if it is certain that Maria will be successful for the brass audition and Marcus will be successful for the percussion audition? **(2)**

Question 13 continues on next page

Question 13 continued

(d) At time t years the number N of individuals is given by $N = \dfrac{a}{1 + b\,e^{-t}}$ for some constants $a > 0$, $b > 0$. The initial population size is 20 and the limiting population size is 100.

(i) Show that $\dfrac{dN}{dt} = N\left(1 - \dfrac{N}{a}\right)$ **(2)**

(ii) Find the values of a and b. **(2)**

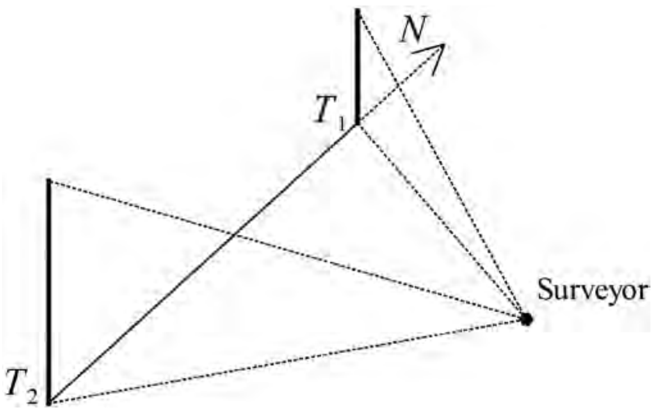
Question 14 — 15 marks

(a) Warehouse A has 100 computers and the probability that of selecting a computer which is defective is 0.02.

Warehouse B has 100 computers, two of which only, are defective.

Joe buys three computers from Warehouse A and three computers from Warehouse B. What is the probability that exactly one of the computers he has bought is defective? **(3)**

(b) Two towers T_1 and T_2 have heights h metres and $2h$ metres respectively. The second tower is due south of the first tower. The bearing of tower T_1 from a surveyor is 292°. The bearing of the tower T_2 from the surveyor is 232°. The angle of elevation from the surveyor to the top of tower T_1 is 30° while the angle of elevation from the surveyor to the top of tower T_2 is 60°.



Show that the distance d between the two towers is given by $d = \dfrac{\sqrt{21}h}{3}$ metres. **(3)**

Question 14 continues on next page

Question 14 continued.

(c) A ball is projected vertically from the ground with a speed of 49 m/s. The height y of the ball at time t is given by $y = -4.9t^2 + 49t.$ (Do NOT show this.)

At the same time, a second ball is projected from the ground into the air with an angle of projection θ. Its horizontal displacement is given by $x = 98t\cos\theta$ and its height is given by $y = -4.9t^2 + 98t\sin\theta$. (Do NOT show this.)

(i) Find the maximum height of the ball that was projected vertically. **(2)**

(ii) Find the value of θ at which the second ball should be projected if it is to hit the first ball when the first ball reaches its maximum height. **(2)**

(iii) Find the horizontal distance between the two balls when they are first projected into the air. Give your answer in exact form. **(1)**

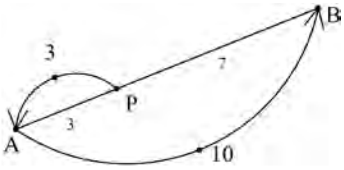
(e) Consider the binomial expansion

$$1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{n}x^n = (1+x)^n$$

Show that, if n is even:

$$4 \times 1 \times \binom{n}{2} + 8 \times 3 \times \binom{n}{4} + 12 \times 5 \times \binom{n}{6} + \ldots + 2n(n-1)\binom{n}{n} = n(n-1)(2)^{n-2}$$ **(4)**

<u>End of Examination</u>

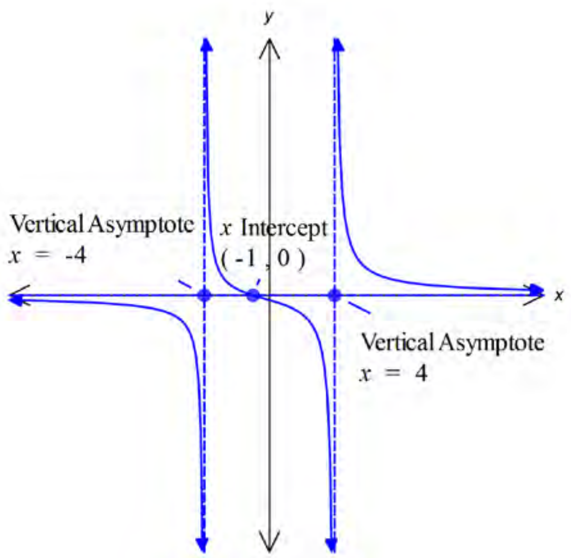
Q1	$OT \perp TP$ radii $\perp$ to tangent at point of contact $5x° + 90° = 180°$ $x = 18$	B
Q2	$\dfrac{\sin 2x}{1-\cos 2x}$ $= \dfrac{2\sin x\cos x}{1-(1-2\sin^2 x)}$ $= \dfrac{\sin x\cos x}{\sin^2 x}$ $= \dfrac{\cos x}{\sin x}$ $= \cot x$	D
Q3		A
Q4	$\int_1^2 \dfrac{1}{\sqrt{4-x^2}}\,dx$ $= \left[\sin^{-1}\dfrac{x}{2}\right]_1^2$ $= \dfrac{\pi}{2} - \dfrac{\pi}{6}$ $= \dfrac{\pi}{3}$	C

Q5	$m_1 = {}^{-}1 \qquad m_2 = \frac{1}{\sqrt{3}}$ $\tan\theta = \left\lvert \frac{m_2 - m_1}{1 + m_1 m_2} \right\rvert$ $= \left\lvert \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} \right\rvert = \left\lvert \frac{1+\sqrt{3}}{\sqrt{3}-1} \right\rvert$ $= \lvert 2+\sqrt{3} \rvert$ $\theta = \tan^{-1}(2+\sqrt{3}) = 75°$ $\therefore$ Obtuse angle = $180° - 75° = 105°$	C
Q6	${}^{6}\mathbf{C}_5 \times {}^{2}\mathbf{P}_1 \times {}^{3}\mathbf{P}_3 = 72$	C
Q7	$\frac{x^5+1}{x^2-1}$ $(x^5+1) = (x^2-1)Q(x) + (ax+b)$ $= (x+1)(x-1)Q(x) + (ax+b)$ $x = 1 \Rightarrow \quad 2 = a+b$ $x = {}^{-}1 \ = .\ 0 = b - a$ $\therefore \quad a = 1 \ \ b = 1$ $R(x) = x + 1$	D
Q8	${}^{9}\mathbf{C}_6\left(\frac{4x}{5}\right)^3\left(-\frac{8}{5x}\right)^6$ $= {}^{9}\mathbf{C}_6\left(\frac{4}{5}\right)^3\left(-\frac{8}{5}\right)^6 x^3 x^{-6}$ $= {}^{9}\mathbf{C}_6\left(\frac{4}{5}\right)^3\left(-\frac{8}{5}\right)^6 x^{-3}$	B

Q9	$v = \sqrt{4 + 4x} = 2\sqrt{1 + x}$ $\frac{dx}{dt} = 2\sqrt{1 + x}$ $\frac{dt}{dx} = \frac{1}{2\sqrt{1 + x}}$ $t = \frac{1}{2}\int (1 + x)^{-\frac{1}{2}}\, dx$ $t = \frac{1}{2} \times 2 \times \sqrt{1 + x} + C$ $t = 0 \Rightarrow x = 0$ $\therefore \quad C = {}^{-}1$ $t = \sqrt{1 + x} - 1$ $1 + x = (t + 1)^2$ $x = (t + 1)^2 - 1$ $t = 3$ $x = (1 + 3)^2 - 1 = 15$	C
Q10	Cubic of form $y = -ax^2(x - b)$ $\therefore y = -x^2(x + 2)$	C

Q11

	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\sin^2 x + \cos^2 x \ dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x + \cos^2 x \ + \ \sin^2 x \, dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 \ + \ \sin^2 x \, dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 \ dx \ + \frac{1}{2} - \frac{1}{2}\cos(2x) dx$ $= \left[\frac{3x}{2} - \frac{\sin(2x)}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \frac{3\pi}{4} - \frac{3\pi}{8} - \frac{\sin\pi}{4} - \frac{\sin\frac{\pi}{2}}{4}$ $= \frac{3\pi}{8} + \frac{1}{4}$	3 marks – correct solution 2 marks - correct integrand 1 mark – simplification to $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 + \sin^2 x \ dx$
	$(\alpha\beta + \alpha\gamma + \beta\gamma)^2$ $= \alpha^2\beta^2 + \alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha^2\beta\gamma + \alpha^2\gamma^2 + \alpha\beta\gamma^2 + \alpha\beta^2\gamma + \alpha\beta\gamma^2 + \beta^2\gamma^2$ $= \alpha^2\beta^2 + \beta^2\gamma^2\alpha^2\gamma^2 + 2(\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2)$ $= \alpha^2\beta^2 + \beta^2\gamma^2\alpha^2\gamma^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$	1 mark – expansion demonstrated

	$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$ $=\frac{\beta^2\gamma^2+\alpha^2\gamma^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$ $=\frac{(\alpha\beta+\alpha\gamma+\beta\gamma)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha^2\beta^2\gamma^2}$ $\alpha+\beta+\;+\gamma=-\frac{b}{a}=3$ $\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}=^-1$ $\alpha\beta\gamma=-\frac{d}{a}=^-3$ $\therefore \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{(^-1)^2-2\times{}^-3\times 3}{(^-3)^2}$ $=\frac{19}{9}$	3 marks – correct solution 2 marks – correct rearrangement of initial term 1 mark – correct values for $\alpha+\beta+\gamma$ $\alpha\beta+\alpha\gamma+\beta\gamma$ $\alpha\,\beta\,\gamma$
	Vert Asymtotes – denominator = 0 $x^2-16=0$ $x=\pm 4$	2 marks – correct answer 1 mark – equating denominator to zero
		2 marks - shape - x-intercept -

	$$\begin{aligned}\cot\theta\sec^2\theta - \cot\theta - \sqrt{3}\sec^2\theta + \sqrt{3} &= 0\\ \cot\theta(\sec^2\theta - 1) - \sqrt{3}(\sec^2\theta - 1) &= 0\\ (\cot\theta - \sqrt{3})\tan^2\theta &= 0\\ \cot\theta &= \sqrt{3}\\ \tan\theta &= \frac{1}{\sqrt{3}}\\ \theta &= n\pi + \frac{\pi}{6}\\ \tan\theta &= 0\\ \text{but if } \tan\theta &= 0\end{aligned}$$ "then" cot θ is undefined in original equation ie. no solution $\therefore \quad \theta = n\pi + \frac{\pi}{6}$	4 marks - correct solution 3 marks -correct solution for without stating reason for excluding cot Θ 2 marks – fully factorised and use of trig identity 1 mark –first factorisation

Q12

	$\int_0^1 \sqrt{4-x^2}\ dx$ $x = 2\sin\theta \Rightarrow dx = 2\cos\theta d\theta$ $x = 0 \Rightarrow \theta = 0$ $x = 1 \Rightarrow \theta = \frac{\pi}{6}$ $\int_0^{\frac{\pi}{6}} \sqrt{4-4\sin^2\theta} \times 2\cos\theta d\theta$ $= 4\int_0^{\frac{\pi}{6}} \cos^2\theta\ d\theta$ $= 2\int_0^{\frac{\pi}{6}} \cos(2\theta) + 1 d\theta$ $= \left[\sin(2\theta) + 2\theta\right]_0^{\frac{\pi}{6}}$ $= \sin\frac{\pi}{3} + \frac{2\pi}{6} - 0 - 0$ $= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$	3 marks – correct solution 2 marks – final integral simplified to cos2Θ 1 mark – correct initial substitution including limits and dΘ

	$\ddot{x} = -\frac{1}{4\sqrt{x}} = -\frac{1}{4}x^{-\frac{1}{2}}$ $\frac{d\left(\frac{1}{2}v^2\right)}{dx} = -\frac{1}{4}x^{-\frac{1}{2}}$ $\frac{1}{2}v^2 = -\frac{1}{4} \times 2 \times \sqrt{x} + C_1$ $v^2 = -\sqrt{x} + C_2$ $t = 0 \quad v = {}^-4$ $16 = 0 + C_2$ $C_2 = 15$ $v^2 = -(\sqrt{x}) + 15$ $v = 0$ $\sqrt{x} = 15$ $x = 15^2 = 225m$	3 marks – correct solution 2 marks – attaining constant correctly 1 mark – use of $\frac{d\left(\frac{1}{2}v^2\right)}{dx} = \ddot{x}$
c-i	$\angle BAQ = \angle DBA = \alpha$ (alt angles on || lines are equal) $\angle BDA = \angle BAQ - \alpha$ (angle between tangent and chord is equal to $\angle$ in alternate segment) $\therefore$ $\Delta\, BAD$ is isosceles (two equal angles)	2 correct solution 1 mark – one correct use of geometrical principle.
c-ii	$\angle DAB = 180° - 2\alpha$ ($\angle$ sum of Δ) $\angle DCB + \angle DAB = 180°$ (opp $\angle$ of cylic quad are supplementary) $\therefore$ $\angle DCB = 2\alpha$	1 – correct solution

c-iii	$BQ = AQ$ (common tangents from a point are equal) $\therefore$ $\Delta\ BAQ$ is isosceles (2 sides equal) $\therefore$ $\angle ABQ = \angle BAQ = \alpha$ ($\angle$'s opp equal sides in isoc Δ are equal) $\therefore$ $\angle BQA = 180° - 2\alpha$ ($\angle$ sum of Δ) $\therefore$ $\angle BQA + \angle DCB = 180° - 2\alpha + 2\alpha = 180°$ $PCBQ$ form cyclic quad (opp $\angle$ s supplementary) $\therefore PCBQ$ are concyclic	2 marks – correct solution 1 mark – determination of size of $\angle BQA$
d-i	$y = 2\sin(1\cdot 8) = 1\cdot 947$ $2\sin x - x = 1\cdot 947 - 1\cdot 8 = 0\cdot 147$ The solution is close to zero therefore reasonable approximation.	1mark – correct solution
d-ii	$x_a = x_0 - \dfrac{f(x)}{f'(x)}$ $= x_0 - \dfrac{2\sin x - x}{2\cos x - 1}$ $= 1\cdot 8 - \dfrac{2\sin 1\cdot 8 - 1\cdot 8}{(2\cos 1\cdot 8 - 1)}$ $= 1\cdot 9$	3 marks – correct solution 2 marks – correct expression for x_a 1 mark – incorrect substitution into initial formula

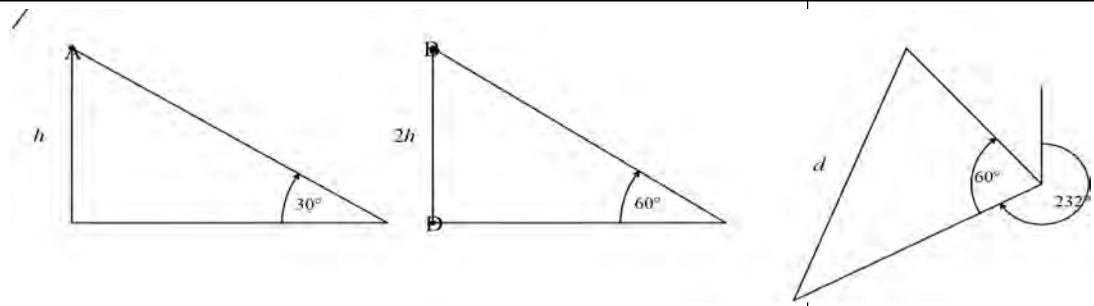
Q13

a)i)	eqn normal: $y - ap^2 = -\frac{1}{p}(x - 2ap)$ $yp - ap^3 = -x + 2apy$ $y = \frac{-x + 2ap + ap^3}{p}$ N lies on normal and y axis $(x = 0)$ $y = \frac{-0 + 2ap + ap^3}{p}$ $= 2a + ap^2$ $= a(p^2 + 2)$ $\therefore N(0, a(p^2+2))$	2 marks-correct solution showing all steps 1 mark- correct equation of normal
ii)	T lies on tangent and x-axis $0 = px - ap^2$ $x = \frac{ap^2}{p}$ $= ap$ midpoint NT $= \left[\frac{0+ap}{2}, \left(0 + \frac{a(p^2+2)}{2}\right)\right]$ $= \left(\frac{ap}{2}, \frac{a(p^2+2)}{2}\right)$ now $x = \frac{ap}{2}$ $p = \frac{2x}{a}$ $y = \frac{a}{2}\left(\left(\frac{2x}{a}\right)^2 + 2\right)$ $= \frac{4x^2}{a} + 2a$	3 marks- correct equation showing all steps 2 marks-correct values for midpoint 1 mark- correct x value for T

b)	prove true for $n = 1$ $(n+1)^2 + n - 1 = (2)^2 + 1 - 1$ $= 4$ which is divisible by 2 $\therefore$ true for n=1 assume true for $n = k$ $(k+1)^2 + k - 1 = 2M$ for some integer M prove true for n=k+1 $ie\ (k = 2)^2 + (k+1) - 1 = 2N$ $\text{LHS} = (k+2)^2 + k$ $= \left[(k+1)^2 + 2k + 3\right] + k$ $= (k+1)^2 + 3k + 3$ $= 2M + 1 - k + 3k + 3$ using assumption $(k+1)^2 = 2M + 1 - k$ $= 2M + 4 - 2k$ $= 2(M + 2 - k)$ $= 2N$ where N is the integer $M + 2 - k$ since $k \geq 1$ and M $\therefore$ true for n=k+1 $\therefore$ statement true as shown by mathematical induction	3 marks- correct solution showing all steps 2 marks-partial correct with insertion of assumption into S(k+1) 1 mark- correct for n=1
c)i)	${}^{12}C_8 \times {}^{10}C_4 = 495 \times 210$ $= 103950$	1 mark- correct solution
ii)	${}^{11}C_7 \times {}^{9}C_3 = 27720$	2 marks- correct solution 1 mark- correct selections for brass/percussion only
d)i)	$N = \dfrac{a}{1 + b e^{-t}}$ $\dfrac{dN}{dt} = \dfrac{\left(1 + b e^{-t}\right)^{80} - a\left(-b e^{-t}\right)}{\left(1 + b e^{-t}\right)^2}$ $= \dfrac{a\left(b e^{-t}\right)}{\left(1 + b e^{-t}\right)^2}$ Now $N = \dfrac{a}{1 + b e^{-t}}$ $\dfrac{N}{a} = \dfrac{1}{1 + b e^{-t}}$ $\therefore \dfrac{dN}{dt} = N\left(\dfrac{b e^{-t}}{1 + b e^{-t}}\right)$ $= N\left(\dfrac{1 + b e^{-t} - 1}{1 = b e^{-t}}\right)$ $= N\left(1 - \dfrac{N}{a}\right)$	2 marks –correct solution 1 mark- correct initial expression for dN/dt

ii)	$t \to \infty$ $N = 100$ $be^{-t} \to 0$ $\therefore \quad 100 = \dfrac{a}{1+0}$ $a = 100$ $t = 0$ $N = 20$ $\therefore \quad 20 = \dfrac{a}{1+be^{0}}$ $20 = \dfrac{a}{1+b}$ $20 = \dfrac{100}{1+b}$ $20 + 20b = 100$ $b = 4$	2 marks- correct solution 1 mark- only one correct value

Question 14

a	P(exactly one computer defective) =P(1 defective from A, 0 from B) +P(0 from A, 1 from B) ${}^3\mathbf{C}_1(0.02)^1(0.98)^2 \times \frac{98}{100} \times \frac{97}{99} \times \frac{96}{98} + {}^3\mathbf{C}_0(0.98)^3 \times \frac{2}{100} \times \frac{98}{99} \times \frac{97}{98} \times 3$ $= 0.1095$	3 marks: correct solution 2 marks: obtaining a correct binomial prob. 1 mark: 1st line – showing understanding of options required.
b		
	In Δ AT_1S, $\tan 30° = \dfrac{h}{(T_1S)}$ $\dfrac{1}{\sqrt{3}} = \dfrac{h}{T_1S}$ $T_1S = \sqrt{3}h$ In Δ BT_2S, $\tan 60 = \dfrac{2h}{T_2S}$ $T_2S = \dfrac{2h}{\sqrt{3}}$	3 marks: correct solution 2 marks:correct expressions for triangles, attempting to use them in cos rule 1 mark: correct expressions for triangles

	In $\Delta\, T_1 T_2 S$ $\quad d^2 = (\sqrt{3}h)^2 + \left(\frac{2h}{\sqrt{3}}\right)^2 - 2\times\sqrt{3}h\times\frac{2h}{\sqrt{3}}\cos 60°$ $= 3h^2 + \frac{4h^2}{3} - 2h^2$ $= \frac{7h^2}{3}$ $d = \frac{\sqrt{7}h}{\sqrt{3}}$ $= \frac{\sqrt{7}h}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{\sqrt{21}h}{3}$ as required	
i	$y = -4.9t^2 + 49t$ $\dot{y} = -9.8t + 49$ max ht occurs when $\dot{y} = 0$ i.e. $\quad 0 = -9.8t + 49$ $9.8t = 49$ $t = 5$ when t=5, $\quad y = -4.9\times 5^2 + 49\times 5$ $= 122.5$ max ht is 122.5 m	2 marks: correct solution 1 mark: finding $\dot{y}$ and solving $\dot{y} = 0$
ii	For 2nd ball $\quad x = 98t\cos\theta$ $y = -4.9t^2 + 98t\sin\theta$ At t=5, y=122.5 $\therefore \quad 122.5 = -4.9\times 5^2 + 98\times 5\sin\theta$ $245 = 490\sin\theta$ $\sin\theta = \frac{1}{2}$ $\theta = 30°$ 2nd ball should be projected at angle of 30° to hit first ball	2 marks: correct solution 1 mark: substituting t=5 and y=122.5 into2nd equation
iii	$x = 98t\cos\theta$ when $\quad t = 5\ \theta = 30°$ $x = 98\times 5\times\cos 30°$ $= 490\times\frac{\sqrt{3}}{2} = 245\sqrt{3}$ $\therefore$ Balls are $\quad 245\sqrt{3}\ m$ apart when projected	1 mark: correct solution

d	$1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{n}x^n$ $= (1+x)^n$ differentiate both sides wrt x $\binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + 4\binom{n}{4}x^3 + \ldots + n\binom{n}{n}x^{n-1}$ $= n(1+x)^{n-1}$ differentiate both sides wrt x $2\binom{n}{2} + 2 \times 3\binom{n}{3}x + 3 \times 4\binom{n}{3}x^2 + \ldots. + (n-1)n\binom{n}{n}x^{n-2}$ $= (n-1)n(1+x)^{n-2}$ let x=1 $\therefore 2\binom{n}{2} + 2 \times 3\binom{n}{3} + 3 \times 4\binom{n}{4} + \ldots + (n-1)n\binom{n}{n}$ $= (n-1)n \times 2^{n-2}$ let x=-1 $2\binom{n}{2} - 2 \times 3\binom{n}{3} + 3 \times 4\binom{n}{4} - \ldots + (^-1)^{n-2}(n-1)n\binom{n}{n}$ $= 0$ $(^-1)^{n-2} = 1$ since n is even $\therefore 2\binom{n}{2} - 3 \times 2\binom{n}{3} + 3 \times 4\binom{n}{4} - \ldots + (n-1)n\binom{n}{n} = 0$ adding these 2 equations $4 \times \binom{n}{2} + 8 \times 3\binom{n}{4} + \ldots + 2n(n-1)\binom{n}{n}$ $= (n-1)n \times 2^{n-2}$	4 marks: correct solution incl. explanation of $(-1)^{n-2} = 1$ for n even 3 marks: correct solution without justifying sign of last term 2 marks: differentiating twice and substituting either x=1 or x=-1 1 mark: differentiating twice