NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

2016

Mathematics Extension I

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Write your Student Number at the top of each page
- Answer Section I- Multiple Choice on Answer Sheet provided
- Answer Section II – Free Response in a separate booklet for each question.
- Board approved calculators and templates may be used.

Section I Multiple Choice

- 10 marks
- Attempt all questions
- Allow about 15 minutes for this section

Section II – Free Response

- 60 marks
- Each question is of equal value
- All necessary working should be shown in every question.
- Allow about 1 hour 45 minutes for this section

Weighting: 40%

Student Number: ____

Section 1: Multiple Choice (10 marks)

Indicate your answer on answer sheet provided.
Allow approximately 15 minutes for this section.

Q1. The binomial $(x + 2)$ is a factor of $-2 + 5x + k\,x^2 + 2\,x^3$.
What is the value of k?

A -7

B 7

C -12

D 12

Q2. Mark, Greg and four friends arrange themselves at random in a circle.
What is the probability that Mark and Greg are not together?

A $\frac{1}{120}$

B $\frac{2}{5}$

C $\frac{3}{5}$

D $\frac{119}{120}$

Q3. A projectile is fired at an angle of 35^0 above the horizontal. At the highest point in its trajectory, its speed is 200m/s. If air resistance is neglected, what is the initial horizontal component of the projectile's motion (m/s)?

A 200

B $200\cos 35^0$

C $200\sin 25^0$

D $\dfrac{200}{\cos 35°}$

Student Number: ______________________________

Q4. It is known that $x^3 + 3x = 7$ has a root between $x = 1$ and $x = 2$. If the method of halving the interval is used twice, between which two values does the root lie?

A $x = 1$ and $x = 1\cdot25$

B $x = 1\cdot25$ and $x = 1\cdot5$

C $x = 1\cdot5$ and $x = 1\cdot75$

D $x = 1\cdot75$ and $x = 2$

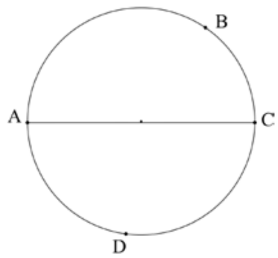
Q5. A, B, C and D are points on a circle as shown. The circle has centre O and AC is a diameter of the circle. If $\angle ABD = 75°$ and $\angle BDC = 25°$. The magnitude of $\angle BCA$ is :

A 15^0

B 25^0

C 65^0

D 75^0



Q6. The point $P(17,13)$ divides the interval joining AB, $A(-1,4)$ and $B(5,7)$ in the ratio $m:n$. The value of $\dfrac{m}{n}$ is:

A $\dfrac{2}{3}$

B $-\dfrac{2}{3}$

C $\dfrac{3}{2}$

D $-\dfrac{3}{2}$

Student Number: ______________________________

Q7. A hospital flu ward has 10 beds. In a flu season, a person has a 40% chance of contracting the flu if exposed to the virus. If 12 people are exposed to the virus, what is the probability that the ward will not be able to cater for all the patients with the flu?

A $3 \cdot 19 \times 10^{-4}$

B $5 \cdot 2 \times 10^{-4}$

C $4 \cdot 19 \times 10^{-5}$

D $5 \cdot 87 \times 10^{-5}$

Q8. The expression $5\cos x + 7\sin x$ can be written in $R\cos(x + \alpha)$ form as which of the following.

A $\sqrt{74}\cos(x + 125°32')$

B $\sqrt{74}\cos(x + 54°28')$

C $\sqrt{74}\cos(x + 144°28')$

D $\sqrt{74}\cos(x + 305°32')$

Q9. The velocity of a particle moving in a straight line is given by $v = 2x + 5$, where x metres is the distance from a fixed point O and v is in metres per second. What is the acceleration of the particle when it is 1 metre from O?

A $a = 7ms^{-2}$

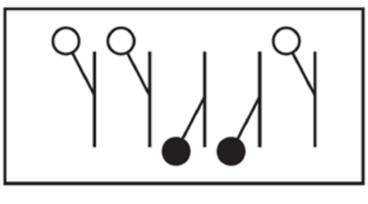
B $a = 12ms^{-2}$

C $a = 14ms^{-2}$

D $a = 24ms^{-2}$

Q10. An electrical panel has five switches. In how many ways can the switches be positioned up or down if three of the switches must be in the up position and two switches in the down position.

One possible switch arrangement.



A 10
B 24
C 48
D 120

End of Multiple Choice

Student Number: ___________________________

Section II Total Marks is 60

Attempt Questions 11 – 14.
Allow approximately 1 hour & 45 minutes for this section.

Answer all questions, starting each new question in a new booklet with your **student ID number** in the top right hand corner and the question number on the left hand side of your paper. All necessary working must be shown in each and every question.

Question 11 Start New Booklet **15 marks**

a) Find $\lim\limits_{x \to 0} \dfrac{\sin^2 2x}{2x^2}$. **2**

b) Find the term independent of x in the expansion $\left(\dfrac{x}{3} + \dfrac{3}{x^2}\right)^9$. **3**

c) Solve $\dfrac{x^2 + 3}{x} \geq 4$. **3**

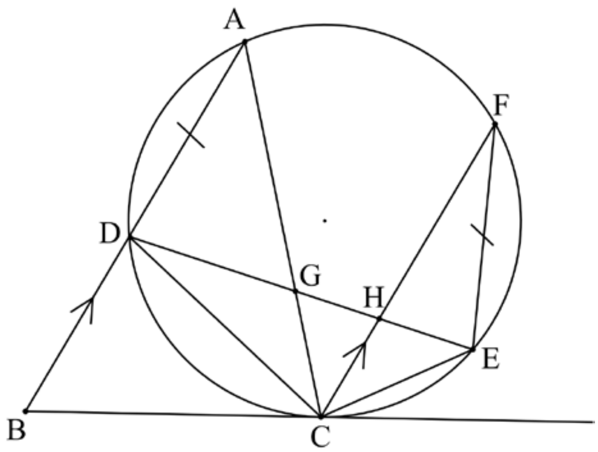
d) Evaluate $\sin^{-1}\left(\cos \dfrac{5\pi}{3}\right)$. Leave your answer in exact form. **2**

e) Eight cars (3 red, 3 blue and 2 yellow) are to be parked in line. How many arrangements are possible if the yellow cars are not to be parked together? (Assume cars of the same colour are identical.) **2**

f) In the diagram below, AB is parallel to CF, $AD = EF$ and BC is a tangent to the curve at C.

(i) Copy the diagram into your booklet. Your diagram should be a minimum of one third of a page.

(ii) Show that $\angle BDC = \angle GHC$ **3**



Student Number: ____

Question 12 Start New Booklet **15 Marks**

a) (i) Find the gradient of the tangent to

$y = 2\cos^{-1}(-x) - 2\sin^{-1}(-x)$ at $x = 0$. **2**

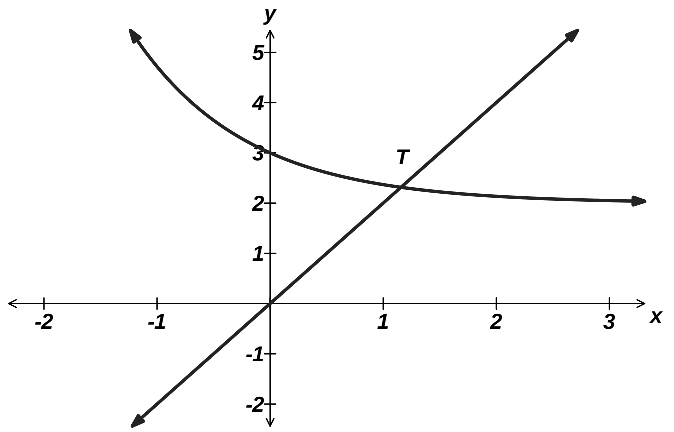
(ii) Hence or otherwise, find the acute angle between the curve and the line $y = x + \pi$ at $x = 0$. Give your answer to the nearest degree. **2**

b) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} [1 - \cos^2(2y)]\ dy$ **3**

c) Use the substitution $u = \log_e x$ to find $\displaystyle\int_e^{e^2} \frac{1}{x\left(\log_e x\right)^2} dx$ **3**

(Leave your answer in exact form)

d) The diagram shows $y = e^{-x} + 2$ and the line $y = 2x$.



The graphs intersect at the point T where $x_T \approx 1$.
Use one application of Newton's method to find x_T correct to one decimal place. **3**

e) The polynomial $Q(x) = r + qx + px^2 + x^3$ has roots -3, 3 and 0.
What are the values of p, q and r ? **2**

Student Number: ____

Question 13 – Start New Booklet **15 Marks**

a) The three numbers a, b and c are consecutive terms in an arithmetic progression. Show that the three numbers e^a , e^b , e^c are consecutive terms in a geometric progression. **2**

b) $T\,(2at, at^2)$ is a point on $x^2 = 4ay$. The tangent to the parabola at T has the equation $y = tx - at^2$.

Find, in simplest form, in terms of t:

(i) The perpendicular distance d from the focus F to the tangent T. **1**

(ii) The ratio $\frac{d}{\mathrm{FT}}$. **2**

c) Use mathematical induction to prove that $2^{n+1}(5 + 7^n)$ is divisible by 6 for all positive integers n. **3**

d) Sketch the curve $y = \frac{x^2}{x^2 - 2x + 1}$ including all asymptotes and intercepts on the coordinate axes. **3**

e) The total surface area of a tent (base and walls) is made in the shape of a cone. The *curved* surface area, S, has slant height l and semi-vertical angle α. The size of this *curved* surface area is given by $S = \pi l^2 \sin\alpha$.

The conical tent is made of elastic material which stretches when subjected to stresses such as heat. The slant height remains constant and the tent maintains its conical shape when the semi-vertical angle increases at a rate of 0.1 radians per second.

Show that the rate at which the tent's *total* surface area is changing when its semi-vertical angle $\alpha = \frac{\pi}{4}$ is $\pi l^2 \frac{(1 + \sqrt{2})}{10\sqrt{2}}$ units^2 per second **4**

Student Number: ______________________________

Question 14 Start New Booklet **15 Marks**

a) Three integers are chosen at random from the integers 1 to 20 (inclusive).

In how many ways can the numbers be chosen if the sum of the integers is even. **2**

b) The velocity v, in metres per second, of a particle moving in a straight line is given by $\frac{dx}{dt} = 10 - x$ where x metres is the displacement from a fixed point O. Initially the particle is at O.

(i) Show that the acceleration of the particle is given by $\frac{dv}{dt} = x - 10$. **1**

(ii) Express the displacement x in terms of time t. **2**

(iii) Hence, or otherwise, find the limiting position of the particle. **1**

c) When a projectile is fired with velocity $V\ ms^{-1}$ at an angle θ above the horizontal the horizontal and vertical displacements (in metres) from the point of projection at time t seconds are given by $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$ respectively (where g is the acceleration due to gravity).

The particle reaches a maximum height of H metres after T seconds.

(i) Show that when $t = \frac{1}{2}T$ the height of the particle is $\frac{3}{4}H$. **3**

(ii) Show that when $t = \frac{1}{2}T$ the particle is moving on a path inclined at an angle α to the horizontal such that $\tan\alpha = \frac{1}{2}\tan\theta$. **2**

d) A cold drink is taken from a refrigerator and placed outside where the temperature is 32^0. After 25 minutes its temperature is 14^0C , after 50 minutes outside its temperature is 20^0C. Assuming the temperature T of the drink satisfies the equation $\frac{dT}{dt} = -k(T - 32)$.

(i) Show that $T = 32 - Ae^{-kt}$ is solution to above equation . **1**

(ii) Determine the initial temperature of the drink. **3**

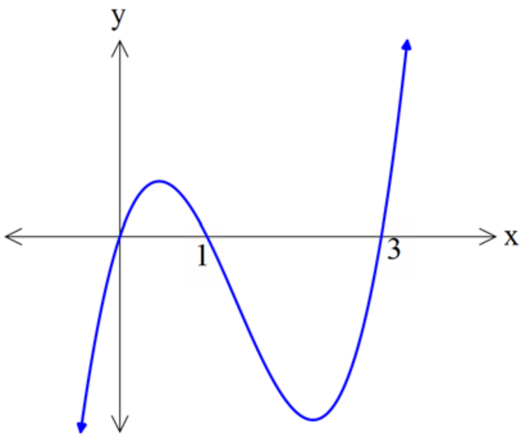
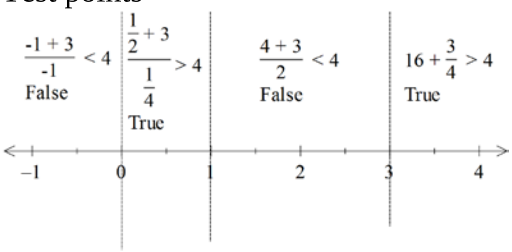
End of Examination

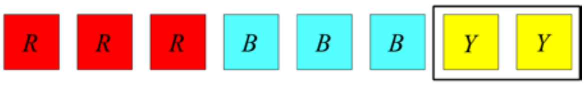
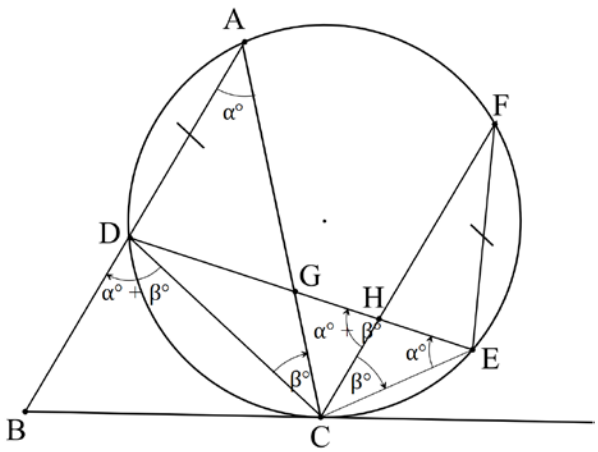
Q1	$-2+5(-2)+(-2)^2k+(2)(-2)^3=0$ $4k=28$ $k=7$	B
Q2	$\dfrac{5!-(2!4!)}{5!}=\dfrac{3}{5}$	C
Q3	at the max height: $v\sin\theta=0$ tf at max height : speed = $v\cos\theta$ (horizontal component) tf $v\sin\theta = 200\ ms^{-1}$	A
Q4	$f(x) = x^3 + 3x - 7$ $f(1) = -3 < 0$, $f(2) = 7 > 0$ $f(1.5) = 0.875 > 0$ $f(1.25) = -1.296875$ $\therefore\ 1.25 < x < 1.5$	B
Q5	$< BAC = < BDC$ (angles in same sector) $< ABC = 90°$ (angle in a semicircle) $\therefore\ < ACB = 65°$ (angle sum of triangle)	C
Q6	$\dfrac{(-1)n+5m}{m+n}=17$ $17m+17n=5m-n$ $12m=-18n$ $\dfrac{m}{n}=-\dfrac{3}{2}$	D
Q7	$(p+q)^{12} \Rightarrow p=0\cdot4 \qquad q=0\cdot6$ 11 or 12 people unhealthy ${}^{12}\mathbf{C}_{11}\,(0\cdot4)^{11}\,(0\cdot6)+{}^{12}\mathbf{C}_{12}(0\cdot4)^{12}$ $=3\cdot187\times10^{-4}$	A
Q8	$R\cos(x+\alpha)=R\cos x\cos\alpha-R\sin x\sin\alpha$ $\therefore 5\cos\ +7\sin x=5\cos x-({}^-7)\sin x$ $\therefore\ \sin\alpha={}^-7$ $\cos\alpha=5$ $\therefore \alpha$ is in Quadrant 4 $\tan\alpha=-\dfrac{7}{5}$ $\alpha=305°32'$	D

Q9	$v = (2x+5)$ $\frac{1}{2}v^2 = \frac{1}{2}(2x+5)^2$ $\frac{d\left(\frac{1}{2}v^2\right)}{dx} = \frac{1}{2} \times 2 \times 2(2x+5)$ at $x = 1$ $\ddot{x} = 14\ ms^{-2}$	C
Q10	Choose 3 places for up switches – 2 down are selected concurrently. ${}^5\mathbf{C}_3 = 10$	A

Q11

a	$\lim_{x \to 0} \frac{\sin^2 2x}{2x}$ $= \lim_{x \to 0} 2 \times \frac{\sin^2 2x}{(2x)^2}$ $= \lim_{x \to 0} 2 \times \left(\frac{\sin 2x}{2x}\right)^2$ $= 2 \times 1 = 1$	2 – correct answer showing derivation. 1 – correct answer without appropriate working. – Indication of use of $\lim_{x \to 0} \frac{\sin x}{x} = 1$
b	$\left(\frac{x}{3} + \frac{3}{x^2}\right)^9$ ${}^9\mathbf{C}_n \left(\frac{x}{3}\right)^{9-n} \left(\frac{3}{x^2}\right)^n$ Term independent of x $\left(\frac{1}{3}\right)^{9-n} 3^n x^{9-n} x^{-2n} = kx^0$ $\therefore \quad 9 - 3n = 0$ $n = 3$ Check ${}^9\mathbf{C}_6 \frac{x^6}{3^6} \times \frac{3^3}{x^6} = {}^9\mathbf{C}_6 \times \frac{1}{27} = \frac{28}{9}$	3 marks – correct solution by correct method. 2 marks – equating coefficient of general term to zero and solving for n. 1 mark - correct general term.

c	Method 1 $\frac{x^2+3}{x} \geq 4$ nb $x \neq 0$ $\frac{(x^2+3)x^2}{x} \geq 4x^2$ $x^3 - 4x^2 + 3x \geq 0$ $x(x^2 - 4x + 3) \geq 0$ $x(x-3)(x-1) \geq 0$ From graph $0 < x \leq 1$ or $x \geq 3$  Method 2 – Critical Points $x \neq 0$ $x^2 + 3 = 4x$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $x = 3$ or $x = 1$ Test points  $0 < x \leq 1$ or $x \geq 3$	3 marks – correct solution 2 marks $0 \leq x \leq 1$ or $x \geq 3$ $x < 0$ or $1 \leq x \leq 3$ 1 mark – Acknowledging - $x \neq 0$ - correct use of x^2 - deriving $x = 1$ or 3

d	$-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$ $\cos\frac{5\pi}{3} = \cos\frac{\pi}{3} = \frac{1}{2} >$ both positive $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ given above domain	2 marks – correct solution 1 mark – a correct solution but incorrect domain eg. $\frac{5\pi}{6}$
e	 Yellow cars together $\frac{7!}{3! \times 3!} = 140$ $\therefore \frac{8!}{3! \times 3! \times 2!} - \frac{7!}{3! \times 3!} = 420$	2 marks – correct answer 1 mark – either correct total or correct for yellow cars together. - Demonstrating that result is Total 1 – Total 2 =
f	 $\angle BAC = \angle CED = \alpha$ ($\angle$'s in same segment are equal) $\angle DCA = \angle ECF = \beta$ ($\angle$'s standing on equal chords are equal) $\angle BDC = \alpha + \beta$ (ext $\angle$ of triangle = sum of opp.int $\angle$'s) $\angle GHC = \alpha + \beta$ (ext $\angle$ of triangle = sum of opp.int $\angle$'s) $\therefore \angle BDC = \angle GHC$	3 marks – correct solution 2 marks – correct use of circle geo theorems demonstrating 2 pairs of equal angles leading towards solution. 1 mark - correct use of circle geo theorems demonstrating 1 pair of equal angles leading towards solution.

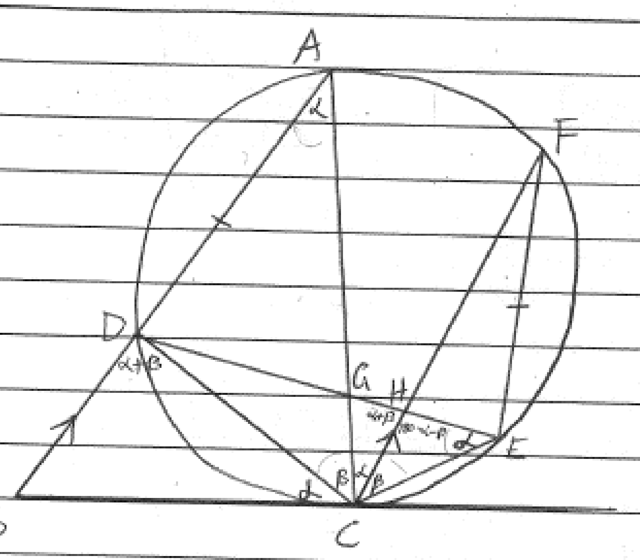
Markers Comment.

Circle Geometry is made easier if you use the diagram well – see below for a very well presented solution with clear use of diagram to explain the answer.

The reason you are asked to copy the diagram is to make use of it.

- 3 -

i-



ii- Let $\angle DCB = \alpha$, $\therefore \angle DAC = \alpha$ (~~alternate~~ $\angle$ between a tangent and a ~~line~~ chord is equal to the $\angle$ in the alternate segment)

$\therefore \angle DEC = \alpha$ ($\angle$'s subtended by the same chord on the circumference are equal)

let $\angle FCE = \beta$

$\because AD = FE$ (given) $\therefore \angle ACD = \beta$ ($\angle$'s subtended by equal chords on the circumference are equal)

$\therefore \angle EHC = 180 - \alpha - \beta$ ($\angle$ sum of a $\Delta = 180°$)

$\angle GHC = 180 - \angle EHC$ ($\angle$ sum of a line $= 180°$)

BA ∥ CF (given) $\quad = 180 - (180 - \alpha - \beta)$

$\boxed{\angle GHC = \alpha + \beta}$

$\angle FCA = \alpha$ (alternate angles on ∥ lines are equal)

$\therefore \angle BDC = \angle DCF$ (alternate angles on ∥ lines are equal)

$= \alpha + \beta \quad \therefore \boxed{\angle BDC = \alpha + \beta = \angle GHC}$

3

☐ ⇦ Tick this box if you have continued this answer in another writing booklet.

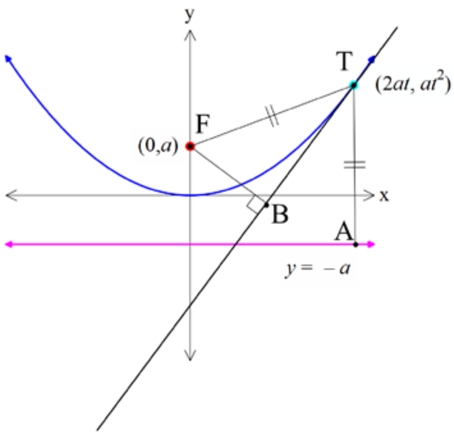
Q12

a i)	$y' = 2\left[\frac{-1}{\sqrt{1-x^2}} \times -1\right] - 2\left[\frac{1}{\sqrt{1-x^2}} \times -1\right]$ $= \frac{2}{\sqrt{1-x^2}} + \frac{2}{\sqrt{1-x^2}}$ $= \frac{4}{\sqrt{1-x^2}}$ for $x = 0$ $y' = 4$	2 marks correct solution 1 mark only one error
a ii)	$y = x + \pi$ then $m_1 = 1$ and $m_2 = 4$ $\tan\theta = \left\lvert\frac{1-4}{1+(4)(1)}\right\rvert$ $\theta = \tan^{-1}\left(\frac{3}{5}\right)$ $\theta = 30° \, 58'$	2 marks correct solution 1 mark only one error
b)	$\cos 4y = 2\cos^2 2y - 1$ $\cos^2 2y = \frac{1}{2}(1 + \cos 4y)$ $1 - \cos^2 2y = \frac{1}{2}(1 - \cos 4y)$ $\int_0^{\frac{\pi}{6}} (1 - \cos^2 \ 2y)\, dy = \int_0^{\frac{\pi}{6}} \frac{1}{2}(1 - \cos \ 4y)\, dy$ $= \frac{1}{2}\left[y - \frac{1}{4}\sin 4y\right]_0^{\frac{\pi}{6}}$ $= \frac{1}{2}\left[\left(\frac{\pi}{6} - \frac{1}{4} \times \frac{\sqrt{3}}{2}\right) - 0\right]$ $= \frac{\pi}{12} - \frac{\sqrt{3}}{16}$	3 marks correct solution 2 marks correct integration in terms of cos 4y 1 mark correct integration and evaluation of an equivalent integral

c)	let $u = \ln x$ then $\frac{du}{dx} = \frac{1}{x}$ $du = \frac{dx}{x}$ for $x = e$, $u = \ln e = 1$ for $x = e^2$, $u = \ln e^2 = 2$ $\int_e^{e^2} \frac{1}{x(\log_e x)^2}\,dx$ $= \int_1^2 \frac{du}{u^2}$ $= \left[-u^{-1} \right]_1^2$ $= -(2^{-1} - 1^{-1})$ $= -\left(\frac{1}{2} - 1\right)$ $= \frac{1}{2}$	3 marks correct solution 2 marks rewrites correct integral in terms of u 1 mark Some progress towards correct substitution
d)	pt intersection: $e^{-x} + 2 = 2x$ $\therefore 2x - e^{-x} - 2 = 0$ let $f(x) = 2x - e^{-x} - 2$ $f'(x) = 2 + e^{-x}$ using newton's method $x_1 = 1 - \left[\frac{-e^{-1}}{2 + e^{-1}} \right]$ $= 1\cdot15536...$ $= 1\cdot155$	3 marks correct solution 2 marks significant progress in using newton's method with correct equation. 1 mark recognises the need to solve $e^{-x} + 2 - 2x = 0$ and makes some progress or 1 mark correct substitutions and evaluation using newtons method.

e)	$Q(0) = r + q(0) + p(0)^2 + (0)^3 = 0$ $\therefore r = 0$ $Q(3) = 0 + q(3) + p(3)^2 + (3)^3 = 0$ now $3q + 9p + 27 = 0$ $Q(-3) = r + q(-3) + p(-3)^2 + \ = 0(-3)^3$ now $3q - 9p + 27$ solving simultaneously" p=0 3q-9(0)=27 q=3 tf r=0 , p=0 ,q=3	2 marks correct solution 1 mark only one error

Q13

a	AP therefore $T_2 - T_1 = T_3 - T_2$ $\therefore b - a = c - b$ GP therefore: $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ $\frac{e^b}{e^a} = \frac{e^c}{e^b}$ $e^{b-a} = e^{c-b}$ True if $b - a = c - b$ which was given as true initially	2 marks – correct solution from correct process. 1 mark – AP test
b-i	 $Dist = \frac{\|ax + by + c\|}{\sqrt{a^2 + b^2}}$ $= \frac{\|tx - y - at^2\|}{\sqrt{a^2 + b^2}}$ $= \frac{\|-a(1 + t^2)\|}{\sqrt{t^2 + (^-1)^2}}$ $= a\sqrt{1 + t^2}$	

b-ii	$\text{FT} = \text{AT} = a + at^2$ $\therefore \dfrac{d}{\text{FT}} = \dfrac{a\left(\sqrt{1+t^2}\right)}{a(1+t^2)}$ $= \dfrac{1}{\sqrt{1+t^2}}$	2 marks – fully simplified solution. 1 mark – correct FT
c	Prove $2^{n+1}(5+7^n) = 6Q \quad n > 0$ where Q is an integer. Let $n = 1$ $\text{LHS} = 2^2(5+7)$ $= 48 = 6 \times 8$ $\therefore$ true for $n = 1$ Assume true for $n = k$ $2^{k+1}(5+7^k) = 6M$ $\therefore \quad 5 \times 2^{k+1} = 6M - 7^k(2^{k+1})$ RTP true for $n = k+1$ $\text{LHS} = 2^{k+2}(5+7^{k+1})$ $= 2 \times 2^{k+1}(5 + 7 \times 7^k)$ $= 2 \times 5 \times 2^{k+1} + 2 \times 2^{k+1}(7 \times 7^k)$ $= 2 \times \left[6M - 7^k(2^{k+1})\right] + 2 \times 2^{k+1}(7 \times 7^k)$ $= 12M - 2 \times 7^k(2^{k+1}) + 14 \times 2^{k+1}(7^k)$ $= 12M + 12 \times 7^k(2^{k+1})$ $= 6 \times 2\left(M + 7^k(2^{k+1})\right)$ $= 6Q$ where Q is an integer as k and M are integers	

<table>
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$$y = \frac{x^2}{x^2 - 2x + 1}$$
$$= \frac{x^2}{(x-1)^2}$$

$\therefore\ y \geq 0$ for all x

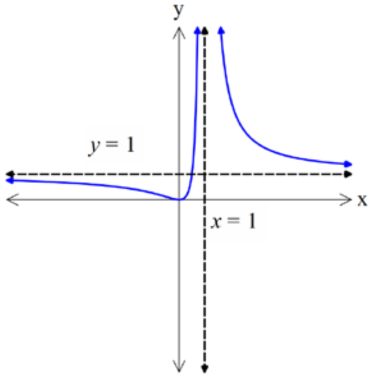
Intercepts $x = 0 \Rightarrow y = 0$
from above this is minimum value of function

Vert asymptote $x = 1$

Horizonatl Asymptote

$$\lim_{x \to \pm\infty} \frac{x^2}{x^2 - 2x + 1}$$

$$\lim_{x \to \pm\infty} \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}} = 1$$



</td>
<td>3 marks – correct shape, asymptotes and intercept

2 marks -

1 mark -</td>
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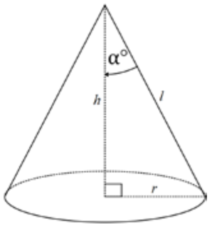
d	$\text{SA} = \pi r^2 + \pi l^2 \sin\alpha$  $\sin\alpha = \dfrac{r}{l}$ $r = l\sin\alpha$ $\text{SA} = \pi l^2 \sin^2\alpha + \pi l^2 \sin\alpha$ $= \pi l^2(\sin^2\alpha + \sin\alpha)$ $\dfrac{d(\text{SA})}{d\alpha} = \pi l^2(2\cos\alpha\sin\alpha + \cos\alpha)$ $\dfrac{d\alpha}{dt} = 0\cdot 1$ $\dfrac{d(\text{SA})}{dt} = \dfrac{d(\text{SA})}{d\alpha} \times \dfrac{d\alpha}{dt}$ $= \pi l^2(2\cos\alpha\sin\alpha + \cos\alpha) \times 0\cdot 1$ at $\alpha = \dfrac{\pi}{4}$ $\dfrac{d(\text{SA})}{dt} = \pi l^2\left(2 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\right) \times \dfrac{1}{10}$ $= \pi^2\left(1 + \dfrac{1}{\sqrt{2}}\right) \times \dfrac{1}{10}$ $= \pi l^2\left(\dfrac{\sqrt{2}+1}{\sqrt{2}}\right) \times \dfrac{1}{10}$ $= \pi l^2 \dfrac{(1+\sqrt{2})}{10\sqrt{2}}$ ... *QED*	4 marks – correct solution 3 marks – correct expression for $\dfrac{d(\text{SA})}{dt}$ 2 marks – correct $\dfrac{d(\text{SA})}{d\alpha}$ 1 mark – correct expression for SA

2016 MSC Trial Mathematics Extension 1 Solutions

Q14

a	No. ways for 3 even or 2 odd and 1 even: $^{10}C_3 + {}^{10}C_2 \,.\, {}^{10}C_1$ $= 120 + 450$ $= 570$	2 marks – correct answer 1 mark – use of 3 even or 2 odd and 1 even OR either 120 or 450
b i)	$\frac{dv}{dt} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dx}\left(\frac{1}{2}(1-x)^2\right)$ $= -(10-x)$ $= x - 10$	1 mark – correct solution from correct working
b ii)	$\frac{dx}{dt} = 10 - x$ $\therefore \quad \frac{dt}{dx} = \frac{1}{10-x}$ $\therefore \quad t = \int \frac{dx}{10-x}$ $t = -\ln(10-x) + c$ $10 - x = e^{-t-c}$ when $t = 0, x = 0$ $\therefore \quad 10 = e^{-c}$ $x = 10 - 10e^{-t}$	2 marks – correct expression with x the subject 1 mark – correct integration resulting in $t = -\ln(10-x)$
b iii)	as $t \to \infty, x \to 10 - 0$ $\therefore$ limiting position is 10	1 mark – correct limiting position from correct working

c i)	Maximum height occurs when $\dot{y} = 0$ $V\sin\theta - gt = 0$ $T = \dfrac{V\sin\theta}{g}$ $y_{\max} = H = V\left(\dfrac{V\sin\theta}{g}\right)\sin\theta - \dfrac{1}{2}g\left(\dfrac{V\sin\theta}{g}\right)^2$ $= \dfrac{V^2\sin^2\theta}{g} - \dfrac{V^2\sin^2\theta}{2g}$ $H = \dfrac{V^2\sin^2\theta}{2g}$ when $t = \dfrac{1}{2}T = \dfrac{V\sin\theta}{2g}$ $y = V\left(\dfrac{V\sin\theta}{2g}\right)\sin\theta - \dfrac{1}{2}g\left(\dfrac{V\sin\theta}{2g}\right)^2$ $= \dfrac{V^2\sin^2\theta}{2g} - \dfrac{V^2\sin^2\theta}{8g}$ $= \dfrac{3V^2\sin^2\theta}{8g}$ $= \dfrac{3}{4}\left(\dfrac{V^2\sin^2\theta}{2g}\right)$ $= \dfrac{3}{4}H$	2 marks - correctly finds the maximum height of the particle in terms of θ. 1 mark - correctly shows result.
c ii)	When $t = \dfrac{1}{2}T = \dfrac{V\sin\theta}{2g}$ $\dot{x} = V\cos\theta$ $\dot{y} = V\sin\theta - g\left(\dfrac{V\sin\theta}{2g}\right)$ $= \dfrac{V\sin\theta}{2}$ horizontal component of velocity $= V\cos\theta$ vertical component of velocity $= \dfrac{V\sin\theta}{2}$ $\tan\alpha = \dfrac{\frac{V\sin\theta}{2}}{V\cos\theta}$ $= \dfrac{1}{2}\tan\theta$	3 marks – correctly shows result. 2 marks - finds horizontal and vertical component of velocity and makes some attempt to find $\tan\alpha$. 1 mark - finds horizontal or vertical component of velocity.

d i)	$T = 32 - Ae^{-kt} \rightarrow Ae^{-kt} = 32 - T$ $\frac{dT}{dt} = kAe^{-kt}$ $= k(32 - T)$ $= -k(T - 32)$	1 mark – correct solution
d ii)	$14 = 32 - Ae^{-25k} \rightarrow Ae^{-25k} = 18 \quad - 1$ $20 = 32 - Ae^{-50k} \rightarrow Ae^{-50k} = 12 \quad - 2$ $1 \div 2 \quad e^{25k} = \frac{3}{2}$ $k = \frac{\ln(1.5)}{25}$ in $1 \quad 18 = Ae^{-\ln(1.5)}$ $\therefore \quad 18 = A \cdot \frac{2}{3}$ ie A = 27 $\therefore \quad T = 32 - 27e^{-kt}$ $t = 0, \ T = 32 - 27$ $= 5$	3 marks – correct initial temperature from correct working 2 marks – correct value for *A* or *k* 1 mark – attempt to solve 2 correct simultaneous equations