NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

Year 12

Trial Examination

2018

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Write your name on the front of every booklet.
- In Questions 11 to 14 show relevant mathematical reasoning and/or calculations.
- NESA approved calculators and templates may be used.
- Weighting: 40%

Section I Multiple Choice

- 10 marks
- Attempt all questions.
- Answer Sheet provided
- Allow about 15 minutes for this section

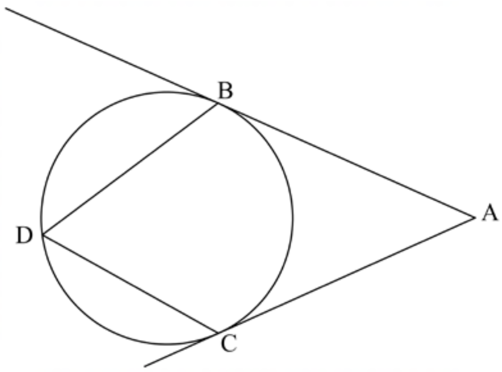
Section II Free Response

- 60 marks
- Start a separate booklet for each question.
- Each question is of equal value.
- All necessary working should be shown in every question.
- Allow about 1 hour and 45 minutes for this section.

Section I
10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use a multiple-choice answer sheet for Questions 1 – 10.

Q1. The diagram below shows a circle with tangents *AB* and *AC*. *D* is a point on the circle such that $\angle BDC = 50°$



Which of the following is true?

A. $\angle BAC = 40°$

B. $\angle BAC = 50°$

C. $\angle BAC = 65°$

D. $\angle BAC = 80°$

Q2. The points $P\left(-\frac{2}{3}, -\frac{4}{3}\right)$, $A\ (1,2)$ and B (6, 12) are collinear. Given $\mathrm{AP} = \frac{1}{k}\mathrm{AB}$, which of the following is the value of k ?

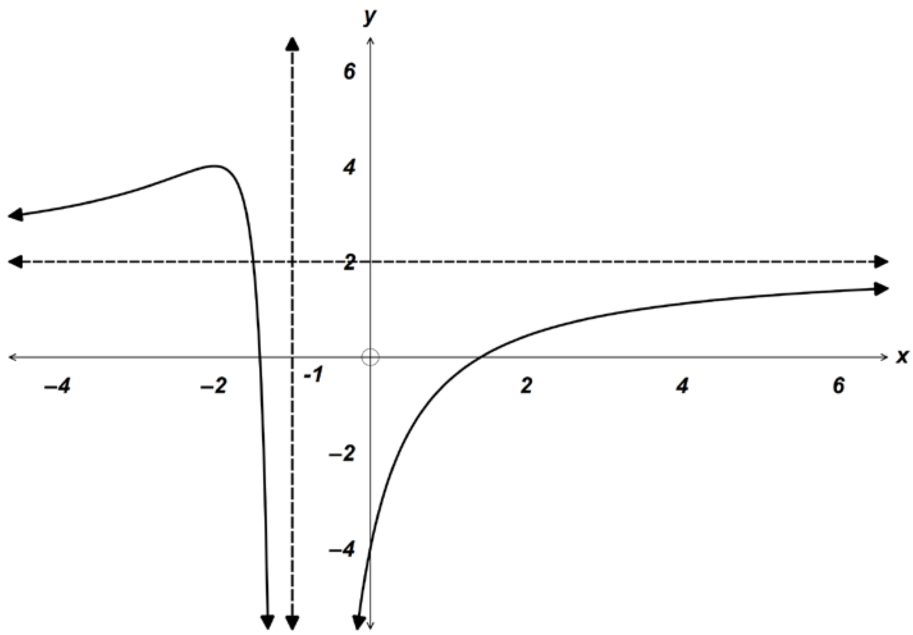
A. 3

B. 4

C. – 3

D. – 4

Q3. The graph shown below has equation of the form $y = \dfrac{ax^2 - b}{(x + c)^2}$



The values of a, b and c are?

A. $a = 2, b = 4, c = -1$

B. $a = 2, b = 4, c = 1$

C. $a = \dfrac{1}{2}, b = 4, c = 1$

D. $a = 2, b = -4, c = 1$

Q4. A family of 6 adults and 3 children are randomly sitting around a circular table. What is the number of possible seating arrangements if none of the children sit together?

A. 720

B. 2400

C. 14 400

D. 86 400

Q5. The roots of the polynomial $P(x) = 2x^3 - 4x + 1$ are α, β and γ.
What is the value of $\alpha\beta(\alpha + \beta)$?

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 1

D. 2

Q6. Which integral is the result of using the substitution $u = x^2 + 4$ in the integral $\int \frac{x}{(x^2 + 4)^2}dx$?

A. $\int \frac{1}{u^2}du$

B. $\frac{1}{2}\int \frac{1}{u}du$

C. $2\int \frac{1}{u^2}du$

D. $\frac{1}{2}\int \frac{1}{u^2}du$

Q7. Find the coefficient of x^2 in the expansion of $\left(5x + \frac{2}{x}\right)^8$

A. 112 000

B. 1 400 000

C. 700 000

D. 224 000

Q8. A crate of toy cars contains 10 working cars and 4 defective cars. How many ways can 5 cars be selected if only 3 work?

A. 6 B. 56 C. 720 D. 3003

Q9. The derivative of $y = 2\sin^{-1}\left(\frac{x}{2}\right)$ is:

A. $\frac{1}{\sqrt{1-x^2}}$

B. $\frac{2}{\sqrt{1-x^2}}$

C. $\frac{2}{\sqrt{4-x^2}}$

D. $\frac{8}{\sqrt{4-x^2}}$

Q10. A particle is moving along the x – axis, its velocity v at position x is given by $v = \sqrt{8x - x^2}$ m/sec.

What is the acceleration of the particle when $x = 2$ metres?

A. $\frac{1}{\sqrt{3}}$ m/sec^2

B. 1 m/sec^2

C. $\frac{1}{\sqrt{2}}$ m/sec^2

D. 2 m/sec^2

END OF MULTIPLE CHOICE

Section II

60 Marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 – Start new booklet (15 marks)

a) Find the remainder when $P(x) = (3x+1)^3 - 5$ is divided by $(x-1)$. **1**

b) The polynomial equation $2x^3 - 21x^2 + 63x - 54 = 0$ has roots $x = \alpha, x = 2\alpha, x = 4\alpha$. Find the value of α **1**

c) Solve the inequality $\dfrac{x}{x-3} \geq 2$. **2**

d) Show $\lim\limits_{x \to 0} \dfrac{3\sin 2x}{x \cos 3x} = 6$ **2**

e) The point *P* (1, 4) divides the line segment joining *A* (-1, 8) and *B* (*x*, *y*) internally in the ratio 2 : 3. Find the coordinates of the point *B*. **2**

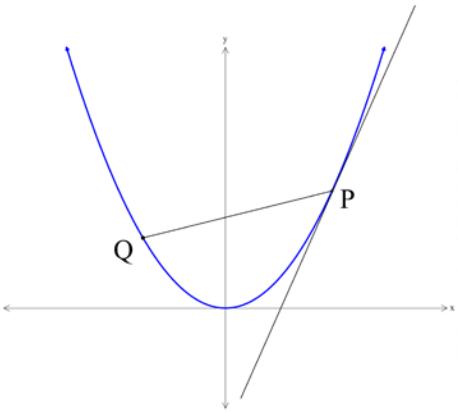
f) By making the substitution $t = \tan\dfrac{\theta}{2}$, show that

$$\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}$$ **2**

Question 11 continues on the next page

Question 11 (continued)

g) The diagram below shows the variable points $P(2ap\,,\,ap^2)$ and $Q\,(2aq\,,\,aq^2)$ on the parabola $x^2 = 4ay$. M is the midpoint of the chord PQ. P and Q move such that the gradient of the tangent at P is four times the gradient of the chord PQ



(i) Show that $q = -\frac{1}{2}p$. **2**

(ii) Show that, as P varies, M moves on a parabola. **3**

End of Question 11

Question 12 – Start new booklet (15 marks)

a) The function $f(x)=\cos(2x)-x$ has a zero near $x=\dfrac{1}{2}$.

Use one application of Newton's method to obtain another approximation to this zero. Give your answer correct to two decimal places. **2**

b) Prove by mathematical induction that $\cos(x+n\pi)=(-1)^n\cos x$ for all positive integers. **3**

c) A tea enthusiast believes that black tea should be prepared with boiling water and consumed at 70°C. She wants to model the temperature of her tea with respect to time. She pours hot water into her favourite mug. The initial temperature of the water is 100°C and the ambient temperature is 25°C. After 5 minutes, the temperature of the water has reached 85°C.

The temperature T of the water after t minutes can be modelled by the equation

$$T=25+Ae^{kt}.$$

(i) Find the value of A. **1**

(ii) Find the value of k, leaving your answer in exact form. **1**

(iii) How long will it take for the temperature to reach 70°C? Give your answer to the nearest minute. **1**

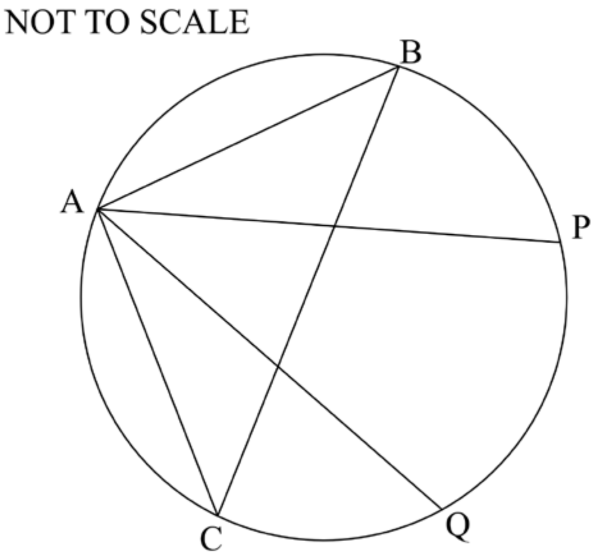
d) The term independent of x in the expansion $x^2\left(2x^2+\dfrac{a}{x}\right)^5$ is 810. **3**

Find the value of a given that $a>0$.

Question 12 continues on the next page

Question 12 (continued)

e) In the diagram below A,B,C,Q and P are concylic. AP meets BC at X and AQ meets BC at Y. $AB=AC$. Let $\angle BAP = \alpha$ and $\angle ABC = \beta$



Copy or trace the diagram into your writing booklet.

(i) Show that $\angle BQP = \alpha$. **1**

(ii) Prove that PQYX is a cyclic quadrilateral. **3**

End of Question 12

Question 13 – Start new booklet (15 marks)

a) Determine the general solution for $\sin 5x = \cos 2x$. **2**

b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \, dx$ using the substitution $u = \cos x$. **3**

c) (i) Show $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$. **1**

(ii) Hence, or otherwise sketch $y = \sin^{-1}(3x) + 3\cos^{-1}(3x)$. **3**

d) A particle is moving along the x – axis. Initially the particle is 1 metre to the right of the origin, travelling at a velocity of 3 metres per second and its acceleration is given by $\ddot{x} = 2x^3 + 4x$, where x is the displacement of the particle at time t.

(i) Show that $\dot{x} = x^2 + 2$. **3**

(ii) Hence, show that $x = \sqrt{2}\tan\left(\sqrt{2}\,t + \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$. **3**

End of Question 13

Question 14 – Start new booklet (15 marks)

a) Consider the function $f(x) = \dfrac{x^4 + 3x^2}{x^4 + 3}$

(i) Show that *f (x)* is an even function. **1**

(ii) What is the equation of the horizontal asymptote to the graph $y = f(x)$? **1**

(iii) Find the *x* – coordinates of all stationary points for the graph $y = f(x)$. **3**

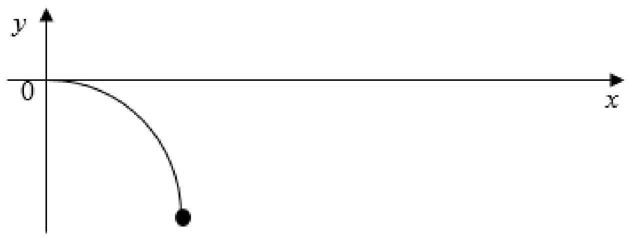
(iv) Sketch the graph $y = f(x)$. You are not required to find any points of inflexion. **2**

b) (i) Use the binomial theorem to obtain an expansion for **1**

$$(1 + x)^{2n} + (1 - x)^{2n} .$$

(ii) Hence evaluate $1 + {}^{20}\mathbf{C}_2 + {}^{20}\mathbf{C}_4 + ... + {}^{20}\mathbf{C}_{20}$. **1**

c) A supply chain management service is testing a new delivery system of goods by unmanned aerial vehicle (UAV). In one particular test, the operator is flying the UAV at a height of 220 metres above the ground with a horizontal velocity of 40 m/s when it releases its parcel.



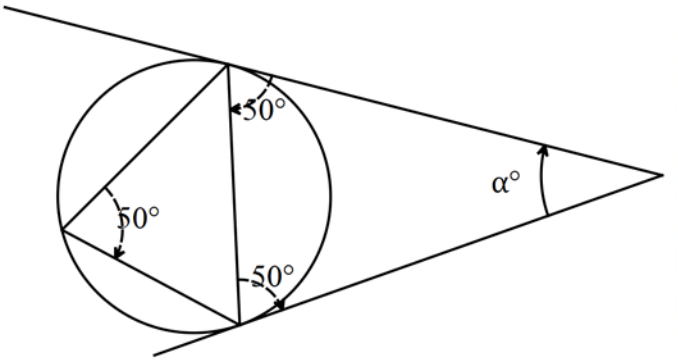
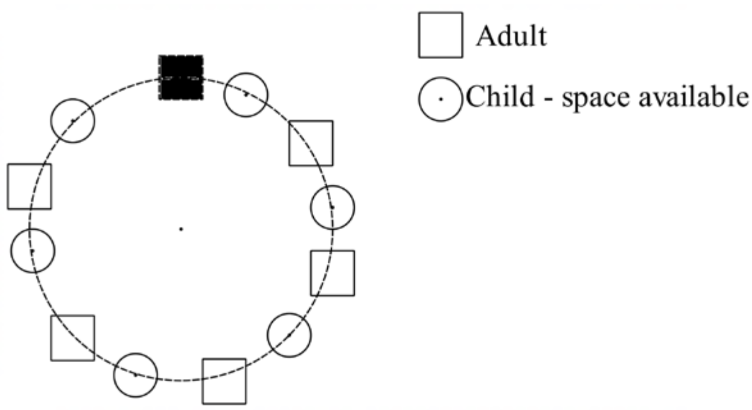
(i) If a parachute activates when the parcel reaches a speed of 50 m/s, find the horizontal distance the parcel has travelled when the parachute activates. (Use *g* = 10 m/s). **3**

(ii) Once the parachute activates, its horizontal velocity reduces to 5 m/s and its vertical acceleration changes to -2 m/s.

Find the total horizontal distance travelled from the time the UAV releases the parcel to the time it hits the ground. **3**

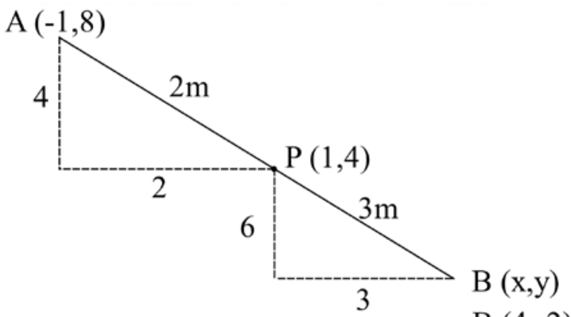
End of Paper

MC

1	 $\angle BAC = 180 - 2 \times 50 = 80°$	D
2	$\text{AP} = \sqrt{\left(1 + \frac{2}{3}\right)^2 + \left(2 + \frac{4}{3}\right)^2} = \frac{5\sqrt{5}}{3}$ $\text{AB} = \sqrt{25 + 100} = 5\sqrt{5}$ $\therefore \frac{5\sqrt{5}}{3} = \frac{1}{k} \times 5\sqrt{5}$ $k = 3$	A
3	$\lim_{x \to \infty} = a \therefore a = 2$ $x + c \neq 0$ $\therefore$ for vertica asymptote $-1 + c = 0$ $c = 1$ $x = 0$ $-4 = -\frac{b}{1}$ $b = 4$	B
4	 Therefore 5! Ways of arranging adults after fixing one, then children must pick 3 from 6 available places. $5! \times {}^6\mathbf{P}_3 = 14400$	C

5	$P(x) = ax^3 + bx^2 + cx + d$ $\alpha + \beta + \gamma = -\frac{b}{a} = \frac{0}{2} = 0$ $\therefore \quad \gamma = -(\alpha + \beta)$ ① $\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{2}$ ② sub ① into ② $\alpha\beta[-(\alpha + \beta)] = -\frac{1}{2}$ $\therefore \quad \alpha\beta(\alpha + \beta) = \frac{1}{2}$	A
6	$\frac{du}{dx} = 2x$ $\frac{du}{2} = xdx$ $\int \frac{x}{(x^2+4)^2}dx = \int \frac{1}{2} \times \frac{1}{u^2}du = \frac{1}{2}\int \frac{1}{u^2}$ du	D
7	General term ${}^8\mathbf{C}_k\,(5x)^{8-k}\left(\frac{2}{x}\right)^k$ $2k - 8 = 2$ $k = 3$ Coefficient: ${}^8\mathbf{C}_3 \times 5^5 \times 2^3 = 1\ 400\ 000$	B
8	${}^{10}\mathbf{C}_3 \times {}^4\mathbf{C}_2 = 720$	C
9	$\frac{dy}{dx} = 2 \times \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \times \frac{1}{2} = \frac{1}{\sqrt{\frac{4-x^2}{4}}}$ $= \frac{2}{\sqrt{4-x^2}}$	C
10	$\ddot{x} = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ $\frac{1}{2}v^2 = \frac{8x - x^2}{2}$ $\therefore \quad \ddot{x} = 4 - x$ at $x = 2$ $\ddot{x} = 2m\ \sec^{-1}$	D

Question 11

a	$P(1) = (3 \times 1 + 1)^3 - 5 = 64 - 5 = 59$	1mark – correct answer
b	$\alpha\beta\gamma = -\frac{d}{a}$ $8\alpha^3 = 27$ $\alpha = \sqrt[3]{\frac{27}{8}}$ $= \frac{3}{2}$	I mark – correct answer
c	$x \neq 3$ $x(x-3) \geq 2(x-3)^2$ $2(x-3)^2 - x(x-3) \leq 0$ $(x-3)[2(x-3)-x] \leq 0$ $(x-3)(x-6) \leq 0$ $3 \leq x \leq 6$ nb. restriction $\therefore$ $3 < x \leq 6$	2 marks – correct solution 1 mark – correct values for significant points
d	$\lim_{x \to 0} \frac{3\sin 2x}{x\cos 3x}$ $= \lim_{x \to 0} \frac{2 \times 3\sin 2x}{2x} \times \frac{1}{\cos 3x}$ $= 2 \times 3 \times 1 \times 1$ $= 6$	2 marks – correct demonstration 1 mark – lim of one term only demontrated
e	 B (4,-2)	2 marks – correct solution 1 mark – either *x* or *y* value correct

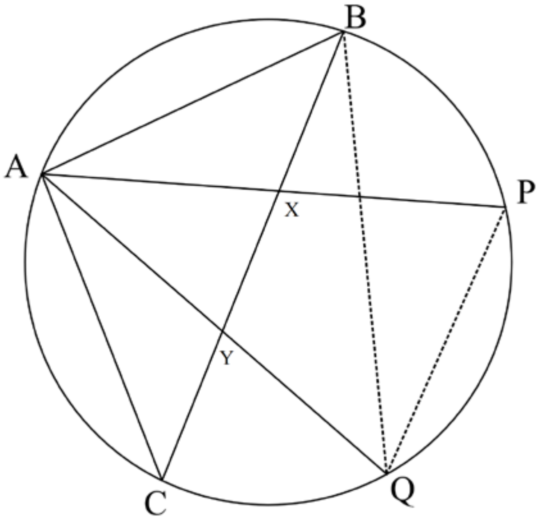
f	$t = \tan\frac{\theta}{2}$ $\therefore \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1+t^2}{2t}$ $\cot\theta = \frac{1}{\tan\theta} = \frac{1-t^2}{2t}$ $\frac{1+t^2}{2t} + \frac{1-t^2}{2t} = \frac{2}{2t}$ $= \frac{1}{t}$ $= \frac{1}{\tan\theta}$ $= \cot\theta$	2 correct demonstration using *t* expression . 1 mark – one correct *t* expression for either sin or cot
g-i	$m_{\text{PQ}} = \frac{a(p^2-q^2)}{2a(p-q)}$ $= \frac{(p-q)(p+q)}{2(p-q)}$ $= \frac{p+q}{2}$ $\qquad p \neq q$ $\frac{dy}{dx} = \frac{2x}{4a}$ $m_{\text{tangent}} = \frac{2 \times 2ap}{4a} = p$ $\therefore \quad p = 4 \times \frac{p+q}{2}$ $\frac{p}{2} = p + q$ $q = \frac{p}{2} - p = -\frac{1}{2}p$	2 marks – correct solution fully demonstrated 1 mark – derivation of both gradients.

g-ii	$M = \left(\dfrac{2a(p+q)}{2}, \dfrac{a(p^2+q^2)}{2}\right)$ $x = a(p+q)$ $\quad$ $y = \dfrac{a}{2}\left(p^2 + \left(\dfrac{-p}{2}\right)^2\right)$ $\dfrac{x}{a} = p - \dfrac{p}{2}$ $\quad$ $y = \dfrac{a}{2} \times \dfrac{5p^2}{4}$ $\dfrac{x}{a} = \dfrac{p}{2}$ $\quad$ $y = \dfrac{a}{2} \times \dfrac{5}{4} \times \dfrac{4x^2}{a^2}$ $\dfrac{2x}{a} = p$ $\quad$ $y = \dfrac{5x^2}{2a}$ $\dfrac{4x^2}{a^2} = p^2$ $\therefore$ M moves on parabola $y = \dfrac{5x^2}{2a}$	3 marks – correct solution 2 marks - Correct in p in terms of x 1 mark - Correct midpoint

Question 12

a	$f(x) = \cos 2x - x$ $f'(x) = -2\sin 2x - 1$ $f(x_0) = f\left(\dfrac{1}{2}\right) = \cos 1 - \dfrac{1}{2} = 0.0403...$ $f'(x_0) = f'\left(\dfrac{1}{2}\right) = -2\sin 1 - 1 = -2.6829...$ $x_1 = x_0 - \dfrac{f x_0}{f'(x_0)}$ $= \dfrac{1}{2} - \dfrac{0.0403...}{-2.6829...}$ $= 0.515$ $= 0.52$	2 marks: Correct solution 1 mark: Correct expression for $\dfrac{f\left(\frac{1}{2}\right)}{f'\left(\frac{1}{2}\right)}$

b	$$\cos(x + n\pi) = (-1)^n \cos x$$ $S(1)$: LHS $= \cos(x + \pi)$ $= \cos x \cos\pi - \sin x \sin\pi$ $= \cos x \times -1 - \sin x \times 0$ $= -\cos x$ RHS $= (-1)^1 \cos x$ $= -\cos x$ $=$ LHS $\therefore$ $S(1)$ true $S(k)$: $\cos(x + k\pi) = (-1)^k \cos x$ $S(k + 1)$: $\cos[x + (k + 1)\pi] = (-1)^{k+1} \cos x$ LHS $= \cos[x + k\pi + \pi]$ $= \cos[(x + k\pi) + \pi]$ $= \cos(x + k\pi)\cos\pi - \sin(x + k\pi)\sin\pi$ $= -\cos(x + k\pi)$ since $\cos\pi = -1$ and $\sin\pi = 0$ $= (-1) \times (-1)^k \cos x$ by $S(k)$ $= (-1)^{k+1} \cos x$ $\therefore$ $S(k) \Rightarrow S(k + 1)$ $\therefore$ result is proved by method of mathematical induction	3 marks: correct solution 2 marks: significant progress 1 mark: some relevant progress
c(i)	$t = 0, T = 100$ $T = 25 + Ae^{kt}$ $100 = 25 + Ae^0$ $A = 75$	1 mark: correct answer
c(ii)	$t = 5, T = 85$ $85 = 25 + 75e^{5k}$ $\frac{60}{75} = e^{5k}$ $5k = \log_e\left(\frac{4}{5}\right)$ $k = \frac{\log_e\left(\frac{4}{5}\right)}{5}$ $= -0.0446...$	1 mark: correct answer

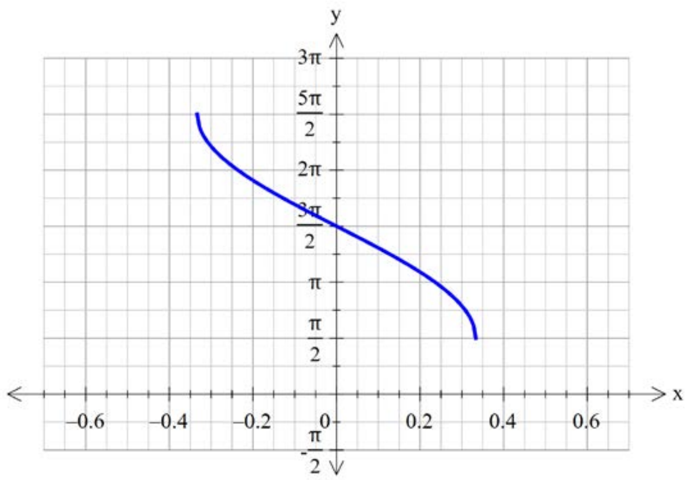
c(iii)	$T = 25 + 75e^{(-0.0446...)t}$ $\frac{70-25}{75} = e^{(-0.0446...)t}$ $0.6 = e^{(-0.0446...)t}$ $(-0.0446...)t = \log_e 0.6$ $t = \frac{\log_e 0.6}{-0.0446...}$ $t = 11.446...\text{ min}$ $= 11\text{ minutes}$	1 mark: correct answer
d	$x^2\left\{2x^2 + \frac{a}{x}\right\}^5$ $T_{k+1} = {}^5\mathbf{C}_k(2x^2)^{5-k}\left(\frac{a}{x}\right)^k$ $= {}^5\mathbf{C}_k \times 2^{5-k} \times a^k \times x^{10-2k} \times x^{-k}$ $= {}^5\mathbf{C}_k \times 2^{5-k} \times a^k \times x^{10-3k}$ $x^2T_{k+1} = {}^5\mathbf{C}_k \times 2^{5-k} \times a^k \times x^{12-3k}$ $12 - 3k = 0 \Rightarrow k = 4$ $810 = {}^5\mathbf{C}_k \times 2^1 \times a^4$ $810 = 10a^4$ $81 = a^4$ $a = 3 \text{ as } a > 0$	3 marks: correct solution 2 marks: correct evaluation 1 mark: some relevant progress
e-i	$\angle BQP = \angle BAP = \alpha$ (equal angles on arc BP)	1 mark: correct response
e-ii	NOT TO SCALE 	3 marks: correct solution with sufficient correct reasons 2 marks: identifies 2 of at least 3 relevant facts 1 mark: identifies 1 of at least 3 relevant facts

	$AB = BC$ (given) ΔABC is isosceles $\angle ACB = \angle ABC$ (equal base angles, isosceles ΔABC) $\angle ACB = \beta$ $\angle AQB = \beta$ (equal angles on arc AB) $\angle AXB = 180 - (\alpha + \beta)$ ($\angle$ sum ΔAXB) $\angle PXY = 180 - (\alpha + \beta)$ (*vert* opp $\angle$'s) $\angle PXY + \angle PQY = 180 - (\alpha + \beta) + (\alpha + \beta)$ $= 180$ $\therefore$ $PQYX$ is cyclic (opposite angles supplementary) or $\angle PXB = \alpha + \beta$ (ext. $\angle$ theorem ΔABX) $= \angle PQY$ $PQYX$ is cyclic (ext. $\angle$ equal opp "int." $\angle$)	

Question 13

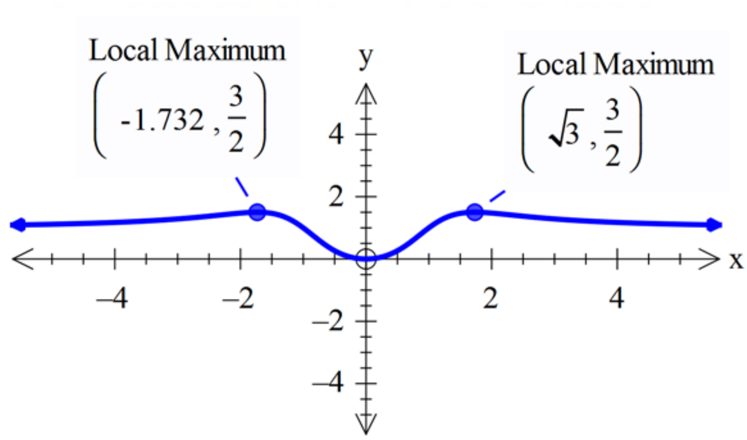
a)	$\sin(5x) = \cos(2x)$ $\cos\left(\frac{\pi}{2} - 5x\right) = \cos(2x)$ $\therefore$ $2x = 2n\pi \pm \cos^{-1}\cos\left(\frac{\pi}{2} - 5x\right)$ $2x = 2n\pi \pm \left(\frac{\pi}{2} - 5x\right)$ $2x = 2n\pi + \frac{\pi}{2} - 5x$ OR $2x = 2n\pi - \frac{\pi}{2} + 5x$ $7x = 2n\pi + \frac{\pi}{2}$ $\quad$ $3x = \frac{\pi}{2} - 2n\pi$ $x = \frac{2n\pi}{7} + \frac{\pi}{14}$ $\quad$ $x = \frac{\pi}{6} - \frac{2n\pi}{3}$ Using $\frac{\pi}{2} - 5x = 2n\pi \pm \cos^{-1}\cos 2x$ resulted in $x = \frac{\pi}{14} - \frac{2n\pi}{7}$ OR $x = \frac{\pi}{6} - \frac{2n\pi}{3}$ which is equivalent to above Changing the equation to sin on both sides also produced equivalent expressions	2 marks for correct solution 1 mark for complementary angle

b)	$u = \cos x$ $\frac{du}{dx} = -\sin x$ $-du = \sin x\, dx$ when $x = \frac{\pi}{2}, u = \cos\left(\frac{\pi}{2}\right) = 0$ when $x = 0, u = \cos 0 = 1$ $\sin^2 x = 1 - \cos^2 x = 1 - u^2$ $\int_0^{\frac{\pi}{2}} \sin^3 x\, dx = \int_0^{\frac{\pi}{2}} \sin x \times \sin^2 x\, dx$ $= \int_1^0 -(1 - u^2)\, \mathrm{du}$ $= \int_1^0 u^2 - 1\, \mathrm{du}$ $= \left[\frac{u^3}{3} - u\right]_1^0$ $= -\left(\frac{1}{3} - 1\right) = \frac{2}{3}$	3 marks for correct solution 1 mark for differentiating cosx and change in bounds 1 mark for substitution 1 mark for correct integration
c)(i)	let $y = \sin^{-1} x$ $x = \sin y$ $x = \cos\left(\frac{\pi}{2} - y\right)$ $\cos^{-1} x = \frac{\pi}{2} - y$ $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$ $\therefore \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$	1 mark for correct solution

c)(ii)	$\sin^{-1}(3x) + \cos^{-1}(3x) = \frac{\pi}{2}$ $\sin^{-1}(3x) = \frac{\pi}{2} - \cos^{-1}(3x)$ $y = \sin^{-1}(3x) + 3\cos^{-1}(3x)$ $= \frac{\pi}{2} - \cos^{-1}(3x) + 3\cos^{-1}(3x)$ $= \frac{\pi}{2} + 2\cos^{-1}(3x)$ Domain $-1 \leq 3x \leq 1 \Rightarrow -\frac{1}{3} \leq x \leq \frac{1}{3}$ Range $0 \leq \frac{y - \frac{\pi}{2}}{2} \leq \pi \Rightarrow \frac{\pi}{2} \leq y \leq \frac{5\pi}{2}$ 	3 marks for correct solution (need to show domain, range, intercept and have correct shape) 1 mark for equation 1 mark for domain 1 mark for range
d)(i)	$\dot{x} = x^2 + 2$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x^3 + 4x$ $\frac{1}{2}v^2 = \frac{2x^4}{4} + \frac{4x^2}{2} + c$ $= \frac{x^4}{2} + 2x^2 + c$ at $t = 0, x = 1, v = 3$ $\frac{9}{2} = \frac{1}{2} + 2 + c$ $c = 2$ $\frac{1}{2}v^2 = \frac{x^4}{2} + 2x^2 + 2$ $v^2 = x^4 + 4x^2 + 4 = (x^2 + 2)^2$ $v = \pm(x^2 + 2)$ but due to initial conditions, take positive v $\therefore \quad \dot{x} = x^2 + 2$	3 marks for correct solution 2 marks for velocity 1 mark for correct integration 1 mark for initial condition (need to make some statement explaining why pos v taken only)

d)(ii)	$\frac{dx}{dt} = x^2 + 2$ $\frac{dt}{dx} = \frac{1}{x^2 + 2}$ $t = \int \frac{1}{x^2 + 2}$ $t = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$ $t = 0, x = 1$ $0 = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) + c$ $c = -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $\therefore \quad t = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $\sqrt{2}\, t = \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$ $\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) = \sqrt{2}\, t + \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$ $\frac{x}{\sqrt{2}} = \tan\left(\sqrt{2}\, t + \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$ $x = (\sqrt{2})\tan\left(\sqrt{2}\, t + \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$	3 marks for correct solution 2 marks for t equation with constant found 1 mark for integration (no constant)

Question 14

a) (i)	$f(-x)=\dfrac{(-x)^4+3(-x)}{(-x)^4+3}$ $=\dfrac{x^4+3x^2}{x^4+3}$ $=f(x)$ $\therefore$ an even function	1 mark for showing substitution of $-x$ and simplifying to show it equals *f (x).*
(ii)	y = 1	1 mark for solution.
(iii)	$f'(x)=\dfrac{(x^4+3)(4x^3+6x)-4x^3(x^4+3x^2)}{(x^4+3)^2}$ $=\dfrac{-6x^5+12x^3+18x}{(x^4+3)^2}$ $f'(x)=0$ when $-6x^5+12x^3+18x=0$ $-6x(x^2-3)(x^2+1)=0$ $x=0, x=\pm\sqrt{3}$	1 mark for obtaining correct derivative. 1 mark for equating the correct equation (first derivative) to zero. 1 mark for all **three** correct solutions.
(iv)		1 mark for showing asymptote and turning points. 1 mark for correct shape.
b) (i)	$(1+x)^{2n}+(1-x)^{2n}$ $={}^{2n}\mathbf{C}_0+{}^{2n}\mathbf{C}_1\,x+\ldots+{}^{2n}\mathbf{C}_{2n}\,x^{2n}+{}^{2n}\mathbf{C}_0-{}^{2n}\mathbf{C}_1\,x+\ldots+{}^{2n}\mathbf{C}_{2n}\,x^{2n}$ $=2\left({}^{2n}\mathbf{C}_0+{}^{2n}\mathbf{C}_2\,x^2+{}^{2n}\mathbf{C}_4\,x^4+\ldots+{}^{2n}\mathbf{C}_{2n}\,x^{2n}\right)$	1 mark for expansion. Did not have to simplify. NOTE: Must expand the expressions to obtain the mark. If it was written in general form and not expanded, no mark was awarded.

(ii)	$1 + {}^{20}\mathbf{C}_2 + {}^{20}\mathbf{C}_4 + \ldots + {}^{20}\mathbf{C}_{20}$ from part (i) substitute $x = 1$ $2\,(1 + {}^{20}\mathbf{C}_2 + {}^{20}\mathbf{C}_4 + \ldots + {}^{20}\mathbf{C}_{20}) = (1+1)^{20} + (1-1)^{20}$ $\therefore\ 1 + {}^{20}\mathbf{C}_2 + {}^{20}\mathbf{C}_4 + \ldots + {}^{20}\mathbf{C}_{20} = \frac{1}{2} \times 2^{20}$ $= 2^{19}$ $= 524288$	1 mark for showing working and correct solution 524288 or 2^{19}. No mark was awarded for just having 524288 or 2^{19} without working.
(c) (i)	$speed = \sqrt{\dot{x}^2 + \dot{y}^2}$ take the origin O to be the point of projection. when $t = 0$, $x = 0$, $y = 0$, $\dot{x} = 40$, $\dot{y} = 0$ Horizontally: $\ddot{x} = 0$ $\dot{x} = 40$ $x = 40t + C$ using initial conditions $C = 0$, $x = 40t$ Vertically: $\ddot{y} = -10$ $\dot{y} = -10t + C_1$ using initial conditions $\dot{y} = -10t$ $y = -5t^2 + C_2$ $y = -5t^2$ $50 = \sqrt{40^2 + (-10t)^2}$ $2500 = 1600 + 100t^2$ $t^2 = 9$ $t = 3$, since $t > 0$ 3 seconds for the patrachute to reach a speed of $\frac{50m}{s}$ now $x = 40 \times 3 = 120m$	1 mark for correct formulas 1 mark for correct time of 3 seconds 1 mark for final solution of 120m NOTE: students were given a carry on error for incorrect time, but correct use of horizontal formula. Also, you must have both horizontal and vertical formulas correct to receive the first mark. Students who made too many errors or were not progressing towards the correct solution were not given any marks.

(ii)	after 3 seconds $\dot{y} = -30$ $y = -45$ $\dot{x} = 5$ $x = 120$ using these from $t = 0$ $\ddot{x} = 0$ $\dot{x} = 5$ $x = 5t + C$ when $t = 0$ $x = 120$ $x = 5t + 120$ $\ddot{y} = -2$ $\dot{y} = -2t + C_2$ $t = 0$ $C_2 = -30$ $\dot{y} = -2t - 30$ $y = -t^2 - 30t + C_3$ when $t = 0$, $C_3 = -45$ $y = -t^2 - 30t - 45$ parcel hits the ground, when $y = -220$ $-t^2 - 30t - 45 = -220$ $(t + 35)(t - 5) = 0$ $t = 5$, since $t > 0$ after 5 seconds parcel hits the ground $x = 5 \times 5 + 120$ $x = 145m$	1 mark for correct formulas. 1 mark for writing correct equation to solve for time. 1 mark for correct solution. NOTE: You must have both horizontal and vertical formulas correct to receive the first mark. Students who made too many errors or were not progressing towards the correct solution were not given any marks.