NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

Year 12

Trial Examination

2019

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Write your name on the front of every booklet.
- In Questions 11 to 14 show relevant mathematical reasoning and/or calculations.
- NESA approved calculators and templates may be used.
- Weighting: 30%

Section I Multiple Choice

- 10 marks
- Attempt all questions.
- Answer Sheet provided
- Allow about 15 minutes for this section

Section II Free Response

- 60 marks
- Start a separate booklet for each question.
- Each question is of equal value.
- All necessary working should be shown in every question.
- Allow about 1 hour and 45 minutes for this section.

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

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1. The acute angle between the lines $y = 2x$ and $y = 6x$ is θ.

   What is the value of $\tan\theta$?

   A. $\frac{4}{13}$

   B. $\frac{8}{11}$

   C. $\frac{1}{8}$

   D. $\frac{4}{9}$

2. From 5 mathematics books and 6 English books, 3 mathematics books and 2 English books are to be selected.

   Which expression represents the number of combinations possible?

   A. ${}^{30}C_6$

   B. ${}^{11}C_5$

   C. ${}^{5}C_3 \times {}^{6}C_2$

   D. ${}^{5}C_3 + {}^{6}C_2$

3. Let $f(x) = \dfrac{1 - x^2}{x}$. What is the $\lim\limits_{x \to \infty} f(x)$?

   A. -1

   B. $-x$

   C. 1

   D. 0

4. Using the substitution $u = x^2 + 1$, which of the following is equivalent to $\int_0^1 x\sqrt{x^2+1}\ dx$?

A. $\int_0^1 \sqrt{u}\ \mathrm{du}$

B. $\frac{1}{2}\int_0^1 \sqrt{u}\ \mathrm{du}$

C. $\int_1^2 \sqrt{u}\ \mathrm{du}$

D. $\frac{1}{2}\int_1^2 \sqrt{u}\ \mathrm{du}$

5. Five distinct green books, four distinct red books and three distinct yellow books are placed randomly along a shelf. What is the probability that the yellow books are all next to each other?

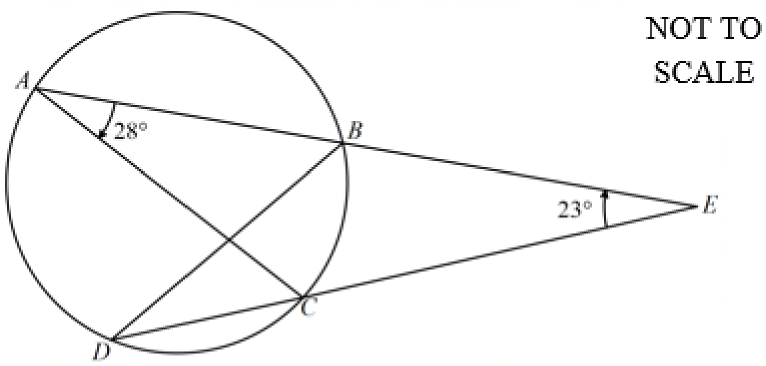
A. $\frac{3!\ 4!\ 5!}{12!}$

B. $\frac{3!\ 10!}{12!}$

C. $\frac{3!}{12!}$

D. $\frac{10!}{12!}$

6. In the diagram, ABC is a circle. D is on the perimeter of the circle, with AB and DC produced to meet at E. $\angle BAC = 28°$ and $\angle DEA = 23°$.
What is the measure of $\angle DBE$?



A. 129°

B. 134°

C. 152°

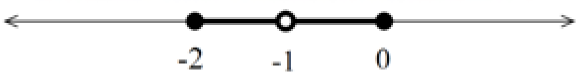
D. 141°

7. Which diagram represents the domain of the function $f(x) = \cos^{-1}\left(\frac{1}{x+1}\right)$?

A.

B.

C.



D.

8. The remainder when the polynomial $P(x) = x^4 - 8x^3 - 7x^2 + 3$ is divided by $x^2 + x$ is $kx + 3$.
What is the value of k ?

A. -14

B. -11

C. -2

D. 5

9. What is the constant term in the binomial expansion of $\left(2x - \frac{3}{x^2}\right)^9$?

A. ${}^9C_6 \times 2^6 \times 3^3$

B. ${}^9C_3 \times 2^3 \times 3^6$

C. $-\,{}^9C_6 \times 2^6 \times 3^3$

D. $-\,{}^9C_3 \times 2^3 \times 3^6$

10. Projectiles A and B are launched at the same time at velocity V and angle α. However projectile A is launched from a higher position. The two projectiles land in the same horizontal plane. Which of the following is always true?

A. A and B will reach the ground at the same time.

B. A and B will have the same range.

C. A will reach its maximum height earlier than B.

D. The maximum speed of A is greater than the maximum speed of B.

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

a) Solve $\dfrac{3x}{2x+3} \geq -1$ **2**

b) In how many ways can three cards be selected from a standard pack of 52 playing cards if no more than one selected card is an Ace? **2**

c) (i) Show that a solution of the equation $x \ln x = 1$ lies between $x = 1$ and $x = 2$. **1**

(ii) Using $x = 1.5$ as a first approximation, apply Newton's Method once to obtain a better estimate of the exact solution.
State your answer correct to one decimal place. **2**

d) Use the substitution $u = 3x + 2$ to find $\displaystyle\int \frac{x\,dx}{(3x+2)^2}$. **2**

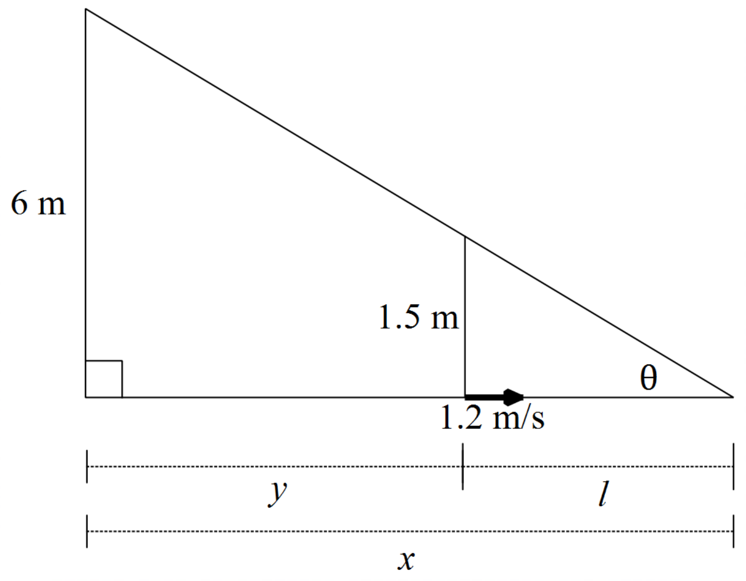
e) A survey of the weather pattern in Cropp River in New Zealand was conducted over a period of 160 consecutive days. It was found that it rained on 96 of those days. Ten of the 160 days are selected at random. Find the probability that it rained on at least eight of those days. State your answer correct to three decimal places. **3**

f) Use mathematical induction to prove $2^{3n} - 3^n$ is divisible by 5 for integers $n \geq 1$. **3**

End of Question 11

Question 12 (15 marks)

a) A person who is 1.5 metres tall is walking away from a light positioned on a pole that is 6 metres tall. The person walks at a constant speed of 1.2 metres per second at the time they are y metres from the base of the pole.



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(i) Using similar triangles, or otherwise, show that the rate at which l, the length of the shadow, is increasing with respect to time is 0.4m/s. **1**

(ii) By differentiating $l = \dfrac{1.5}{\tan\theta}$, or otherwise, find the rate at which the angle θ is changing with respect to time when the person is 5 metres from the base of the light pole. **3**

b) A particle is moving in a straight line such that its velocity v ms^{-1} is given by $v = 2e^{-2x}$, where x is the displacement of the particle (in metres) from the origin. The particle is initially at O.

(i) Find an expression for the acceleration of the particle in terms of x. **1**

(ii) Find an expression for the displacement of the particle in terms of time t. **3**

Question 12 continues on the next page

c) The rate of change of the temperature of a cool item placed in a warmer environment is proportional to the difference between the temperature T of the cool item and the temperature S of the warmer environment.

That is, $\frac{dT}{dt} = k(S - T)$

(i) Prove that $T = S - Ae^{-kt}$, where A and k are constants, is a solution of the above differential equation. **1**

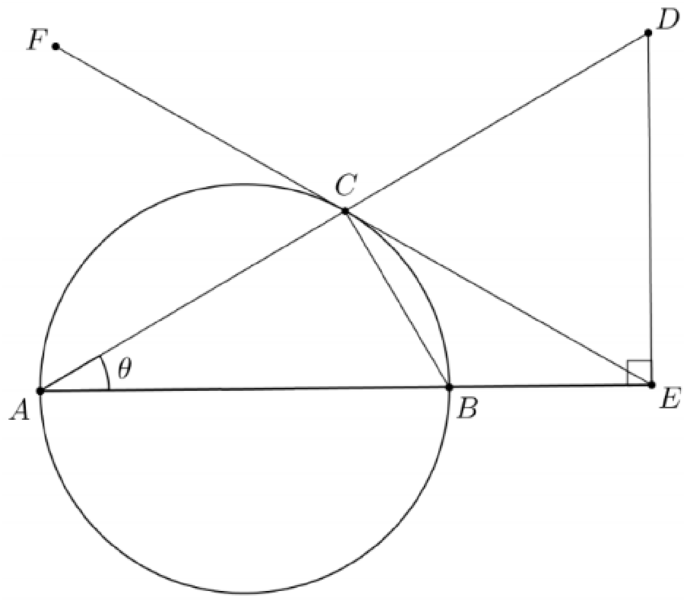
(ii) Anna is cooking a large roast in an oven set to 160°C. The roast will be cooked when the temperature at the centre of the roast reaches 150°C. When Anna started cooking, the temperature of the roast was 4°C and thirty minutes later the temperature was 60°C.
How long to the nearest minute will it take for the roast to be cooked? **3**

d) Use the substitution $x = 3\tan\theta$ to find $\int \frac{dx}{\sqrt{(x^2+9)^3}}$ **3**

End of Question 12

Question 13 (15 marks)

a) The diagram shows a circle with diameter AB. EF is a tangent to the circle at C. AC is extended to D such that DE is perpendicular to AE. Let $\angle BAC = \theta$.



(i) Copy the diagram and prove that $BCDE$ is a cyclic quadrilateral **1**

(ii) Prove that $EC = ED$. **3**

b) (i) Sketch $y = \sin^{-1}(1 - x)$ and state the domain. **2**

(ii) Given that $\sin\left(\sin^{-1}x - \cos^{-1}x\right) = 2x^2 - 1$,

hence solve $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(1 - x)$. **2**

c) Consider the function $f(x) = \dfrac{x - 3}{x^2}$

(i) Find the coordinates of the stationary point(s) on the curve $y = f(x)$, and determine the nature. **3**

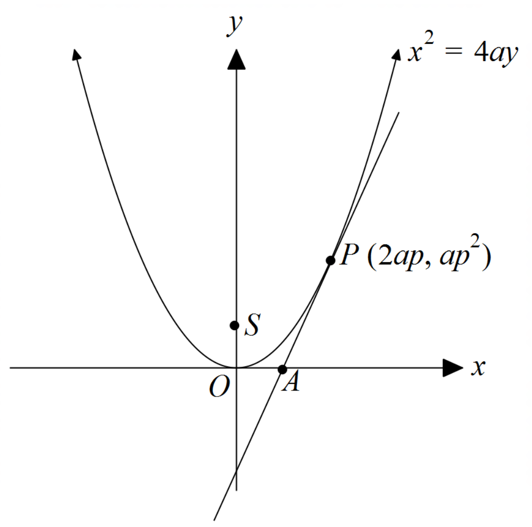
(ii) Determine the vertical and horizontal asymptotes for the curve $y = f(x)$. **2**

(iii) Sketch the graph of $y = f(x)$, showing the stationary points, asymptotes and x- intercept. **2**

End of Question 13

Question 14 (15 marks)

a) A point $P\ (2ap, ap^2)$ lies on the parabola $x^2 = 4ay$. The focus of the parabola is at S. The tangent to the parabola at point P meets the x-axis at point A.



(i) Find the coordinates of A. **1**

(ii) Prove that $\angle PAS$ is a right angle. **1**

(iii) Show that the locus of the centre of the circle whose circumference passes through the points P, S and A is a parabola, giving its vertex and focal length. **3**

b) Let n be an **even** positive integer.

(i) Show that $\displaystyle\sum_{k=0}^{n} \frac{{}^n\mathrm{C}_k}{k+1} x^{k+1} = \frac{(1+x)^{n+1} - 1}{n+1}$ **2**

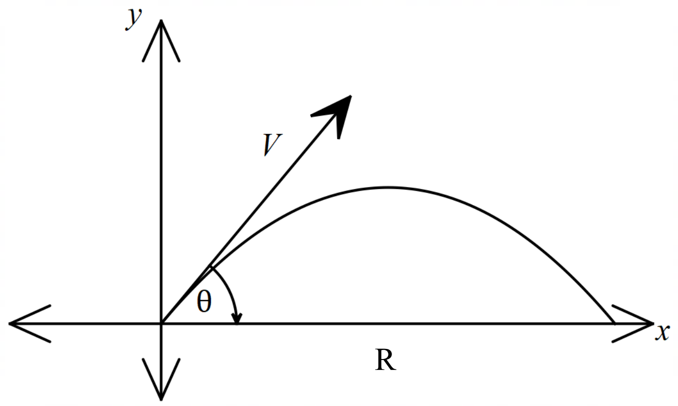
(ii) Find a similar expression for $\displaystyle\sum_{k=0}^{n} \frac{(-1)^k\, {}^n\mathrm{C}_k}{k+1}$ **1**

(iii) Hence, or otherwise, show that

$${}^n\mathrm{C}_0 + \frac{1}{3}\,{}^n\mathrm{C}_2 + \frac{1}{5}\,{}^n\mathrm{C}_4 + \dots + \frac{1}{n+1}\,{}^n\mathrm{C}_n = \frac{2^n}{n+1}$$ **2**

Question 14 continues on the next page

c) A projectile is fired from O with velocity V ms^{-1} at an angle θ ($0° < \theta < 90°$) above horizontal ground and strikes the ground R metres from O. The horizontal and vertical displacement (in metres) of the particle from O at time t seconds are given by $x = Vt\cos\theta$ and $y = -\frac{1}{2}gt^2 + Vt\sin\theta$ respectively (where g is the acceleration due to gravity).



(i) Show that $R = \dfrac{V^2 \sin 2\theta}{g}$ metres. **2**

(ii) A second projectile is fired from O at an angle α ($0° < \alpha < 90°$) with velocity V ms^{-1} and strikes the ground at a distance $2R$ metres from O.

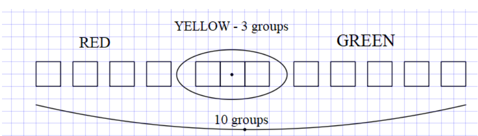
Explain why $0 < \sin 2\theta < \frac{1}{2}$. **1**

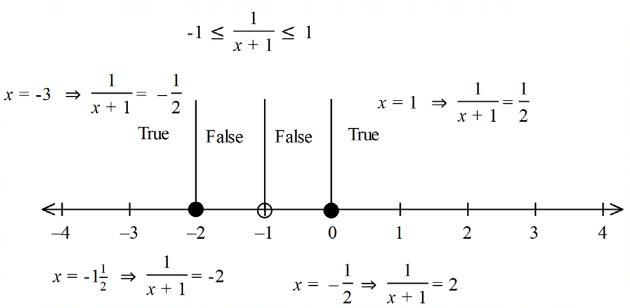
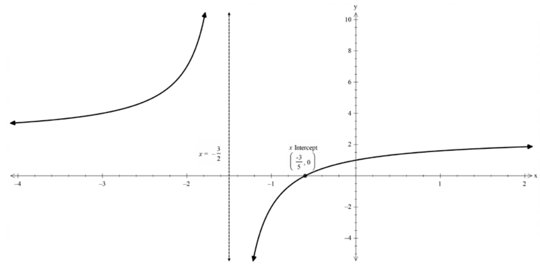
(iii) Show that this is only possible if $0° < \theta < 15°$ or $75° < \theta < 90°$. **2**

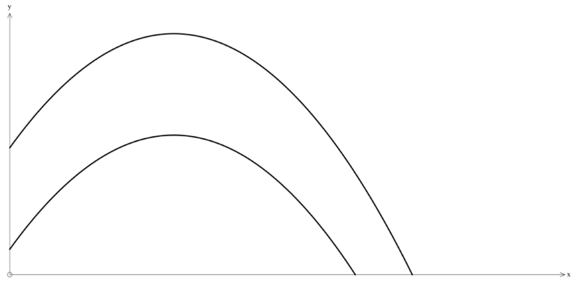
End of paper

MSC HSC Mathematics X1 Trial 2019

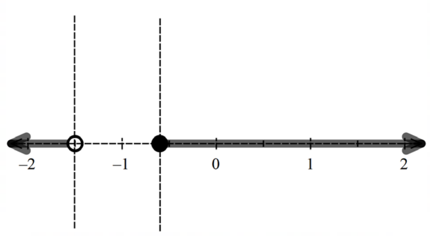
Multiple Choice Q1-10

Q1	$tan\theta = \left\lvert\frac{2-6}{1+12}\right\rvert$ $= \left\lvert\frac{-4}{13}\right\rvert$ $= \frac{4}{13}$	A
Q2	${}^5C_3 \times {}^6C_2$	C
Q3	$f(x) = \frac{1-x^2}{x}$ $\lim\limits_{x\to\propto} \frac{1-x^2}{x}$ $\lim\limits_{x\to\propto} \frac{\frac{1}{x}-\frac{x^2}{x}}{\frac{x}{x}}$ $\lim\limits_{x\to\propto} \frac{1}{x} - x$ $0 - x$ $y \to -x$	B
Q4	$u = x^2 + 1 \quad \Rightarrow \quad du = 2x\,dx$ $x = 0 \quad \Rightarrow \quad u = 1$ $x = 1 \quad \Rightarrow \quad u = 2$ $\int_0^1 x\sqrt{x^2+1}\,dx$ $= \frac{1}{2}\int_0^2 \sqrt{u}\ 2x\,dx$ $= 1.2\int_1^2 \sqrt{u}\ \mathrm{du}$	D
Q5	Each book is distinct therefore no repetitions.  	B

	$P = \dfrac{3!10!}{12!}$	
Q6	$\angle BDC = 28°$ angles in the same segment $\angle DBE = 180 - 23 - 28 = 129°$	A
Q7	$-1 \le \dfrac{1}{x+1} \le 1$ nb $x \ne -1$ $-1 = \dfrac{1}{x+1}$ $1 = \dfrac{1}{x+1}$ $-x - 1 = 1$ $x + 1 = 1$ $-x = 2$ $x = 0$ $x = -2$ Test regions  $\therefore x \le -2$ or $x \ge 0$ 	D
Q8	$P(x) = x(x+1)Qx + kx + 3$ $P(0) = 3$ $P(-1) = (-1)^4 - 8(-1)^3 - 7(-1)^2 + 3 = kx + 3$ $12 - 7 = k(-1) + 3$ $k = -2$	C

Q9	${}^9\mathbf{C}_k(2x)^{9-k}(-3x^{-2})^k = ax^0$ $x^{9-3k} = x^0$ $k = 3$ ${}^9\mathbf{C}_3\ 2^6\ (-3)^3 = -{}^9\mathbf{C}_3\ 2^6\ (3)^3$	C
Q10		D

Question 11

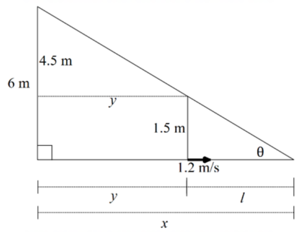
a	Method 1 $$\frac{3x}{2x+3} \geq -1$$ $$x \neq -\frac{3}{2}$$ $$\frac{3x}{2x+3} \times (2x+3)^2 \geq -1(2x+3)^2$$ $$3x(2x+3) + (2x+3)^2 \geq 0$$ $$(2x+3)[(2x+3)+3x] \geq 0$$ $$(2x+3)(5x+3) \geq 0$$ $$x < -\frac{3}{2} \text{ or } x \geq -\frac{3}{5}$$	2 marks – correct answer 1 mark – correct values but incorrect in inequalities or statement that $x \neq -\frac{3}{2}$
a	Method 2 $$\frac{3x}{2x+3} \geq -1$$ $$\therefore \quad x \neq -\frac{3}{2}$$ Solve $\frac{3x}{2x+3} = -1$ $$3x = -2x - 3$$ $$5x + 3 = 0$$ $$x = -\frac{3}{5}$$ Test regions  	
b	Number of ways = No aces selected + One Ace Selected $^{48}C_3 + {}^{4}C_1 \times {}^{48}C_2 = 21808$ OR $^{52}C_3 - ({}^{4}C_2 \,.\, {}^{48}C_1 + {}^{4}C_3 \,.\, {}^{48}C_0) = 21808$	2 marks – correct solution 1 mark – either condition

c i	$x = 1$ $f(x) = x\log_e x - 1$ $1 \times 0 - 1 = -1$ If $x = 2$ $f(x) = x\log_e x - 1$ $2 \times \log_e 2 - 1 = 0.386$ If curve is continuous, then it must cross x-axis between 1 and 2 as $f(x)$ changes sign.	1 mark if both demonstration of change of sign and **including reference to graph must be continuous**
c ii	$f(x) = x\log_e x$ $f\,'(x) = \log_e x + x \times \frac{1}{x}$ $= \log_e x + 1$ $a_1 = a_0 - \frac{f(a_0)}{f\,'(a_0)}$ $= 1.5 - \frac{1.5\log_e 1.5 - 1}{\log_e 1.5 + 1}$ $= 1.778$ $f(1.778) = -0.137$ $f(1.5) = 0.608$ Therefore better solution	2 marks – correct solution 1 mark – correct a1 expression

d	$\int \frac{x\,dx}{(3x+2)^2}$ let $u = 3x + 2$ $\frac{du}{dx} = 3$ $x = \frac{u-2}{3}$ $dx = \frac{1}{3}\text{du}$ $\therefore \quad = \int \frac{u-2}{3}\, u^{-2}\, \frac{1}{3}\text{du}$ $= \frac{1}{9}\int u^{-1} - 2u^{-2}\text{ du}$ $= \frac{1}{9}\int \frac{1}{u} - 2u^{-2}$ $= \frac{1}{9}\left[\log u - \frac{2}{-1}u^{-1}\right] + C$ $= \frac{1}{9}\left[\log u + \frac{2}{u}\right] + C$ $= \frac{1}{9}\left[\log(3x+2) + \frac{2}{3x+2}\right] + C$	2 marks - correct solution 1 mark – correct transformation into integral involving u with correct substitution of $\frac{1}{3}$ du in place of dx
e	Rained on at least 8 = rain on 8, 9 or 10 days. $(p+q)^n$ success = $p = \frac{96}{160} = \frac{3}{5}$ $\therefore q = \frac{2}{5}$ ${}^{10}\mathbf{C}_2\, p^8 q^2 + {}^{10}\mathbf{C}_1\, p^9 q^1 + {}^{10}\mathbf{C}_{10}\, p^{10}$ $= 0.16729$	3 marks – correct solution 2 mark – two correct day probability. 1 mark – correct binomial expansion expression.

f	1. Prove true for $n = 1$ $T_1 = 2^3 - 3^1$ $= 8 - 3 = 5$ therefore divisible by 5 True for $n = 1$ 2. Assume true for $n = $ k $2^{3k} - 3^k = 5P$ (P is an integer) $2^{3k} = 5P + 3^k$ or $3^k = 2^{3k} - 5P$ 3. Prove true to $n = $ k + 1 $T_{k+1} = 2^{3(k+1)} - 3^{k+1}$ $= 2^3\,2^{3k} - 3 \times 3^k$ since $2^{3k} = 5P + 3^k$ from above $= 8(5P + 3^k) - 3.3^k$ $= 5 \times 8P + (8-3)\,3^k$ $= 5(8P + 3^k)$ $= 5M$ M is an integers P and k are integers Therefore $2^{3n} - 3^n$ is divisible by 5 where n is an integer.	3 marks – correct solution 2 marks – correct substitution in part 3 but no further progress 1 mark – part 1 and part 2 correct.

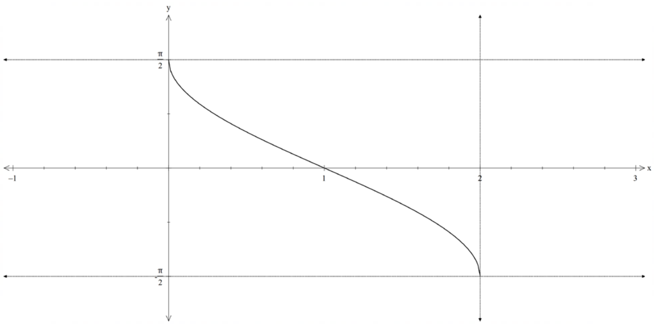
Question 12

a-i	 We can equate using similar triangles: $\frac{l}{1.5} = \frac{y}{4.5}$ $l = \frac{1}{3}y$ Differentiate both sides w.r.t. t: $\frac{dl}{dt} = \frac{1}{3}\frac{dy}{dt}$ Substituting $\frac{dy}{dt} = 1.2$: $\frac{dl}{dt} = \frac{1}{3} \times 1.2 = 0.4$ m/s	1 mark correct solution
a-ii	$l \tan\theta = 1.5$ Differentiating w.r.t. t: $\frac{dl}{dt}\tan\theta + l\sec^2\theta\,\frac{d\theta}{dt} = 0$ Solving for $\frac{d\theta}{dt}$: $\frac{d\theta}{dt} = -\frac{\frac{dl}{dt}\tan\theta}{l\sec^2\theta}$ We need values of l and θ when $y = 5$: $\frac{l}{1.5} = \frac{5+l}{6}$ $6l = 1.5(5 + l)$ $6l = 7.5 + 1.5l$ $4.5l = 7.5$ $l = \frac{5}{3}$ m $\tan\theta = \frac{1.5}{5/3} = 0.9$ $\theta = \tan^{-1} 0.9 \approx 42°$ Substituting (including $\frac{dl}{dt} = 0.4$): $\frac{d\theta}{dt} = -\frac{0.4\times\tan(42°)}{\frac{5}{3}\times\sec^2(42°)} = -\frac{0.4\times\tan(42°)}{\frac{5}{3}\times\frac{1}{\cos^2(42°)}} = -0.119\ldots \approx -0.12°/\text{s}$	3 marks, correct solution including correct final numerical value 2 marks, correct expressions for $\frac{dl}{d\theta}$ *and* $\frac{d\theta}{dt}$ (or equivalent progress) 1 mark, obtains at least one relevant, significant result
b-i	$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dx}\left[\frac{1}{2}\left(2e^{-2x}\right)^2\right]$ $= \frac{d}{dx}\left[2e^{-4x}\right]$ $= -8e^{-4x}$	1 mark correct solution

b-ii	$v = 2e^{-2x}$ $\frac{dx}{dt} = \frac{2}{e^{2x}}$ $\frac{dt}{dx} = \frac{e^{2x}}{2}$ $t = \int \frac{e^{2x}}{2} dx$ $= \frac{1}{4}e^{2x} + c$ $\text{when } t = 0, x = 0 \Rightarrow c = -\frac{1}{4}$ $t = \frac{1}{4}e^{2x} - \frac{1}{4}$ $4t = e^{2x} - 1$ $x = \frac{1}{2}\ln(4t+1)$	3 marks correct solution 2 marks correct expression for t 1 mark for correct primitive function for t, including a constant.
c-i	$T = S - Ae^{-kt}$ $\therefore Ae^{-kt} = S - T \quad \ldots (1)$ $\frac{dT}{dt} = kAe^{-kt}$ $= k(S-T) \quad \ldots \text{ from } (1)$	1 mark correct solution
c-ii	$T = S - Ae^{-kt}$ $S = 160°$ at $t = 0$ $4 = 160 - Ae^{0}$ $A = 154$ $60 = 160 - 154e^{-30k}$ $e^{-30k} = \frac{100}{154}$ $30k = \ln\left(\frac{100}{154}\right)$ $k = \frac{1}{30}\ln\left(\frac{100}{154}\right)$	3 marks correct solution 2 marks, obtains correct values for A and k or obtains an answer through correct working with one incorrect value. 1 mark, finds a correct value for A or k.

	$150 = 160 - 154e^{-tk}$ $e^{-tk} = \frac{10}{154}$ $-tk = \ln\left(\frac{10}{154}\right)$ $t = -\frac{1}{k}\ln\left(\frac{10}{154}\right)$ $= 189.98 \cong 190 \text{ min}$	
d	$x = 3\tan\theta$ $dx = 3\sec^2\theta\, d\theta$ $\int \frac{dx}{\sqrt{(x^2+9)^3}}$ $= \int \frac{3\sec^2\theta\, d\theta}{\sqrt{(9\tan^2\theta + 9)^3}}$ $= \int \frac{3\sec^2\theta\, d\theta}{\left(\sqrt{9(\tan^2\theta+1)}\right)^3}$ $= \int \frac{3\sec^2\theta\, d\theta}{(3\sec\theta)^3}$ $= \frac{1}{9}\int \cos\theta \;\; d\theta$ $= \frac{1}{9}\sin\theta + C$ $= \frac{x}{9\sqrt{x^2+9}} + C$ since $\tan\theta = \frac{x}{3}$	3 marks correct solution 2 marks obtains a correct primitive function in terms of theta. 1 mark for dx

Question 13

a	$\angle\, ACB = 90°$ (angle in a semicircle) $\angle\, BED = 90°$ (given) $\therefore$ $BEDC$ is a cyclic quad ($ext\,\angle\ =$ opp interior $\angle$)	1 mark – for a correct solution.
a-ii	$\angle\, CBD = 90°$ (straight angle) $\therefore\ \angle DCE = 90° - \angle BCE$ $= 90° - \theta°$ $\angle EDA = 90° - \angle DAE$ $= 90° - \theta°$ $\therefore$ ΔEDC is isosceles (equal base angles) $\therefore$ $EC = ED$ (sides opp equal ang's in isosceles Δ)	3 marks – correct solution 2 marks - 1 mark -
		2 marks – correct diagram 1 mark – correct domain

		2 marks – correct solution including explanation for rejection of x = -1 1 mark – deriving quadratic equation

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$$\sin^{-1} x - \cos^{-1} x = \sin^{-1}(1 - x)$$
$$\sin[\sin^{-1} x - \cos^{-1} x] = \sin[\sin^{-1}(1 - x)]$$
$$2x^2 - 1 = 1 - x$$
$$2x^2 + x - 2 = 0$$
$$(2x - 1)(x + 1) = 0$$
$$x = \frac{1}{2} \text{ or } x = -1$$
$$x = \frac{1}{2} \text{ only lies in domain}$$
$$\therefore \quad x = \frac{1}{2}$$

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<tr><td>c-i</td><td>

$$f(x) = \frac{x-3}{x^2}$$

$u = x - 3$ $v = x^2$
$u' = 1$ $v = 2x$

$$f'(x) = \frac{vu' - uv'}{v^2}$$
$$= \frac{x^2 - 2x(x-3)}{x^4}$$
$$= \frac{6x - x^2}{x^4}$$

$f'(x) = 0$
$6x - x^2 = 0$
$x(6 - x) = 0$
$x = 6 \text{ or } 0$
But $x \neq 0$
$\therefore \quad x = 6 \Rightarrow y = \frac{1}{12}$

x	5	6	7
f '(x)	$\frac{1}{125}$	0	$-\frac{1}{343}$

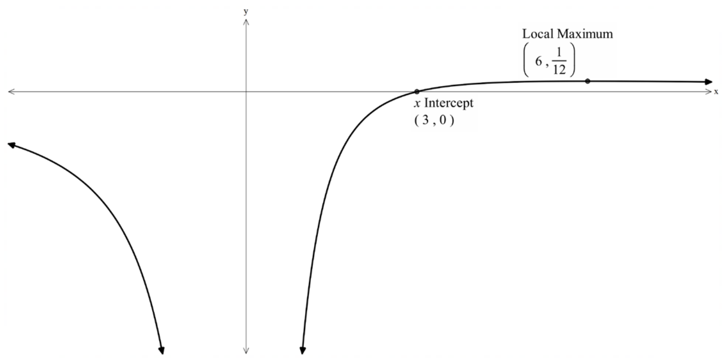
Therefore Maximum at $x = 6$

</td><td>

3 marks – correct solution

2 marks
- Failure to prove nature but correct coordinates
- One arithmetic error in calculation
- Did not explain rejection of $x = 0$

</td></tr>
</table>

c-ii	Vertical Asymptotes $f(x)=\frac{a}{b}$ $\therefore \quad b \neq 0$ vertical asymptote at $x^2=0$ $x=0$ Horizontal Asymptote $\lim_{x \to \pm\infty} f(x)=\frac{x-3}{x^2}$ $=\frac{1}{x}-\frac{3}{x^2}$ $=\frac{1}{\pm\infty}-\frac{3}{\pm\infty^2}$ $=0$ Horizontal Asumptote $y=0$	2 marks – both asymptotes correctly derived 1 mark – one correct asymptote
c-iii		2 marks – correct diagram with all required points 1 mark – correct shape lacking required points

Question 14

a-i	$x^2 = 4ay$ $y = \frac{x^2}{4a}$ $\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$ at $x = 2ap$ $m = \frac{2ap}{2a} = p$ $y - ap^2 = p(x - 2ap)$ $y = xp - ap^2$ at $y = 0$ $x = \frac{ap^2}{p} = ap$ $\therefore A\ (ap, O)$	1 mark – no need to derive equation as should be used from the Reference Sheet.
a-ii	$m_{AP} = p$ $m_{AS} = \frac{0 - a}{ap - 0} = -\frac{a}{ap} = -\frac{1}{p}$ $m_{AP} \times m_{AS} = p \times -\frac{1}{p} = -1$ $\therefore$ perpendicular $\therefore \angle PAS$ is a right angle	1 mark – correct derivation of both gradients and use of $m_1 \times m_2 = -1$

a-iii	$P\ (2ap, ap^2);\ S(0,a);\ A\ (ap,0)$ From part ii – if $\angle PAS = 90°$ then PS is a diameter of circle passing through P, A, and S. (angle in semicircle is a right angle) Centre of circle is midpoint of PS $M_{SP} = \left(\frac{0+2ap}{2}, \frac{a+ap^2}{2}\right)$ $= \left(ap, \frac{a}{2}(1+p^2)\right)$ $x = ap \Rightarrow p = \frac{x}{a}$ $y = \frac{a}{2}\left(1+\frac{x^2}{a^2}\right)$ $= \frac{a}{2} \times \frac{a^2+x^2}{a^2}$ $2ay = x^2 + a^2$ $x^2 = 2ay - a^2$ $x^2 = 2a\left(y - \frac{a}{2}\right)$ $Ver\exists x = \left(0, \frac{a}{2}\right)$ Focal length $= \frac{a}{2}$	3 marks – correct solution 2 – correct equation for locus 1 mark – correct derivation of midpoint

b-i	$$(1+x)^n = {}^n\mathbf{C}_0 + {}^n\mathbf{C}_{1x} + {}^n\mathbf{C}_2\, x^2 + \ \ldots\ + {}^n\mathbf{C}_n\, x^n$$ $$\int (1+x)^n \ dx = \frac{(1+x)^{n+1}}{n+1}$$ $$\therefore \quad \frac{(1+x)^{n+1}}{n+1} = {}^n\mathbf{C}_0 \frac{x}{1} + {}^n\mathbf{C}_1 \frac{x^2}{2} + \ldots {}^n\mathbf{C}_n \frac{x^{n+1}}{n+1} + C$$ Let $x = 0$ $$\frac{1}{n+1} = C$$ $$\frac{(1+x)^{n+1}}{n+1} = \sum_{k=0}^{n} \frac{{}^n\mathbf{C}_k}{k+1}\, x^{k+1} + \frac{1}{n+1}$$ $$\sum_{k=0}^{n} \frac{{}^n\mathbf{C}_k}{k+1}\, x^{k+1} = \frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1}$$ $$\sum_{k=0}^{n} \frac{{}^n\mathbf{C}_k}{k+1}\, x^{k+1} = \frac{(1+x)^{n+1} - 1}{n+1}$$	2 marks – correct solution 1 mark – attempt to use integration

b-ii	$(1-x)^n = {}^n\mathbf{C}_0 + {}^n\mathbf{C}_1(-1)x + {}^n\mathbf{C}_2(-1)^2x^2 + \ldots + {}^n\mathbf{C}_n(-1)^n x^n$ $\int (1-x)^n \, dx = \frac{(1-x)^{n+1}}{-(n+1)}$ $\therefore \quad \frac{(1-x)^{n+1}}{-(n+1)} = {}^n\mathbf{C}_0 \frac{x}{1} + {}^n\mathbf{C}_1(-1)\frac{x^2}{2} + \ldots {}^n\mathbf{C}_n(-1)^n \frac{x^{n+1}}{n+1} + C$ Let $x = 0$ $-\frac{1}{n+1} = C$ $\frac{(1+x)^{n+1}}{-(n+1)} = \sum_{k=0}^{n} \frac{{}^n\mathbf{C}_k}{k+1}(-1)^k x^{k+1} - \frac{1}{n+1}$ $\sum_{k=0}^{n} \frac{{}^n\mathbf{C}_k}{k+1}(-1)^k x^{k+1} = \frac{(1-x)^{n+1}}{-(n+1)} + \frac{1}{n+1}$ $\sum_{k=0}^{n} \frac{{}^n\mathbf{C}_k}{k+1}(-1)^k x^{k+1} = \frac{-(1-x)^{n+1}+1}{n+1}$ Let $x = 1$ $\sum_{k=0}^{n} \frac{{}^n\mathbf{C}_k}{k+1}(-1)^k \;) = \frac{1}{n+1}$	1 mark for solution

b-iii	$\sum_{k=0}^{n}\frac{{}^{n}C_k}{(k+1)\,x^{k+1}} = \frac{(1+x)^{n+1}-1}{n+1}$ let $x = 1$ $\sum_{k=0}^{n}\frac{{}^{n}C_k}{(k+1)} = {}^{n}C_0 + \frac{{}^{n}C_1}{2} + \frac{{}^{n}C_2}{3} + \frac{{}^{n}C_3}{4} + ..\frac{{}^{n}C_n}{n+1}$ ① $\sum_{k=0}^{n}\frac{{}^{n}C_k}{k+1}(-1)^k(x^{k+1}) = \frac{1}{n+1} - \frac{[1-(-x)]^{n+1}}{n+1}$ let $x = 1$ $\sum_{k=0}^{n}\frac{{}^{n}C_k}{k+1}(-1)^k(x^{k+1}) = {}^{n}C_0 + \frac{{}^{n}C_1}{2}(-1) + \frac{{}^{n}C_2}{3}(-1)^2 + \frac{{}^{n}C_3}{4}(-1)^3 + ..\frac{{}^{n}C_n}{n+1}(-1)^n$ ② ① + ② $2\left({}^{n}C_0 + \frac{1}{3}{}^{n}C_2 + \frac{1}{5}{}^{n}C_4 + \ldots\frac{1}{n}{}^{n}C_n\right) = \frac{2^{n+1}-1}{n+1} + \frac{1}{n+1} = \frac{2^{n+1}}{(n+1)}$ ${}^{n}C_0 + \frac{1}{3}{}^{n}C_2 + \frac{1}{5}{}^{n}C_4 + \ldots\frac{1}{n}{}^{n}C_n = \frac{2^n}{(n+1)}$	2 marks for working and solution. MUST mention n is even.
C i	$x = Vt\cos\theta$ $y = -\frac{1}{2}gt^2 + Vt\sin\theta$ when $y = 0$ $V\sin\theta = \frac{1}{2}\text{gt}$ $\frac{2V\sin\theta}{g} = t$ $x = V \times \frac{2V\sin\theta}{g} \times \cos\theta$ $x = \frac{V^2\sin(2\theta)}{g}$ since $2\sin\theta\cos\theta = \sin(2\theta)$	2 marks for working and solution. 1 mark for solving for t.
C ii	$\frac{V^2\sin(2\alpha)}{g} = \frac{2V^2\sin(2\theta)}{g}$ $\sin(2\alpha) = 2\sin(2\theta)$ but $0 < \sin(2\alpha) < 1$ $0 < 2\sin(2\theta) < 1$ $0 < \sin(2\theta) < \frac{1}{2}$	1 mark for working

C iii	$sin^{-1}(0) < 2\theta < sin^{-1}(0.5)$ $0 < 2\theta < 30 \quad or \quad 150 < 2\theta < 180$ $0 < \theta < 15 \quad or \quad 75 < \theta < 90$	2 marks for correct solution and working 1 mark for only one inequality solution.