Newcastle Grammar School

Mathematics Extension 1

2016 HSC Trial Examination

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/or calculations

Total marks – 100
Section I Pages 2 – 4
10 marks
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 5 –11
60 marks
- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Examiner DC

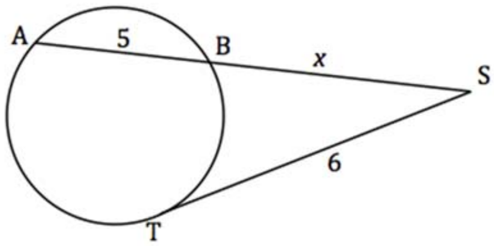
Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the objective response answer sheet for Questions 1–10.

Question 1

The line DT is a tangent to the circle at T and AS is a secant meeting the circle at A and B. Given that ST = 6, AB = 5 and SB = x, which of the following is the value of x?



(A) $x = 4$ (B) $x = 5$ (C) $x = 6$ (D) $x = 9$

Question 2

What is the coefficient of x^5 in the expansion of $(2x + 5)^8$?

(A) 1400000 (B) 224000 (C) 25000 (D) 4000

Question 3

A particle is moving along the x-axis. Its velocity v at position x is given by $v = \sqrt{8x - x^2}$.
What is the acceleration when $x = 3$?

(A) 1 (B) 2 (C) 3 (D) 4

Question 4

What is the domain and range of $y = \cos^{-1}\left(\dfrac{3x}{2}\right)$?

(A) Domain $\dfrac{-2}{3} \le x \le \dfrac{2}{3}$ Range $0 \le y \le \pi$

(B) Domain $-1 \le x \le 1$ Range $0 \le y \le \pi$

(C) Domain $\dfrac{-2}{3} \le x \le \dfrac{2}{3}$ Range $-\pi \le y \le \pi$

(D) Domain $-1 \le x \le 1$ Range $-\pi \le y \le \pi$

Question 5

The parametric equation of a function is $x = 2t^2,\ y = 4 - t$. The Cartesian equation is

(A) $x = 4(2-y)^2$ (B) $x = 2(y-4)^2$ (C) $x = 2(y+4)^2$ (D) $x = 2(4-y)^2$

Question 6

What is the $\lim\limits_{x \to 0} \dfrac{5\sin 3x}{x}$

(A) 15 (B) $\dfrac{5}{3}$ (C) $\dfrac{3}{5}$ (D) $\dfrac{1}{15}$

Question 7

Evaluate $\sum\limits_{n=3}^{10} 8 + 5n$

(A) 283.5 (B) 324 (C) 567 (D) 648

Question 8

The expression $\sin x - \sqrt{3}\cos x$ can be written in the form $2\sin(x+\alpha)$.
Find the value α

(A) $\alpha = \dfrac{\pi}{6}$ (B) $\alpha = -\dfrac{\pi}{6}$ (C) $\alpha = \dfrac{\pi}{3}$ (D) $\alpha = -\dfrac{\pi}{3}$

Question 9

Evaluate $\int_0^1 \frac{e^x}{1+e^x}\,dx$

(A) $\frac{e}{1+e}$ (B) $\frac{e^2}{1+e^2}$ (C) $\log_e(1+e)$ (D) $\log_e\left(\frac{1+e}{2}\right)$

Question 10

A particle moves in a straight line and its position at any time t is given by $x = 3\cos 2t + 4\sin 2t$. The motion is simple harmonic. What is the greatest speed achieved by the particle?

(A) 6 (B) 10 (C) 12 (D) 20

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 START A NEW BOOKLET (15 marks)

(a) Solve $\dfrac{x^2+20}{x-4} < -4$ (3)

(b) Find

i) $\displaystyle\int \frac{1}{\sqrt{\frac{1}{9}-x^2}}\, dx$ (2)

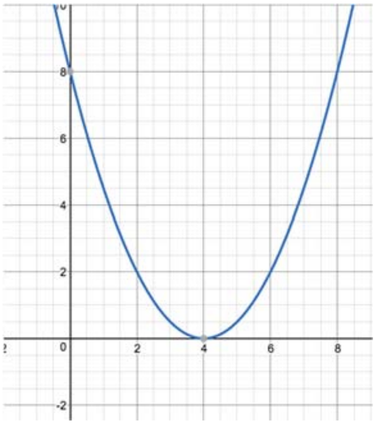
ii) $\displaystyle\int \sin^2 x\ dx$ (2)

(c) i) Find the linear factors of $x^3 - 5x^2 + 8x - 4$ (2)

ii) Hence solve $x^3 - 5x^2 + 8x - 4 > 0$ (2)

Question 11 continued on next page

(d) The function $f(x)=\frac{1}{2}(x-4)^2$ is shown. g(x) = f(x) over the limited domain $x \geq 4$.



i) Find $g^{-1}(x)$, the inverse of function g(x) (2)

ii) Find the point of intersection of g(x) and $g^{-1}(x)$ (2)

Question 12 START A NEW BOOKLET (15 marks)

(a) Write $\tan\left(\cos^{-1}\left(-\frac{1}{3}\right)\right)$ in the form $a\sqrt{b}$ where a and b are rational. (2)

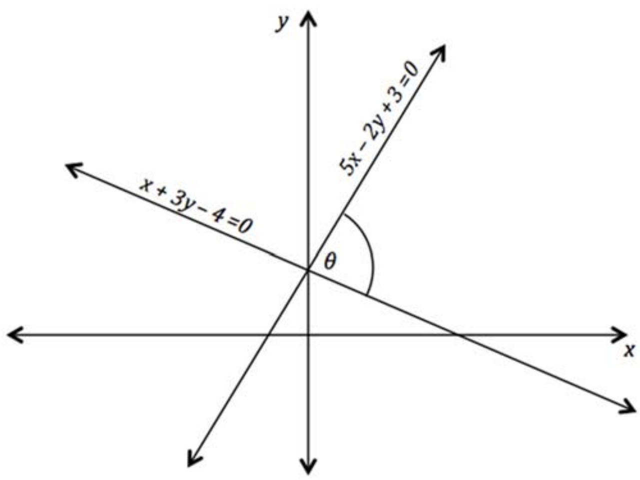
(b) i) Find $\frac{d}{dx}\ln(\cos 2x)$ (1)

ii) Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{6}} \tan 2x \; dx$ (2)

(c) Prove by mathematical induction that

$$\frac{1}{1\times 5}+\frac{1}{5\times 9}+\frac{1}{9\times 13}+....\frac{1}{(4n-3)(4n+1)}=\frac{n}{4n+1}$$ (3)

(d) Calculate the value of θ, correct to the nearest minute. (2)



(e) Evaluate $\int_0^1 x^3\sqrt{x^4+1}\; dx$, using the substitution $u = x^4+1$ (3)

(f) Find $\frac{d}{dx}\tan^{-1}\frac{x}{4}$ (2)

Question 13 START A NEW BOOKLET (15 marks)

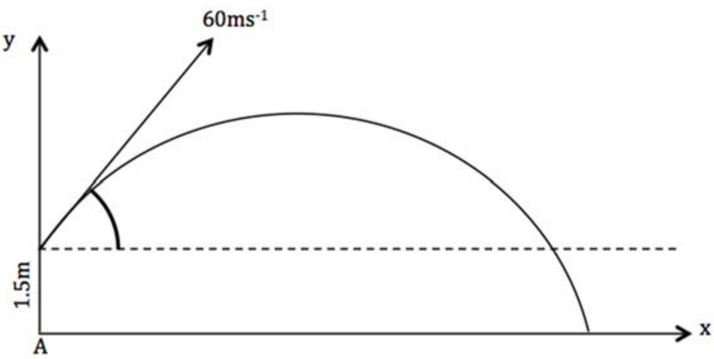
(a) At two points A and B, 400m apart on a straight horizontal road, the top of a hill is observed, with point Q representing the base of the hill, directly below its vertex.

At A, the hill is due north with an elevation of 15º. At B, the hill is due west with an elevation of 17º.

i) Draw a neat sketch showing all of the above information and find an expression for AQ in terms of h, the height of the hill. (2)

ii) Find the height of the hill to the nearest m (1)

(b) An archer shoots an arrow from a bow at an initial velocity of 60ms^{-1}, while standing at point A. The bow is 1.5m above the horizontal ground level at the time of firing and the angle of projection in 30º.



i) Allowing gravity to be 9.8m/s^{-2}, show that the equations of motion are

$x = 30\sqrt{3}t$ and $y = 30t - 4.9t^2 + 1.5$ (2)

ii) The archer is aiming for a tree that is 300m away and 3.4 metres in height. Show calculations that prove the arrow will not hit the tree (2)

iii) The archer has painted a target on the tree at a point 1 metre above the ground. What angle (to the nearest degree) will the archer need to shoot the arrow at in order to hit the target if the initial velocity is 60ms^{-1}? (3)

Question 13 continued on next page

(c) A particle moving in a straight line is performing Simple Harmonic Motion. At time t seconds its displacement x metres from a fixed point 0 is given by $x = 2\sin 3t - 2\sqrt{3}\cos 3t$

i) Express x in the form $x = R\sin(3t - \alpha)$ for some constants R > 0 and $0 < \alpha < \dfrac{\pi}{2}$. (1)

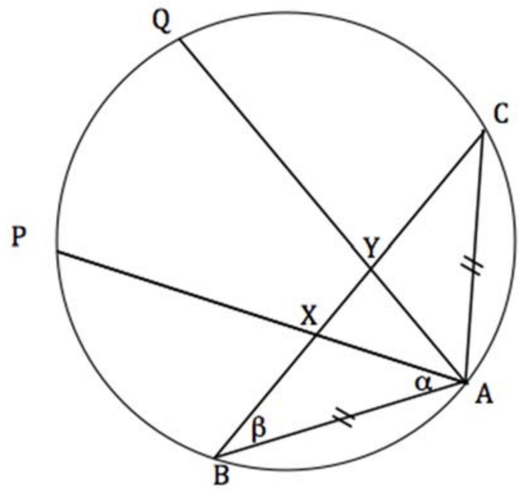
ii) Describe the initial motion of the particle in terms of its initial position, velocity and acceleration. (2)

iii) Find the exact value of the first time the particle is 2 metres to the left of *O* and moving towards *O*. (2)

Question 14 START A NEW BOOKLET (15 marks)

(a) Find the exact value of x if $\log_e(2\log_e x) = 1$ (2)

(b) In the circle below AB = AC. Let $\angle PAB = \alpha$ and $\angle ABC = \beta$.



i) Copy the diagram into your booklet and give a reason why $\angle PQB = \alpha$ (1)

ii) Prove $\angle AQB = \beta$. (1)

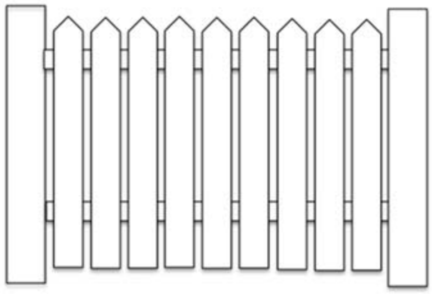
iii) Prove XYQP is a cyclic quadrilateral (3)

(c) The rate at a which a cup of coffee cools in air is proportional to the difference between its temperature T and the constant surrounding air temperature A, ie $\frac{dT}{dt} = k(T - A)$, where t is the time in minutes and k is a constant.

i) Show that $T = A + Be^{kt}$, where B is a constant, is a solution to the differential equation. (1)

ii) The coffee cools from 90ºC to 50ºC in 2 minutes. The surrounding temperature is 25ºC. Find the temperature of the coffee after one further minute has elapsed. Give your answer to the nearest degree. (3)

d) An artist is randomly painting the 9 panel sections of a fence. She paints two panels red, three yellow and four green.



i) How many sections of fence could she paint differently? (1)

ii) What is the probability that the red panels in any section are not next to each other? (2)

END OF EXAMINATION

Student ID ___________________________

OJECTIVE RESPONSE ANSWER SHEET

Question	**1**	A ⬭	B ⬭	C ⬭	D ⬭
	2	A ⬭	B ⬭	C ⬭	D ⬭
	3	A ⬭	B ⬭	C ⬭	D ⬭
	4	A ⬭	B ⬭	C ⬭	D ⬭
	5	A ⬭	B ⬭	C ⬭	D ⬭
	6	A ⬭	B ⬭	C ⬭	D ⬭
	7	A ⬭	B ⬭	C ⬭	D ⬭
	8	A ⬭	B ⬭	C ⬭	D ⬭
	9	A ⬭	B ⬭	C ⬭	D ⬭
	10	A ⬭	B ⬭	C ⬭	D ⬭

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Mathematics Extension 1

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SOLUTIONS

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Examiner D[illegible]

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Section I Pages 2 – 4

10 marks

- Attempt Questions 1–10
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Section II Pages [illegible]–10

60 marks

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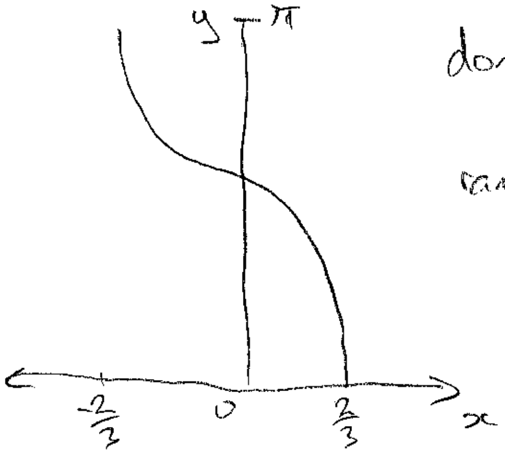
QUESTION M|C EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
1) $x(x+5) = 6^2$ $x^2 + 5x = 36$ $x^2 + 5x - 36 = 0$ $(x+9)(x-4) = 0$ $x = -9$ or 4 neglect negative answer.	A	
2) $(2x+5)^8$ ${}^8C_k (2x)^{8-k} (5)^k$ at $8-k = 5$ $k = 3$ $\therefore {}^8C_3 \, 2^5 \, 5^3 = 224000$	B	
3) $v = \sqrt{8x - x^2}$ $v^2 = 8x - x^2$ $a = \frac{d}{dx} \frac{1}{2} v^2$ $= \frac{1}{2}(8 - 2x)$ $= 4 - x$ at $x = 3$ $a = 1 \text{ m/s}^2$	A	

QUESTION M' | EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
4] $y = \cos^{-1}\left(\frac{3x}{2}\right)$		



domain $-\frac{2}{3} \le x \le \frac{2}{3}$

range $0 \le y \le \pi$

A

5] $x = 2t^2 \quad y = 4 - t$

$t = 4 - y$

$x = 2(4-y)^2$

D

6] $\lim_{x \to 0} \frac{5\sin 3x}{x} = \lim_{x \to 0} \frac{3 \times 5 \sin 3x}{3x}$

$= 3 \times 5 \times \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= 3 \times 5 \times 1$

$= 15$

A

7] $\sum_{n=3}^{10} 8 + 5n$

$= 8+15 + 8+20 + 8+25 + 8+30 \ldots 8+50$

$= 23 + \ldots \quad 58$

$= \frac{n}{2}[a + l]$

$= \frac{8}{2}[23+58] = 324$

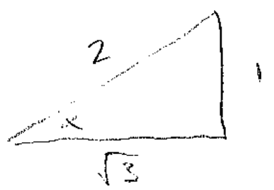
B

QUESTION M|C EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment

8)

$\sin x - \sqrt{3}\cos x$



$\sin\alpha = \frac{1}{2}$

$\cos\alpha = \frac{\sqrt{3}}{2}$

$= 2\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right)$

$= 2\left(\cos\alpha\sin x - \sin\alpha\cos x\right)$

$= 2\sin(x-\alpha)$

$= 2\sin\left(x - \frac{\pi}{3}\right)$

$= 2\sin\left(x + \left(-\frac{\pi}{3}\right)\right)$ — Mark: D

9)

$\int_0^1 \frac{e^x}{1+e^x}\,dx$

$= \left[\ln(1+e^x)\right]_0^1$

$= \ln(1+e) - \ln(1+1)$

$= \ln(1+e) - \ln 2$

$= \ln\left(\frac{1+e}{2}\right)$ — Mark: D

QUESTION M|C EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
$\boxed{10}$		
$x = 3\cos 2t + 4\sin 2t$		
$\dot{x} = -3\sin 2t . 2 + 4\cos 2t . 2$		
$= 8\cos 2t - 6\sin 2t$		
now max velocity at $x = 0$		
at $x = 0$		
$3\cos 2t + 4\sin 2t = 0$		
$4\sin 2t = -3\cos 2t$		
$4\tan 2t = -3$		
$\tan 2t = -\frac{3}{4}$		
$2t = \tan^{-1}\left(\frac{3}{4}\right)$		
max velocity		
$\dot{x} = 8\cos\left(\tan^{-1}\frac{3}{4}\right) - 6\sin\left(\tan^{-1}\frac{3}{4}\right)$		
$= 10$	B	

QUESTION 11 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment

a) $\dfrac{x^2+20}{x-4} < -4$

$(x^2+20)(x-4) < -4(x-4)^2$

$x^3 - 4x^2 + 20x - 80 < -4(x^2 - 8x + 16$

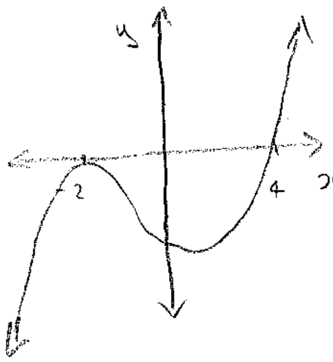
$x^3 - 4x^2 + 20x - 80 < -4x^2 + 32x - 64$

$x^3 - 12x - 16 < 0$

$(x+2)(x^2-2x-8) < 0$

$(x+2)(x-4)(x+2) < 0$

$(x+2)^2(x-4) < 0$



$\therefore \quad x < -2 \text{ or } -2 < x < 4$ — Mark: 3

b) i) $\displaystyle\int \frac{1}{\sqrt{\frac{1}{9} - x^2}}\,dx$

$= \displaystyle\int \frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - x^2}}\,dx$

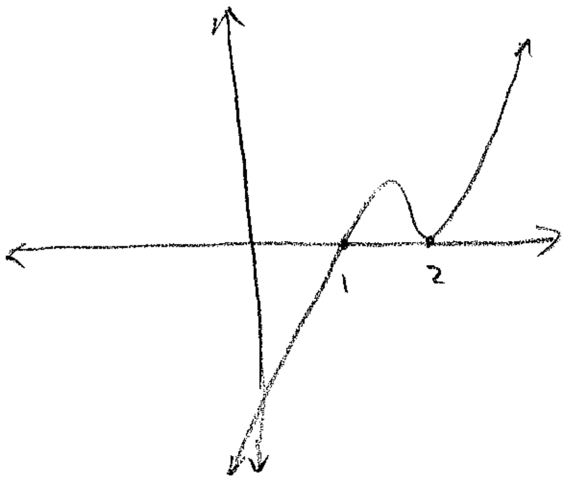
$= \sin^{-1} 3x + C$ — Mark: 2

QUESTION	11	EXAM	2016 HSC Ext 1 Task 4

Solution	Mark	Comment
b) ii) $\int \sin^2 x \, dx$ $= \frac{1}{2}\int 1 - \cos 2x \, dx$ $= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$ $= \frac{x}{2} - \frac{\sin 2x}{4} + c$	2	No, $\cos 2x = \cos^2 x - \sin^2 x$ $= 1 - \sin^2 x - \sin^2 x$ $= 1 - 2\sin^2 x$ $2\sin^2 x = 1 - \cos 2x$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
c) i) let $P(x) = x^3 - 5x^2 + 8x - 4$ $P(1) = 1 - 5 + 8 - 4$ $= 0$ $\therefore x - 1$ is a factor $(x^3 - 5x^2 + 8x - 4) \div (x - 1) = x^2 - 4x + 4$ $x^3 - x^2$ $-4x^2 + 8x$ $-4x^2 + 4x$ $4x - 4$ $4x - 4$ 0 $P(x) = (x - 1)(x^2 - 4x + 4)$ $= (x - 1)(x - 2)^2$	2	

QUESTION 11 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
ii) $x^3 - 5x^2 + 8x - 4 > 0$		



$1 < x < 2$ and $x > 2$ — Mark: 2

d) $g(x) = \frac{1}{2}(x-4)^2$

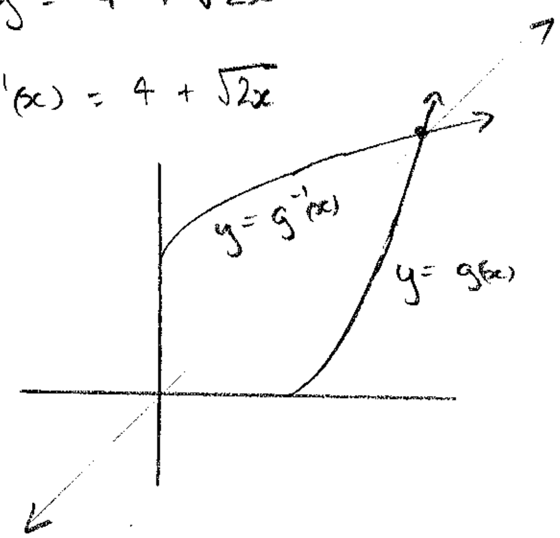
now $x = \frac{1}{2}(y-4)^2$

$2x = (y-4)^2$

$y - 4 = \sqrt{2x}$

$y = 4 + \sqrt{2x}$

$g^{-1}(x) = 4 + \sqrt{2x}$ — Mark: 2



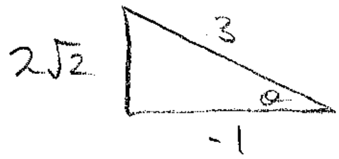
Comment: must intersect on the line $y = x$

QUESTION 11 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
d) con't		
$\therefore \quad x = \frac{1}{2}(x-4)^2$		
$2x = x^2 - 8x + 16$		
$x^2 - 10x + 16 = 0$		
$(x-8)(x-2) = 0$		
$\therefore x = 8$ or 2		
however $x \geq 4$		
$\therefore \quad x = 8$		
$\therefore$ Pt of intersection is $(8,8)$	2	

QUESTION 12 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
a) $\tan\left(\cos^{-1}\left(\frac{-1}{3}\right)\right)$		Note $\cos^{-1}x$ $0 \le x \le \pi$



$\cos\theta = \frac{-1}{3}$

$\tan\theta = \frac{2\sqrt{2}}{-1}$

$= -2\sqrt{2}$

$a = -2 \quad b = 2$

2

b) i) $\frac{d}{dx} \ln(\cos 2x)\, dx$

$= \frac{1}{\cos 2x} \cdot -\sin 2x \cdot 2$

$= -2\frac{\sin 2x}{\cos 2x}$

$= -2\tan 2x$

1

ii) $\int_0^{\frac{\pi}{6}} \tan 2x\, dx = -\frac{1}{2}\int_0^{\frac{\pi}{6}} -2\tan 2x\, dx$

$= -\frac{1}{2}\Big[\ln(\cos 2x)\Big]_0^{\frac{\pi}{6}}$

$= \frac{-1}{2}\left[\ln\frac{1}{2} - \ln 1\right]$

2

$= \frac{-1}{2}\ln\frac{1}{2}$ or $\ln\sqrt{2}$

QUESTION	12	EXAM	2016 HSC Ext 1 Task 4

Solution	Mark	Comment
c) Step 1 Prove true for $n=1$ $LHS = \frac{1}{1\times 5}$ $= \frac{1}{5}$ $RHS = \frac{1}{4+1}$ $= \frac{1}{5}$ $= LHS$ $\therefore$ True for $n=1$ Step 2 Assume true for $n=k$ $\frac{1}{1\times 5} + \frac{1}{5\times 9} + \dots \frac{1}{(4k-3)(4k+1)} = \frac{k}{4k+1}$ Step 3 Prove true for $n=k+1$ ie $\frac{1}{1\times 5} + \frac{1}{5\times 9} + \dots \frac{1}{(4(k+1)-3)(4(k+1)+1)} = \frac{k+1}{4(k+1)+1}$ $LHS = \frac{k}{4k+1} + \frac{1}{(4(k+1)-3)(4(k+1)+1)}$ $= \frac{k}{4k+1} + \frac{1}{(4k+1)(4k+5)}$ $= \frac{k(4k+5)+1}{(4k+1)(4k+5)}$ $= \frac{4k^2+5k+1}{(4k+1)(4k+5)}$ $= \frac{(4k+1)(k+1)}{(4k+1)(4k+5)}$ $= \frac{k+1}{4(k+1)+1}$ $= R.H.S$		

QUESTION 12 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
c) Step 1 Prove true for $n=1$		
$LHS = \frac{1}{1\times 5}$		
$= \frac{1}{5}$		
$RHS = \frac{1}{4+1}$		
$= \frac{1}{5}$		
$= L.H.S$		
$\therefore$ True for $n=1$		
Step 2 Assume true for $n=k$		
$\frac{1}{1\times 5} + \frac{1}{5\times 9} + \frac{1}{9\times 13} \cdots \frac{1}{(4k-3)(4k+1)} = \frac{k}{4k+1}$		
Step 3 Prove true for $n=k+1$		
Required to prove		
$\frac{1}{1\times 5} + \frac{1}{5\times 9} + \cdots \frac{1}{(4(k+1)-3)(4(k+1)+1)} = \frac{k+1}{4(k+1)+1}$		
$LHS = \frac{k}{4k+1} + \frac{1}{(4(k+1)-3)(4(k+1)+1)}$		
$= \frac{k}{4k+1} + \frac{1}{(4k+1)(4k+5)}$		
$= \frac{k(4k+5)+1}{(4k+1)(4k+5)}$		
$= \frac{4k^2+5k+1}{(4k+1)(4k+5)}$		
$= \frac{(4k+1)(k+1)}{(4k+1)(4k+5)}$		

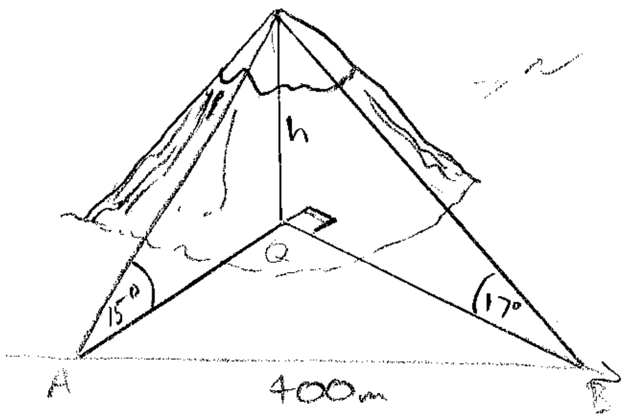
QUESTION 12 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
c) con't Therefore the statement is true for $n=k+1$ if it is true for $n=k$ As the statement is true for $n=1$, by the principle of mathematical induction, the statement is true for all integers $n \geq 1$	3	
d) $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $x + 3y - 4 = 0$ $y = -\frac{x}{3} + \frac{4}{3}$ $m_1 = -\frac{1}{3}$ $5x - 2y + 3 = 0$ $y = \frac{5}{2}x + \frac{3}{2}$ $m_2 = \frac{5}{2}$ $\therefore \tan\theta = \left\lvert \frac{-\frac{1}{3} - \frac{5}{2}}{1 - \frac{5}{6}} \right\rvert$ $\theta = 86^\circ 38'$ (nearest min)	2	

QUESTION 12 EXAM **2016 HSC Ext 1 Task 4**

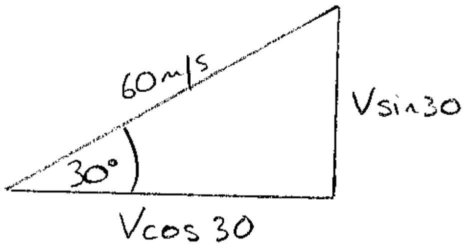
Solution	Mark	Comment
e) $\int_0^1 x^3\sqrt{x^4+1}\,dx$ let $u = x^4+1$ $du = 4x^3\,dx$ at $x=0$ $u=1$ $x=1$ $u=2$ $= \frac{1}{4}\int_1^2 u^{\frac{1}{2}}\,du$ $= \frac{1}{4}\left[\frac{2u^{\frac{3}{2}}}{3}\right]_1^2$ $= \frac{1}{4}\left[\frac{4\sqrt{2}}{3} - \frac{2}{3}\right]$ $= \frac{\sqrt{2}}{3} - \frac{1}{6}$ $= \frac{2\sqrt{2}-1}{6}$ (0.30 2dp).	3/	
f) $\frac{d}{dx}\tan^{-1}\frac{x}{4}$ $= \frac{4}{16+x^2}$	2/	

QUESTION 13 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
		
$\tan 15 = \frac{h}{AQ}$ $\quad$ $\tan 17 = \frac{h}{BQ}$		
$AQ = \frac{h}{\tan 15}$ $\quad$ $BQ = \frac{h}{\tan 17}$	2	
In $\triangle AQB$		
$AB^2 = AQ^2 + BQ^2$		
$160000 = \frac{h^2}{\tan^2 15} + \frac{h^2}{\tan^2 17}$		
$= h^2 \ \frac{1}{\tan^2 15} + \frac{1}{\tan^2 17}$		
$h^2 = \dfrac{160000}{\frac{1}{\tan^2 15} + \frac{1}{\tan^2 17}}$		
$= 6497$		
$h = 80.6$		
$\doteqdot 81$ m (nearest metre)	1	

QUESTION 13 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
b)		



Horizontal $\ddot{x} = 0$

$\dot{x} = c$

at $t = 0$

$\dot{x} = V\cos 30$

$x = V\cos 30 t + c$

at $t = 0$ $x = 0$ $\therefore c = 0$

$x = V\cos 30 t$

$= 60\cos 30 t$

$= 30\sqrt{3}\, t$

Vertically $\ddot{y} = -g$

$\dot{y} = -gt + c$

at $t=0$ $\dot{y} = V\sin 30$ $\therefore c = V\sin 30$

$\dot{y} = V\sin 30 - gt$

$y = V\sin 30 t - \frac{1}{2}gt^2 + c$

at $t = 0$ $y = 1.5$ $\therefore c = 1.5$

$y = 60\sin 30 t - \frac{1}{2}\times 9.8 t^2 + 1.5$

$= 30t - 4.9t^2 + 1.5$

Mark: 2

QUESTION 13 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
b) ii) at $x = 300$		
$300 = 30\sqrt{3}\, t$		
$\frac{10}{\sqrt{3}} = t$		
at $t = \frac{10}{\sqrt{3}}$		
$y = 30\left(\frac{10}{\sqrt{3}}\right) - 4.9\left(\frac{100}{3}\right) + 1.5$		
$= 11.37\text{m}$		
∴ The arrow misses the tree.	2	
iii) $300 = 60\cos\theta\, t$		
$1 = 60\sin\theta\, t - 4.9t^2 + 1.5$		
$t = \frac{5}{\cos\theta}$		
∴ $-0.5 = 60\sin\theta \cdot \frac{5}{\cos\theta} - 4.9 \cdot \frac{25}{\cos^2\theta}$		
$= 300\tan\theta - \frac{122.5}{\cos^2\theta}$		
$0 = 300\tan\theta - 122.5(\sec^2\theta) + 0.5$		
$= 300\tan\theta - 122.5(1 + \tan^2\theta) + 0.5$		
$= 300\tan\theta - 122.5 - 122.5\tan^2\theta + 0.5$		
$122.5\tan^2\theta - 300\tan\theta + 122 = 0$		
$\tan\theta = \frac{300 \pm \sqrt{90000 - 4 \times 122.5 \times 122}}{245}$		
$= 1.93$ or 0.51		
$\theta = 63°$ $27°$	3	

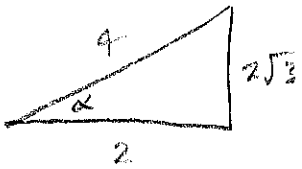
QUESTION 13 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment

c)

$x = 2\sin 3t - 2\sqrt{3}\cos 3t$

i)



$\cos\alpha = \frac{2}{4}$

$\sin\alpha = \frac{2\sqrt{3}}{4}$

$\alpha = \cos^{-1}\frac{1}{2}$

$\alpha = \frac{\pi}{3}$

$x = 4\left(\frac{2}{4}\sin 3t - \frac{2\sqrt{3}}{4}\cos 3t\right)$

$= 4(\cos\alpha \sin 3t - \sin\alpha \cos 3t)$

$= 4\sin(3t - \alpha)$

$= 4\sin\left(3t - \frac{\pi}{3}\right)$ 1

ii) at $t = 0$

$x = 4\sin\left(-\frac{\pi}{3}\right)$

$= -4\sin\frac{\sqrt{3}}{2}$

Approx 3m to the left of the origin

$\dot{x} = 4\cos\left(3t - \frac{\pi}{3}\right).3$

$= 12\cos\left(3t - \frac{\pi}{3}\right)$

at $t = 0$

$\dot{x} = 12\cos\left(-\frac{\pi}{3}\right)$

$= 6\ m/s$

Heading towards the origin

$\ddot{x} = -12\sin\left(3t - \frac{\pi}{3}\right).3$

$= -36\sin\left(3t - \frac{\pi}{3}\right)$

at $t = 0$

$\ddot{x} = -36\sin\left(-\frac{\pi}{3}\right)$

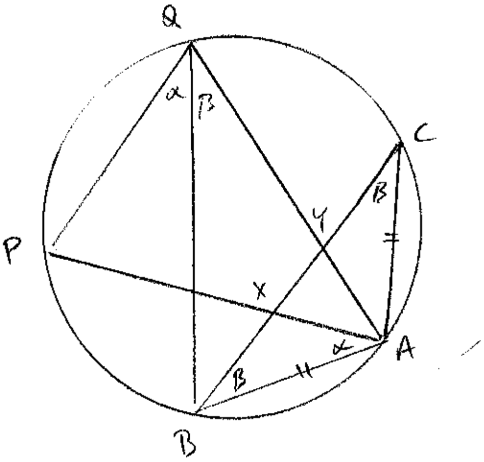
QUESTION 13 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
c) con't $\ddot{x} = -36\sin\left(-\frac{\pi}{3}\right)$ $= 18\sqrt{3}\ m/s^2$	2/	Accelerating towards the origin
ii) at $x = -2$ $-2 = 4\sin\left(3t - \frac{\pi}{3}\right)$ $-\frac{1}{2} = \sin\left(3t - \frac{\pi}{3}\right)$ $-\frac{\pi}{6} = 3t - \frac{\pi}{3}$ $\frac{\pi}{6} = 3t$ $t = \frac{\pi}{18}$ $t = \frac{\pi}{18}$ is the first time the particle is at $x = 2$ moving towards the origin	3/	

QUESTION 14 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
a) $\log_e(2\log_e x) = 1$ $2\log_e x = e$ $\log_e x^2 = e$ $x^2 = e^e$ $x = \sqrt{e^e}$ $= e^{\frac{e}{2}}$	2	
alternatively, $\log_e(2\log_e x) = 1$ $2\log_e x = e$ $\log_e x = \frac{e}{2}$ $x = e^{\frac{e}{2}}$ $= \sqrt{e^e}$		

QUESTION 14 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
b)		
		
i) $\angle PQB = \alpha$ (Angles at the circumfrence subtented by the same arc (PB) are equal)	1	
ii) $BA = CA$ (Given) $\therefore \triangle ACB$ is isosceles $\triangle$ ($BA = CA$) $\angle BCA = \angle ACB$ (Base angles of an isoscreles $\triangle$ are equal). $\therefore \angle AQB = \beta$ (Angles at the circumfrence subtended by the same arc (AB) are equal)	1	
iii) $\angle XBA = \beta$ (Given) $\angle XAB = \alpha$ (Given) $\angle AXB = 180 - (\alpha + \beta)$ (Angle sum of a triangle) $\angle PXY = 180 - (\alpha + \beta)$ (Vertically opposite angles are equal)		

QUESTION 14 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
$\angle PQY = \alpha + \beta$		
$\angle PQY + \angle PXY = \alpha + \beta + 180 - (\alpha + \beta)$		
$= 180$		
$\therefore XYQP$ is a cyclic quad as opposite angles are supplementary.	3/	
c) i) $T = A + Be^{kt}$		
$\frac{dT}{dt} = Be^{kt} \cdot k$		
$= kBe^{kt}$		
now $Be^{kt} = T - A$		
$\therefore \frac{dT}{dt} = k(T - A)$	1	
ii) at $t = 0$ $A = 25$ $T = 90$		
$90 = 25 + Be^{k \times 0}$		
$= 25 + B$		
$B = 65$		
$\therefore T = 25 + 65e^{kt}$		
at $t = 2$ $T = 50$		
$50 = 25 + 65e^{2k}$		
$e^{2k} = \frac{25}{65}$		

QUESTION 14 EXAM 2016 HSC Ext 1 Task 4

Solution	Mark	Comment
$2k = \log_e \frac{25}{65}$		
$k = \frac{1}{2} \log_e \frac{5}{13}$		-0.4778
at $t = 3$		
$T = 25 + 65e^{\left(\frac{3}{2} \log_e \frac{5}{13}\right)}$		
$T = 41°$	3/	
d) i) $\frac{9!}{2! \times 3! \times 4!} = 1260$	1/	
ii) P(red are together) $= \frac{8!}{3! \times 4!}$		
$= 280$		
P(Not together) $= \frac{1260 - 280}{1260}$		
$= \frac{980}{1260}$		
$= \frac{7}{9}$	2/	