QUESTION 1 *(12 marks)*

Marks

(a) By considering the expansion of $\tan(45^0 - 30^0)$, find the exact value of $\tan 15^0$. **2**

(b) Solve the inequality $x^2 + 2x \le 3$. **2**

(c) Given that A is the point (-3,1) and B is the point (4,2), find the co-ordinates of the point that divides AB externally in the ratio 3:4. **3**

(d) Using the substitution $u = e^x$ find the exact value of the definite integral **3**

$$\int_0^{\log_e 3} \frac{e^x dx}{\sqrt{1+e^x}} .$$

(e) Find the indefinite integral $\int \frac{4dx}{\sqrt{1-4x^2}}$ **2**

QUESTION 2 *(12 marks)* **Use a SEPARATE writing booklet**

(a) The polynomial $P(x) = ax^3 + bx^2 - 8x + 3$ has a factor of $(x - 1)$ and leaves a remainder of 15 when divided by $(x + 2)$. Find the values of a and b. **3**

(b) Solve for $0 \le \theta \le 2\pi$, $\sin 2\theta = \cos\theta$ **3**

(c) Solve $\frac{1-t}{1+2t} \le 1$ **3**

(d) If $\cos A = \frac{7}{9}$ and $\sin B = \frac{1}{3}$, where A and B are acute angles, prove that $A = 2B$ without finding the angles A and B. **3**

QUESTION 3 *(12 marks)* **Use a SEPARATE writing booklet**

Marks

(a) Find the equation of the concave upwards parabola with vertex (-1,-1) which passes through the origin and whose axis is parallel to the y axis. **2**

(b) (i) Show that the equation $e^{-x} = \sin 2x$ has a root lying between 1 and 2. **2**

(ii) By taking 1.5 as a first approximation, use Newton's Method once to obtain a better approximation to this root correct to two decimal places. **2**

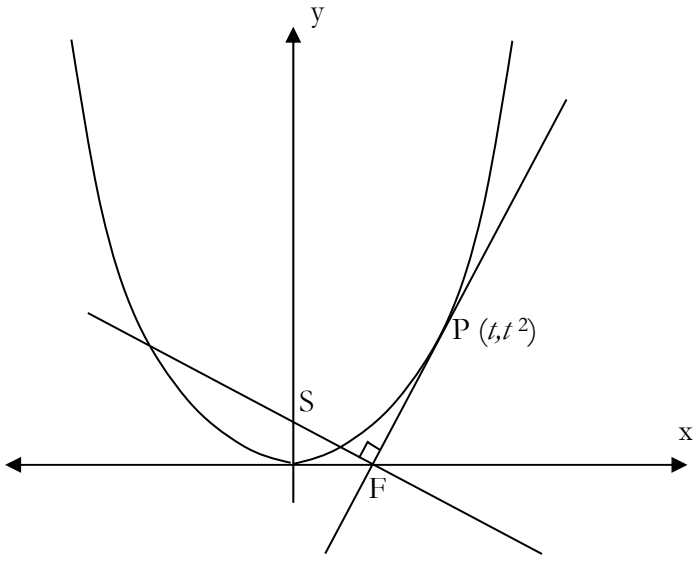
(c) Find the term independent of x in the expansion of $(2x + \frac{1}{x^2})^6$. **3**

(d) The area between the curve $y = \sin x$ and the x axis, for $0 \leq x \leq \pi$, is rotated about the x axis. Find the volume of the solid obtained. **3**

QUESTION 4 *(12 marks)* **Use a SEPARATE writing booklet**

(a)

Consider the parabola $y = x^2$.



(i) Show that the equation of the tangent to this parabola at the point P (t, t^2) is $y = 2tx - t^2$. **2**

/Q4 cont…Page 3

QUESTION 4 Continued

(iii) Show that F, the foot of the perpendicular from the focus to the tangent at P has co-ordinates $(\frac{t}{2}, 0)$. **2**

(iv) Find the cartesian equation of the locus of M, the mid-point of PF. **2**

(b) The velocity v ms^{-1} of a particle moving along the x axis in simple harmonic motion is given by $v^2 = 21 - 4x - x^2$ where x is the position of the particle.

(i) Between which two points on the x axis does the particle oscillate? **2**

(ii) What is the maximum velocity of the particle? **2**

QUESTION 5 *(12 marks)* **Use a SEPARATE writing booklet**

Marks

(a) The acceleration of a particle moving in a straight line is given by

$$\frac{d^2x}{dt^2} = -\frac{72}{x^2}$$

where x metres is the displacement from the origin after t seconds. When t = 0, the particle is 9 metres to the right of the origin with a velocity of 4 metres per second.

(i) Show that the velocity v of the particle in terms of x is given by $v = \frac{12}{\sqrt{x}}$ **4**

(ii) Find an expression for t in terms of x. **3**

(b) (i) Express $\cos\theta - \sin\theta$ in the form $A\cos(\theta + \alpha)$ where A > 0. **3**

(ii) Hence or otherwise solve the equation $\cos\theta - \sin\theta = 1$ for $0 \le \theta \le \pi$. **2**

/Q6…Page 4

QUESTION 6 *(12 marks)* **Use a SEPARATE writing booklet**

(a) In each of the following questions leave your answers as factorials.
Let each different arrangement of all of the letters of D E M A MD be called a word.

(i) How many words are possible? **1**

(ii) In how many of these words will the D's be separated? **2**

(b) (i) Use the Principle of Mathematical Induction to prove that for all integer $n \geq 1$, **4**

$$6\,(1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2) = n\,(n+1)\,(2n+1)$$

(ii) Hence evaluate $\lim_{n\to\infty}\left[\dfrac{1^2+2^2+3^2+.....+n^2}{n^3}\right]$ **1**

(c) A projectile is fired at an angle of 30° to the horizontal with velocity 10 m/sec from a platform 30 metres above ground level. If g is taken as 10 m/sec^2, the displacement of the particle at any time t secs is given by the equations

$x = 5\sqrt{3}\,t$ and $y = -5t^2 + 5t + 30$

(i) Find the speed of the projectile as it hits the ground. **3**

(ii) Also find the tangent of the acute angle at which it strikes the ground. **1**

QUESTION 7 *(12 marks)* **Use a SEPARATE writing booklet**

Marks

(a) (i) Differentiate $y = \tan^{-1}(\frac{1}{x})$, $x \neq 0$, and hence show that **3**

$$\frac{d}{dx}(\tan^{-1}x + \tan^{-1}(\frac{1}{x})) = 0 .$$

(ii) Hence or otherwise, find the value(s) of $\tan^{-1}x + \tan^{-1}(\frac{1}{x})$. **2**

(b) (i) When $(3 + 2x)^n$ is written as a polynomial in x, the coefficients of x^5 and x^6 have the same value. Find the value of n. **3**

(ii) By considering n to have a value of 10 prove that **4**

$$1 + \binom{10}{2}3^2 + \binom{10}{4}3^4 + \binom{10}{6}3^6 + \binom{10}{8}3^8 + 3^{10} = 2^9(2^{10} + 1)$$

END OF PAPER