Question 1 12 Marks **Marks**

(a) Evaluate the following definite integral $\int_{-2}^{2} \frac{dx}{x^2+4}$ **2**

.

(b) Solve $\frac{5}{x-3} \geq 2$. **2**

(c) Show that $\int \frac{dx}{\sqrt{9-4x^2}} = \frac{1}{2}\sin^{-1}\frac{2x}{3} + C$, where C is constant. **2**

(d) Find the general solution for $\cos 2\theta = \frac{\sqrt{3}}{2}$ **2**

(e) (i) Show that the derivative of $\frac{1+\sin x}{\cos x}$ is $\frac{1}{1-\sin x}$. **4**

(ii) Hence, deduce that $\int_{0}^{\pi/4} \frac{dx}{1-\sin x} = \sqrt{2}$

Question 2 12 Marks Start a new booklet

(a) Let $f(x) = x^3 + 5x^2 + 17x - 10$. The equation $f(x) = 0$ has only one real root. **4**

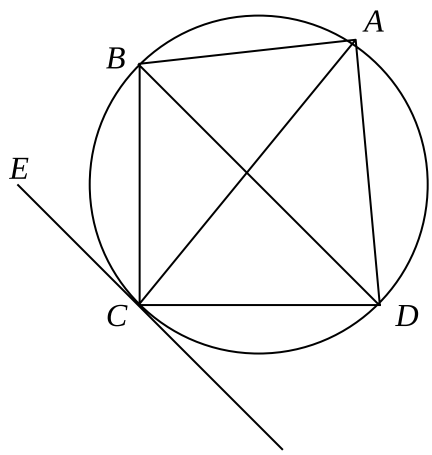
(i) Show that the root lies between 0 and 2.

(ii) Use one application of the 'halving the interval' method to find a smaller interval containing the root.

(iii) Which end of the smaller interval found in part (ii) is closer to the root? Briefly justify your answer.

(b) Evaluate $\int_{-1}^{2} \frac{x\,dx}{\sqrt{3-x}}$ using the substitution $x = 3 - u$. **4**

(c) ABCD is a cyclic quadrilateral in which AC bisects $\angle DAB$. CE is the tangent to the circle at C. Prove $CE \parallel DB$. **4**



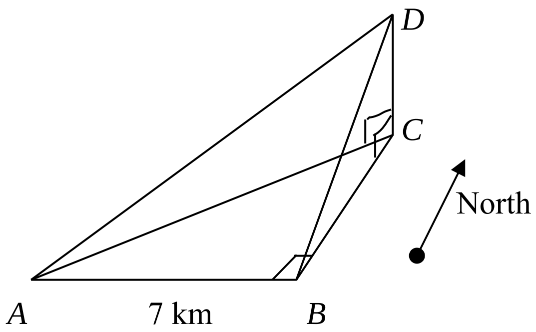
Question 3 page 2.

Question 3 12 Marks Start a new booklet **Marks**

(a) (i) Express $\sin\theta + \sqrt{3}\cos\theta$ in the form $R\sin\left(\theta + \phi\right)$. **4**

(ii) Hence, or otherwise, solve the equation $\sin\theta + \sqrt{3}\cos\theta = 1$ for values of θ between 0 and 2π .

(b) Cadel notices that the angle of elevation of the top of a mountain due north is 14°. Upon riding 7 kilometres due west, he finds that the angle of elevation of the top of the mountain is 10°. How high is the mountain? Give your answer correct to the nearest metre. **4**



$\angle DBC = 14^\circ$, $\angle DAC = 10^\circ$

(c) (i) Show that $\cos\theta = 2\cos^2\dfrac{\theta}{2} - 1$ **4**

(ii) Hence, show that $\dfrac{1}{1+\sec x} = 1 - \dfrac{1}{2}\sec^2\dfrac{x}{2}$.

(iii) Use part (ii) to deduce that $\displaystyle\int_0^{\pi/2} \frac{dx}{1+\sec x} = \frac{\pi}{2} - 1$.

Question 4 page 3.

Question 4 12 Marks Start a new booklet

(a) The sides of a cube are increasing at a rate of 2 cms^{-1}. Find at what rate the surface area is increasing when the sides are each 10 cm. **2**

(b) Prove by induction $9^{n+2}-4^{n}$ is divisible by 5, for $n \geq 1$ **4**

(c) 100 grams of cane sugar in water is converted into dextrose at a rate which is proportional to the amount unconverted at any time. That is, if *m* grams are converted in *t* minutes, then $\frac{dm}{dt}=k(100-m)$, where *k* is constant. **6**

(i) Show that $m=100+Ae^{-kt}$, where *A* is a constant, satisfies this equation.

(ii) Find the value of *A*.

(iii) If 40 grams are converted in the first 10 minutes, find how many grams are converted in the first 30 minutes.

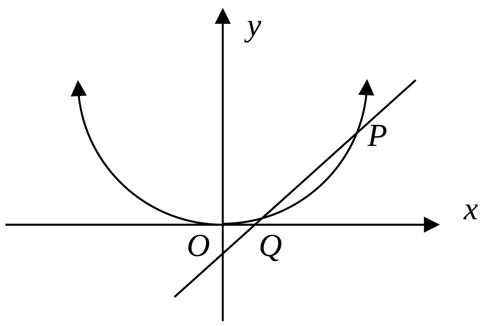
(iv) What is the limiting value of *m* as *t* increases indefinitely?

Question 5 12 Marks Start a new booklet

(a) Solve the equation $6x^3-17x^2-5x+6=0$, given that two of its roots have a product of -2. **3**

(b) Find the values of *a* and *b* so that $x^4+4x^3-x^2+ax+b$ is divisible by $(x-2)(x+1)$. **3**

(c) $P(2ap,ap^2)$ and $Q(2aq,aq^2)$ are points on the parabola $x^2=4ay$. **6**



(i) Show that the equation of the chord *PQ* is $\frac{(p+q)x}{2}-y=apq$

(ii) The line *PQ* passes through the point $(0,-a)$. Show that $pq=1$.

(iv) Hence, or otherwise, if *S* is the focus of the parabola, show that $\frac{1}{SP}+\frac{1}{SQ}=\frac{1}{a}$.

Question 6 page 4.

Question 6 12 Marks Start a new booklet

(a) By considering the expansion of $(1+x)^{2n}$ in ascending powers of x show that ${}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ \dots\ + {}^{2n}C_{2n} = 4^n$ **2**

(b) (i) Write the expansion of $(2+3x)^{20}$ in the form $\sum_{r=0}^{20} c_r x^r$, where c_r is the coefficient of x^r in the expansion. **4**

(ii) Show that $\dfrac{c_{r+1}}{c_r} = \dfrac{60-3r}{2r+2}$

(iii) Hence, or otherwise, find the greatest coefficient in the expansion of $(2+3x)^{20}$. Leave your answer in index form.

(c) Of a set of otherwise similar cards, ten are white, six are red and four are yellow. Three cards are taken at random. What is the probability that: **3**

(i) They are all different colours;

(ii) They are the same colour?

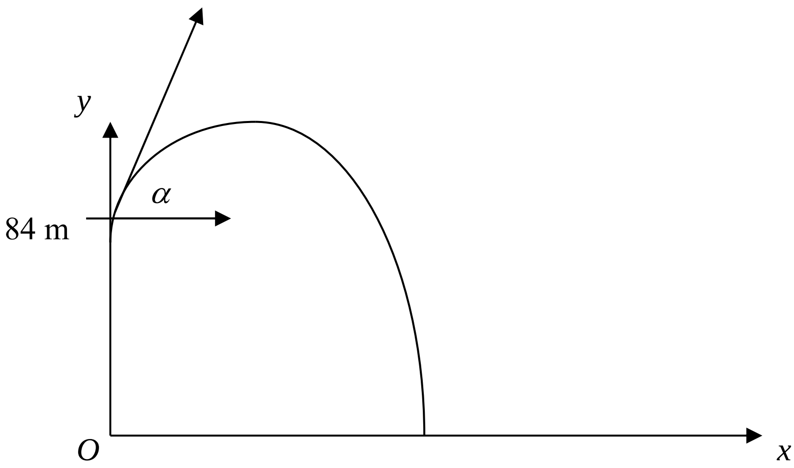
(d) (i) In how many ways can 2 boys and 1 girl be arranged in a row if the selection is made from 4 boys and 3 girls? **3**

(ii) In how many of these arrangements does a girl occupy the middle position?

Question 7 page 5.

Question 7 12 Marks Start a new booklet

(a) A particle is projected with speed 40 ms^{-1} from the top of a cliff 84 metres high at an angle of elevation $\alpha = \tan^{-1}\frac{4}{3}$. Assume that the equations of motion are $\ddot{x} = 0$ and $\ddot{y} = -10\ \text{ms}^{-2}$. **6**



(i) Derive the equations $x = 24t$ and $y = 84 + 32t - 5t^2$.

(ii) Hence or otherwise find the range on the horizontal plane through the foot of the cliff.

(iii) Find the speed of the body when it reaches this plane. Answer correct to two significant figures.

(b) A particle moves in simple harmonic motion about $x = 0$ and its displacement x metres, at time t seconds, is given by $x = a\sin n(t+\alpha)$. The particle moves with a period of 16 seconds. It passes through the centre of motion when $t = 2$ seconds. Its velocity is $4\,\text{ms}^{-1}$ when $t = 4$ seconds. **6**

(i) Show $\ddot{x} = -\frac{\pi^2}{64}x$.

(ii) Find the maximum displacement.

(iii) Find the speed of the particle when $t = 10$ seconds.

END OF EXAM

Question 1

(a) $\int_{-2}^{2} \frac{dx}{x^2+4} = 2\int_{0}^{2} \frac{dx}{x^2+4}$

$= 2\times\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2$

$= \tan^{-1}1 - \tan^{-1}0$

$= \frac{\pi}{4}$

(b) $\frac{5}{x-3} \geq 2, \qquad x \neq 3$

$\frac{5}{x-3}\times(x-3)^2 \geq 2\times(x-3)^2$

$5(x-3) \geq 2(x-3)^2$

$0 \geq 2(x-3)^2 - 5(x-3)$

$0 \geq (x-3)[2(x-3)-5]$

$0 \geq (x-3)(2x-11)$

$3 < x \leq \frac{11}{2}$ Note: $x \neq 3$

(c) $\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{4\left(\frac{9}{4}-x^2\right)}}$

$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{4}-x^2\right)}}$

$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(\left(\frac{3}{2}\right)^2-x^2\right)}}$

$= \frac{1}{2}\sin^{-1}\left(\frac{x}{3/2}\right)+C$

$= \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right)+C$

(d) $\cos 2\theta = \cos\frac{\pi}{6}$

$2\theta = 2n\pi \pm \frac{\pi}{6}, n \in \{\text{integers}\}$

$\theta = n\pi \pm \frac{\pi}{12}$

(e) (i) $\frac{d}{dx}\left(\frac{1+\sin x}{\cos x}\right)=\frac{vu'-uv'}{v^2}$, where $u=1+\sin x,\ v=\cos x$

$$=\frac{\cos x.\cos x-(1+\sin x).-\sin x}{\cos^2 x}$$

$$=\frac{\cos^2 x+\sin x+\sin^2 x}{\cos^2 x}$$

$$=\frac{1+\sin x}{\cos^2 x}$$

Note: $\cos^2 x+\sin^2 x=1$

$$=\frac{1+\sin x}{1-\sin^2 x}$$

$$=\frac{(1+\sin x)}{(1-\sin x)(1+\sin x)}$$

$$=\frac{1}{(1-\sin x)}$$

(ii) $\int_0^{\pi/4}\frac{dx}{1-\sin x}=\left[\frac{1+\sin x}{\cos x}\right]_0^{\pi/4}$

$$=\left[\frac{1+\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}-\frac{1+\sin 0}{\cos 0}\right]$$

$$=\frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}-\frac{1}{1}$$

$$=\frac{2}{\sqrt{2}}\left(1+\frac{\sqrt{2}}{2}\right)-1$$

$$=\frac{2}{\sqrt{2}}+1-1$$

$$=\frac{2}{\sqrt{2}}=\sqrt{2}$$

Question 2

(a) (i) $f(0)=-10, f(2)=52$. $\therefore f(\alpha)=0,\ 0<\alpha<2$

(ii) $f(1)=13,\ \therefore 0<\alpha<1$

(iii) $f\left(\frac{1}{2}\right)=-\frac{1}{8},\ \therefore \frac{1}{2}<\alpha<1$

α is closer to 1.

(b) $x = 3 - u,\ \therefore \dfrac{dx}{du} = -1$ or $dx = -du$

When $x = -1, u = 4$ and when $x = 2, u = 1$

$$-\int_4^1 \frac{(3-u)\,du}{\sqrt{u}} = \int_1^4 u^{-\frac{1}{2}}(3-u)\,du$$

$$= \int_1^4 \left(3u^{-\frac{1}{2}} - u^{\frac{1}{2}}\right) du$$

$$= \left[6u^{\frac{1}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_1^4$$

$$= 6.4^{\frac{1}{2}} - \frac{2}{3}.4^{\frac{3}{2}} - \left(6.1^{\frac{1}{2}} - \frac{2}{3}.1^{\frac{3}{2}}\right)$$

$$= 12 - \frac{16}{3} - \left(6 - \frac{2}{3}\right)$$

$$= \frac{4}{3}$$

(c) Let $\alpha = \angle DAC$

$\angle BAC = \angle DAC = \alpha$ (given)

$\angle BDC = \angle BAC = \alpha$ (angles in the same segment)

$\angle ECB = \angle BDC = \alpha$ (angle in the alternate segment)

$\angle CBD = \angle DAC = \alpha$ (angles in the same segment)

$\therefore \angle ECB = \angle CBD$

$\therefore EC \parallel DB$ (equal alternate angles)

<u>Question 3</u>

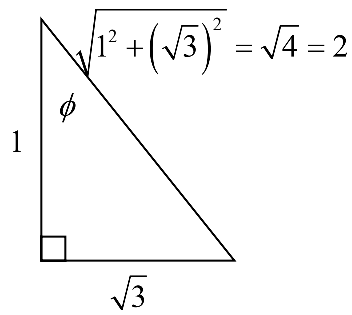
(a) (i)

$$\sin\theta + \sqrt{3}\cos\theta = 1.\sin\theta + \sqrt{3}\cos\theta$$

$$= 2\left(\frac{1}{2}.\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right)$$

$$= 2\left(\cos\frac{\pi}{3}.\sin\theta + \sin\frac{\pi}{3}.\cos\theta\right)$$

$$= 2\sin\left(\theta + \frac{\pi}{3}\right)$$



$$\sin\phi = \frac{\sqrt{3}}{2},\ \cos\phi = \frac{1}{2}$$

$$\therefore \phi = \frac{\pi}{3}$$

(ii) $\sin\theta+\sqrt{3}\cos\theta=1$

$$2\sin\left(\theta+\frac{\pi}{3}\right)=1$$

$$\sin\left(\theta+\frac{\pi}{3}\right)=\frac{1}{2}$$

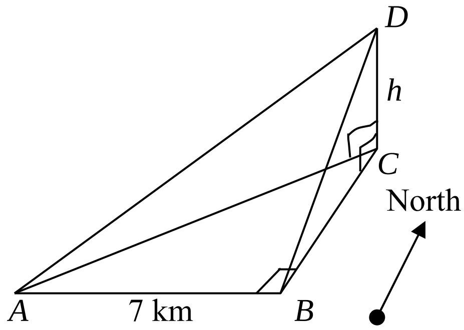
$$\theta+\frac{\pi}{3}=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\ \ldots$$

$$\theta=\frac{\pi}{6}-\frac{\pi}{3},\frac{5\pi}{6}-\frac{\pi}{3},\frac{13\pi}{6}-\frac{\pi}{3},\ \ldots$$

$$\theta=-\frac{\pi}{6},\frac{\pi}{2},\frac{11\pi}{6},\ \ldots$$

$$\theta=\frac{\pi}{2},\frac{11\pi}{6},\ \text{for } 0\le\theta\le2\pi$$

(b)



$\angle DBC=14^\circ,\ \angle DAC=10^\circ$

$$\tan14^\circ=\frac{h}{BC},\ \therefore BC=\frac{h}{\tan14^\circ}$$

$$\tan10^\circ=\frac{h}{AC},\ \therefore AC=\frac{h}{\tan10^\circ}$$

$$AC^2=BC^2+7^2$$

$$\frac{h^2}{\tan^2 10^\circ}-\frac{h^2}{\tan^2 14^\circ}=49$$

$$h^2\left(\frac{1}{\tan^2 10^\circ}-\frac{1}{\tan^2 14^\circ}\right)=49$$

$$h=\sqrt{\frac{49}{\frac{1}{\tan^2 10^\circ}-\frac{1}{\tan^2 14^\circ}}}$$

= 5.432 km (to nearest metre)

(c) (i) $\cos\theta=\cos\left(2\times\frac{\theta}{2}\right)$

$$=2\cos^2\frac{\theta}{2}-1$$

(ii) $\dfrac{1}{1+\sec x}=\dfrac{1}{1+\dfrac{1}{\cos x}}$.

$=\dfrac{1}{\dfrac{\cos x+1}{\cos x}}$

$=\dfrac{\cos x}{\cos x+1}$

$=\dfrac{2\cos^2\dfrac{x}{2}-1}{2\cos^2\dfrac{x}{2}-1+1}$

$=\dfrac{2\cos^2\dfrac{x}{2}}{2\cos^2\dfrac{x}{2}}-\dfrac{1}{2\cos^2\dfrac{x}{2}}$

$=1-\dfrac{1}{2}\sec^2\dfrac{x}{2}$

(ii) $\int_0^{\pi/2}\left(1-\dfrac{1}{2}\sec^2\dfrac{x}{2}\right)dx$

$=\left[x-\tan\dfrac{x}{2}\right]_0^{\pi/2}$

$=\left[\dfrac{\pi}{2}-\tan\dfrac{\left(\dfrac{\pi}{2}\right)}{2}\right]-\left[0-\tan 0\right]$

$=\dfrac{\pi}{2}-\tan\dfrac{\pi}{4}$

$=\dfrac{\pi}{2}-1$

Question 4

(a) Let x = length of one side of the cube. Let S = surface area = $6x^2$.

$\dfrac{dx}{dt}=2cms^{-1}$, $\dfrac{dS}{dx}=12x$

$\dfrac{dS}{dt}=\dfrac{dS}{dx}.\dfrac{dx}{dt}$

$=12x.2$

$=24x,$ at $x=10$

$=240cm^2s^{-1}$

(b) Step 1 Prove result true for $n = 1$

$9^{1+2} - 4^1 = 9^3 - 4$

$= 725$

$= 145 \times 5$

Step 2 Assume the result is true for $n = k$, where k is a positive integer

i.e. $9^{k+2} - 4^k = 5P$, where P is a positive integer

Step 3 Prove the result is true for $n = k + 1$

i.e. Prove $9^{k+1+2} - 4^{k+1} = 5M$

$\text{LHS} = 9.9^{k+2} - 4.4^{k+1}$

$= 9\left(9^{k+2} - 4^k\right) + 5.4^k$

$= 9(5P) + 5.4^k$, from assumption

$= 5\left(9P - 4^k\right)$

$= 5M,\ M = 9P - 4^k$

Step 4 The result has been proved true for $n = 1, n = 1+1 = 2, n = 2+1 = 3,$ etc.

Hence, by the principle of Mathematical Induction, the result is true for all positive integers.

(c) (i) $m = 100 + Ae^{-kt}$

$\frac{dm}{dt} = -ke^{-kt}$

$= -k(m - 100)$

$= k(100 - m)$

(ii) When $t = 0$, $m = 0$.

$\therefore 0 = 100 + Ae^0,\ A = -100$

(iii) When $t = 10$, $m = 40$

$\therefore 40 = 100 - 100e^{-10k}$

$e^{-10k} = \frac{60}{100}$

$-10k = \ln 0.6$

$k = -\frac{1}{10}\ln 0.6$

Let $t = 30$

$m = 100 - 100e^{-\left(-\frac{1}{10}\ln 0.6\right)30}$

$= 100 - 100e^{3\ln 0.6}$

$= 100 - 100e^{\ln\left(0.6^3\right)}$

$= 100 - 100\left(0.6^3\right)$

$= 78.4$ g

(iv) $\lim_{t\to\infty} m = \lim_{t\to\infty}\left(100 - 100e^{\frac{t}{10}\ln 0.6}\right)$

$= 100 - 0$

$= 100$ g

Question 5

(a) $6x^3-17x^2-5x+6=0$. Let the roots be $\alpha, \frac{-2}{\alpha}, \beta$.

$\alpha+\frac{-2}{\alpha}+\beta=\frac{17}{6}$ ———— [1]

$\alpha.\frac{-2}{\alpha}.\beta=\frac{-6}{6}=-1$

$-2\beta=-1$

$\beta=\frac{1}{2}$ ———— [2]

Sub [2] into [1].

$\alpha+\frac{-2}{\alpha}+\frac{1}{2}=\frac{17}{6}$

$6\alpha^2+-12+3\alpha=17\alpha$

$3\alpha^2-7\alpha-6=0$

$(3\alpha+2)(\alpha-3)=0$

$\alpha=\frac{-2}{3}$ or 3

$\therefore$ roots are $\frac{-2}{3}, 3, \frac{1}{2}$

(b) If divisible by $(x-2)(x+1)$ then $f(2)=f(-1)=0$

$\therefore 2^4+4(2)^3-2^2+2a+b=0$ and $(-1)^4+4(-1)^3-(-1)^2-a+b=0$

$2a+b=-44$ ____________ 1.

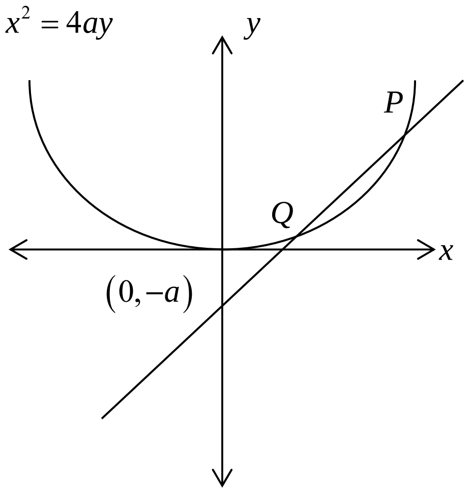
$a-b=-4$ ____________ 2.

Solve simultaneously

$a=-16, b=-12$

(c)

(i) $$m_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$$
$$= \frac{a\left(p^2 - q^2\right)}{2a\left(p - q\right)}$$
$$= \frac{a\left(p - q\right)\left(p + q\right)}{2a\left(p - q\right)}$$
$$= \frac{p + q}{2}$$



Equation PQ:

$$y - ap^2 = \frac{p + q}{2}\left(x - 2ap\right)$$
$$= \left(\frac{p + q}{2}\right)x - \left(\frac{p + q}{2}\right)2ap$$
$$= \left(\frac{p + q}{2}\right)x - ap^2 - apq$$
$$\therefore \left(\frac{p + q}{2}\right)x - y = apq$$

(ii) $$\left(\frac{p + q}{2}\right)x - y = apq$$

Let $x = 0, y = -a$

$$0 - \left(-a\right) = apq$$
$$a = apq$$
$$pq = 1$$

(iii) $SP=\sqrt{\left(2ap-0\right)^2+\left(ap^2-a\right)^2}$

$=\sqrt{4a^2p^2+a^2p^4-2a^2p^2+a^2}$

$=\sqrt{a^2\left(p^4+2p^2+1\right)}$

$=\sqrt{a^2\left(p^2+1\right)^2}$

$=a\left(p^2+1\right)$

Similarly $SQ=a\left(q^2+1\right)$

Now, $\frac{1}{SP}+\frac{1}{SQ}=\frac{1}{a\left(p^2+1\right)}+\frac{1}{a\left(q^2+1\right)}$

$=\frac{1}{a}\left[\frac{1}{\left(p^2+1\right)}+\frac{1}{\left(q^2+1\right)}\right]$

$=\frac{1}{a}\left[\frac{\left(q^2+1\right)+\left(p^2+1\right)}{\left(p^2+1\right)\left(q^2+1\right)}\right]$

$=\frac{1}{a}\left[\frac{p^2+q^2+2}{\left(p^2+1\right)\left(q^2+1\right)}\right]$

$=\frac{1}{a}\left[\frac{p^2+q^2+2}{p^2q^2+p^2+q^2+1}\right]$

$=\frac{1}{a}\left[\frac{p^2+q^2+2}{p^2+q^2+2}\right],\quad \because pq=1$

$=\frac{1}{a}$

Question 6

(a) $\left(1+x\right)^{2n}={}^{2n}C_0+{}^{2n}C_1x+{}^{2n}C_2x^2+......{}^{2n}C_{2n}x^{2n}$

Let $x = 1$

$\left(1+1\right)^{2n}={}^{2n}C_0+{}^{2n}C_1 1+{}^{2n}C_2 1^2+......{}^{2n}C_{2n}1^{2n}$

$\left(2\right)^{2n}=\left(2^2\right)^n={}^{2n}C_0+{}^{2n}C_1+{}^{2n}C_2+......{}^{2n}C_{2n}$

$\therefore 4^n={}^{2n}C_0+{}^{2n}C_1+{}^{2n}C_2+......{}^{2n}C_{2n}$

(b) (i) $\left(2+3x\right)^{20}={}^{20}C_0 2^{20}+{}^{20}C_1 2^{19}\left(3x\right)^1+{}^{20}C_2 2^{18}\left(3x\right)^2+......{}^{20}C_{20}\left(3x\right)^{20}$

$=\sum_{r=0}^{20}{}^{20}C_r 2^{20-r}\left(3x\right)^r$

$=\sum_{r=0}^{20}{}^{20}C_r 2^{20-r}3^r x^r$

$=\sum_{r=0}^{20}c_r x^r$, where $c_r={}^{20}C_r 2^{20-r}3^r$

(ii) $\frac{c_{r+1}}{c_r} = \frac{{}^{20}C_{r+1}2^{20-(r+1)}3^{r+1}}{{}^{20}C_r 2^{20-r}3^r}$

$= \frac{\frac{20!}{(20-(r+1))!(r+1)!}2^{20-r-1}3^{r+1}}{\frac{20!}{(20-r)!r!}2^{20-r}3^r}$

$= \frac{20!}{(19-r)!(r+1)!} \times \frac{(20-r)!r!}{20!} \times \frac{3}{2}$

$= \frac{(20-r)3}{(r+1)2}$

$= \frac{60-3r}{2r+2}$

(iii) Let $\frac{c_{r+1}}{c_r} > 1$

$\therefore \frac{60-3r}{2r+2} > 1$

$60 - 3r > 2r + 2$

$5r < 58$

$r < 11.6$

$\therefore c_{r+1} > c_r$ when $r = 0,1,2,3,4,...,11$

$\therefore c_{12} > c_{11} > c_{10} > c_9 ... > c_2 > c_1 > c_0$

and $\therefore c_{r+1} < c_r$ when $r = 12,13,14,...,19$

$\therefore c_{12} > c_{13} > c_{14} > c_{15} > c_{16} > c_{17} > c_{18} > c_{19}$

Maximum coefficient $= c_{12} = {}^{20}C_{12}2^8 3^{12}$

(c) (i) Number of possible selections $= {}^{20}C_3$

Number of ways of selecting one of each colour

$= {}^{10}C_1.{}^{6}C_1.{}^{4}C_1$

P(different colours) $= \frac{{}^{10}C_1.{}^{6}C_1.{}^{4}C_1}{{}^{20}C_3} = \frac{4}{19}$

(ii) P(same colour) = P(3 white or3 red or 3 yellow)

$= \frac{{}^{10}C_3 + {}^{6}C_3 + {}^{4}C_3}{{}^{20}C_3}$

$= \frac{12}{95}$

(d) (i) ${}^{4}C_2.{}^{3}C_1.3! = 108$

(ii) Girl occupies centre position in one third of arrangements

$= \frac{1}{3} \times 108$

$= 36$

Question 7

(a) (i)

Horizontal:

$\ddot{x} = 0$

$\dot{x} = c$

When $t = 0, \dot{x} = 40\cos\alpha$

$\therefore c = 40\cos\alpha$

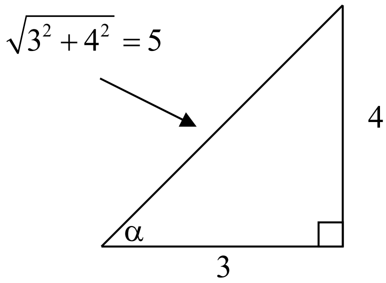
$\therefore \dot{x} = 40\times\frac{3}{5}$

$\therefore \dot{x} = 24$

$x = 24t + c_1$

When $t = 0, x = 0, \therefore c_1 = 0$

$\therefore x = 24t$



$\therefore \cos\alpha = \frac{3}{5},\ \sin\alpha = \frac{4}{5}$

Vertical

$\ddot{y} = -10$

$\dot{y} = -10t + k$

When $t = 0, \dot{y} = 40\sin\alpha$

$\therefore k = 40\sin\alpha$

$= 40\times\frac{4}{5} = 32$

$\therefore \dot{y} = 32 - 10t$

$y = 32t - 5t^2 + k_1$

When $t = 0, y = 84, \therefore k_1 = 84$

$\therefore y = 32t - 5t^2 + 84$

(ii) Let $y = 0$

$40t.\frac{4}{5} - 5t^2 + 84 = 0$

$32t - 5t^2 + 84 = 0$

$5t^2 - 32t - 84 = 0$

$(5t - 42)(t + 2) = 0$

$t = \frac{42}{5}$ or -2, but $t > 0$

$\therefore t = 8.4$ seconds

Now, $x = 24t$ and the time of flight, $t = 8.4$.

Range in horizontal plane $= 24\times 8.4 = 201.6$ m

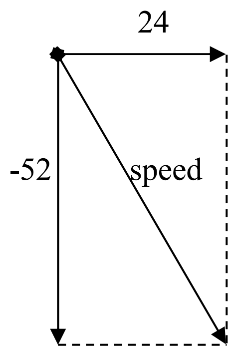
(iii) $\dot{x} = 24 \text{ ms}^{-1}$

$\dot{y} = 32 - 10t$

$= -52\text{ms}^{-1}$

Speed $= \sqrt{24^2 + (-52)^2}$

$= 57\text{ms}^{-1}$ (two sig. fig.)



(b) (i) $\dfrac{2\pi}{n} = 16, \quad \therefore n = \dfrac{\pi}{8}$

$\therefore x = a \sin \dfrac{\pi}{8}(t + \alpha)$

$\therefore \dot{x} = \dfrac{\pi}{8} a \cos \dfrac{\pi}{8}(t + \alpha)$

$\therefore \ddot{x} = -\left(\dfrac{\pi}{8}\right)^2 a \sin \dfrac{\pi}{8}(t + \alpha)$

$= -\left(\dfrac{\pi}{8}\right)^2 x$

$= -\dfrac{\pi^2}{64} x$

(ii) When $t = 2$, $x = 0$ and when $t = 4$, $\dot{x} = 4$

$0 = a \sin \dfrac{\pi}{8}(2 + \alpha)$

$\therefore \dfrac{\pi}{8}(2 + \alpha) = 0$

$\therefore \alpha = -2$

$4 = \dfrac{\pi}{8} a \cos \dfrac{\pi}{8}(4 - 2)$

$4 = \dfrac{\pi}{8} a \cos \dfrac{\pi}{4}$

$4 = \dfrac{\pi a}{8\sqrt{2}}$

$a = \dfrac{32\sqrt{2}}{\pi}$

$\therefore x = \dfrac{32\sqrt{2}}{\pi} \sin \dfrac{\pi}{8}(t - 2)$

Maximum displacement = $\dfrac{32\sqrt{2}}{\pi}$ m

(iii) $\dot{x} = \frac{\pi}{8} \cdot \frac{32\sqrt{2}}{\pi} \cos \frac{\pi}{8}(t-2)$

$= 4\sqrt{2} \cos \frac{\pi}{8}(t-2)$

Now, when $t = 10$, $\dot{x} = 4\sqrt{2} \cos \pi \ \text{ms}^{-1}$

Speed $= \left|\dot{x}\right| = \left|4\sqrt{2} \cos \pi\right| \text{ms}^{-1}$

$= \left|-4\sqrt{2}\right| \text{ms}^{-1}$

$= 4\sqrt{2} \text{ms}^{-1}$