Student Number

NORMANHURST BOYS' HIGH SCHOOL
NEW SOUTH WALES

2014

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Begin each question on a separate writing booklet

Total marks - 70

Section I Pages 2-5

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II Pages 6-11

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

Students are advised that this is a school-based examination only and cannot in any way guarantee the content or format of future Higher School Certificate Examinations.

SECTION I

Use the multiple-choice answer sheet provided for Questions 1 – 10.

Allow about 15 minutes for this section.

1. A particle moves in a straight line. Its position at any time t is given by

$x = 3\cos 2t + 4\sin 2t$.

The acceleration in terms of x is:

(A) $\ddot{x} = -3x$

(B) $\ddot{x} = -4x$

(C) $\ddot{x} = -16x^2$

(D) $\ddot{x} = -6\cos 2x + 8\sin 2x$

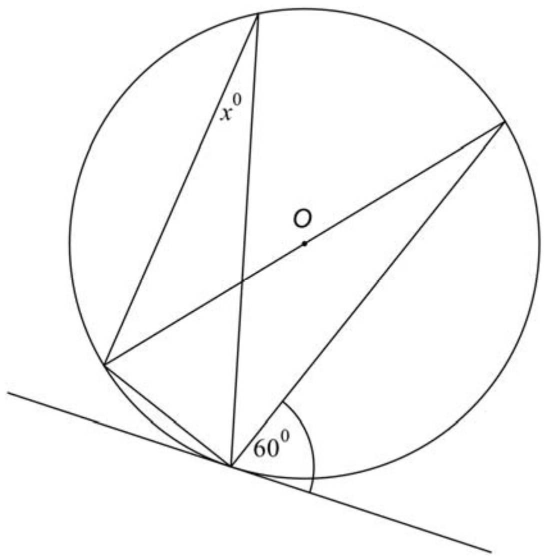
2. O is the centre of the circle. Find the value of x :

(A) 30°

(B) 45°

(C) 60°

(D) 90°



3. The expansion needed to show $\sin 75° = \frac{\sqrt{6} + \sqrt{2}}{4}$ is:

(A) $\sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$

(B) $\sin(45° + 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$

(C) $\sin(100° - 25°) = \sin 100° \cos 25° + \cos 100° \sin 25°$

(D) $\tan(45° + 30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$

4. If α, β and γ are the roots of $x^3 - 7x^2 + 9x - 15 = 0$. Find the value of $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$.

(A) $-\frac{7}{15}$

(B) $\frac{7}{15}$

(C) $\frac{9}{15}$

(D) $\frac{15}{7}$

5. The Cartesian equation of the tangent, at $t = -3$, to the parabola $x = t - 3$, $y = t^2 + 2$ is:

(A) $6x + y + 25 = 0$

(B) $6x + y + 36 = 0$

(C) $6x - y - 25 = 0$

(D) $6x + 2y - 25 = 0$

6. Using one application of Newton's method with a starting value of $x = 3$, find the approximate value of $\sqrt[3]{33}$.

(A) $2\frac{7}{9}$

(B) $3\frac{1}{5}$

(C) $3\frac{2}{9}$

(D) $3\frac{2}{3}$

7. Which of the following is an expression for $\int \cos^2 x \sin x dx$?

(A) $2\cos x \sin x + c$

(B) $\cos^3 x + c$

(C) $\frac{1}{3}\cos^3 x + c$

(D) $-\frac{1}{3}\cos^3 x + c$

8. $\int \frac{dx}{\sqrt{1-3x^2}} =$

(A) $\left(\sin^{-1} 3x\right) + C$

(B) $\left(\tan^{-1} 3x\right) + C$

(C) $\frac{1}{\sqrt{3}}\left(\tan^{-1} \sqrt{3}\ x\right) + C$

(D) $\frac{1}{\sqrt{3}}\left(\sin^{-1} \sqrt{3}\ x\right) + C$

9. Find the derivative of : $e^{1 + ln\ x}$

(A) $e^{1 + ln\ x}$

(B) $e^{1 + \frac{1}{x}}$

(C) $x^{-1}\ e^{1+ ln\ x}$

(D) $x\ e^{1 + ln\ x}$

10. Using the substitution $u = 1 - x^3$, evaluate $\int_0^1 x^2\sqrt{1 - x^3}\ dx.$

(A) $-\frac{1}{9}$

(B) $\frac{1}{9}$

(C) $\frac{2}{9}$

(D) $\frac{1}{3}$

Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) A polynomial is given by $P(x) = x^3 + ax^2 + bx - 18$. **2**
Find the values of a and b if $(x + 2)$ is a factor of $P(x)$ and the remainder when $P(x)$ is divided by $(x - 1)$ is -24.

(b) A and B are the points (1, 4) and (5, 2) respectively. Find the coordinates of the point M which divides the interval AB externally in the ratio 2:3. **1**

(c) Differentiate $y = \cos^{-1}(3x + 2)$. State the domain for which x is defined in the given relationship. **2**

(d) Find the volume of the solid of revolution formed when the curve $y = x^3 + 1$ is rotated about the y-axis from $y = 0$ to $y = a$. **3**

(e) Find the area bounded by the curve $y = \dfrac{1}{9 + x^2}$, the x axis and the lines $x = 0$ and $x = \sqrt{3}$. **3**

(f) Sketch $y = \dfrac{x - 3}{x^2}$ showing all the main features including stationary points, inflexions and asymptotes. **4**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Show that $3\sin x\cos x = \frac{3}{2}\sin 2x$ **1**

(ii) Hence or otherwise, find the exact value of $\int_0^{\frac{\pi}{2}} 9\ \sin^2 x\cos^2 x\,dx$. **2**

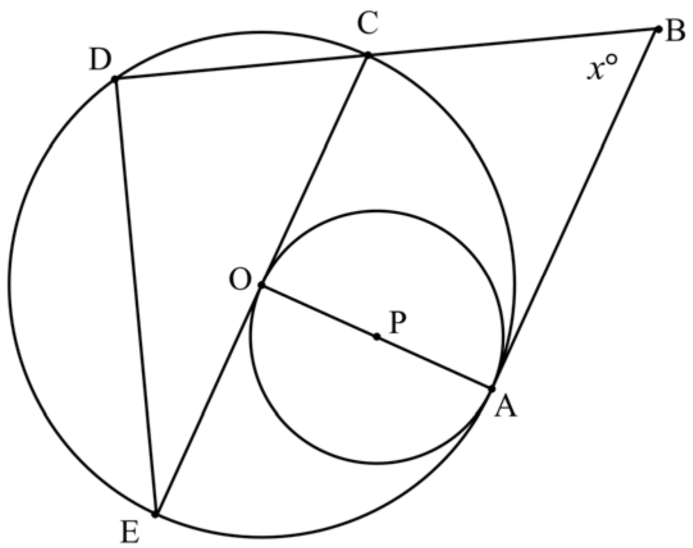
(b) (i) Use the identity $sin(\theta + 2\theta) = sin\theta\, cos2\theta + cos\theta\, sin2\theta$, to prove that **2**

$$sin3\theta = 3sin\theta - 4sin^3\theta.$$

(ii) Hence solve the equation $sin\ 3\theta = 2\ sin\theta$ for $0 \le \theta \le 2\pi$ **3**

(c) A circle, centre O, passes through the points A, C, D and E. **2**
Another circle, centre P, passes through the points A and O.
CE is a tangent to the circle centre P, with point of contact at O.
AB is a tangent to both circles with point of contact at A.
$\angle CBA = x°$.

Show that $\angle CED = (90 - x)°$



Question 12 continues on page 8

Question 12 (continued)

(d) Greg designs a sky-diving simulator for a video game. He simulates the rate of change of the velocity of the skydivers as they fall by:

$$\frac{dV}{dt} = -k(V-P), \text{ where } k \text{ and } P \text{ are constants.}$$

The constant P represents the terminal velocity of the skydiver in the prone position which is 55 m/s.

(i) Show that $V = P + Ae^{-kt}$ is a solution for this rate of change. **1**

(ii) Initially the velocity of the skydiver is 0 m/s and his velocity after 10 seconds is 27 m/s. Find values for A and k. **2**

(iii) Find the velocity of the skydiver after 17 seconds. **1**

(iv) How long does it take the skydiver to reach a velocity of 50 m/s? **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) By expressing $\cos x + \sin x$ in the form of $r \sin\ (x + \alpha)$, solve the equation $\sin x + \cos x = 1$ for $0 \le x \le 2\pi$. **2**

(b) Consider the function $f(x) = \dfrac{e^x}{4 + e^x}$, where $f\,'(x) = \dfrac{4\,e^x}{\left(4 + e^x\right)^2}$.

The function has no stationary points.

(i) Find any points of inflexion. **2**

(ii) Explain why $f(x)$ has an inverse function. **1**

(iii) Find the inverse function $y = f^{-1}(x)$. **2**

(c) A particle moves on a line so that its distance from the origin at time t is x.

(i) Prove that $\dfrac{d^2x}{dt^2} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$ where v denotes velocity. **2**

(ii) Find v^2 in terms of x if $\dfrac{d^2x}{dt^2} = -2x\,(x^2 - 20)$ and $v = 0$ at $x = 2$ **2**

(iii) Considering the positive displacement only for this particle, is the motion simple harmonic? Justify your answer. **1**

(d) Prove by mathematical induction that **3**

$$a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} = \frac{a\left(r^n - 1\right)}{r - 1}$$

for all a and r, where n is a positive integer.

End of Question 13

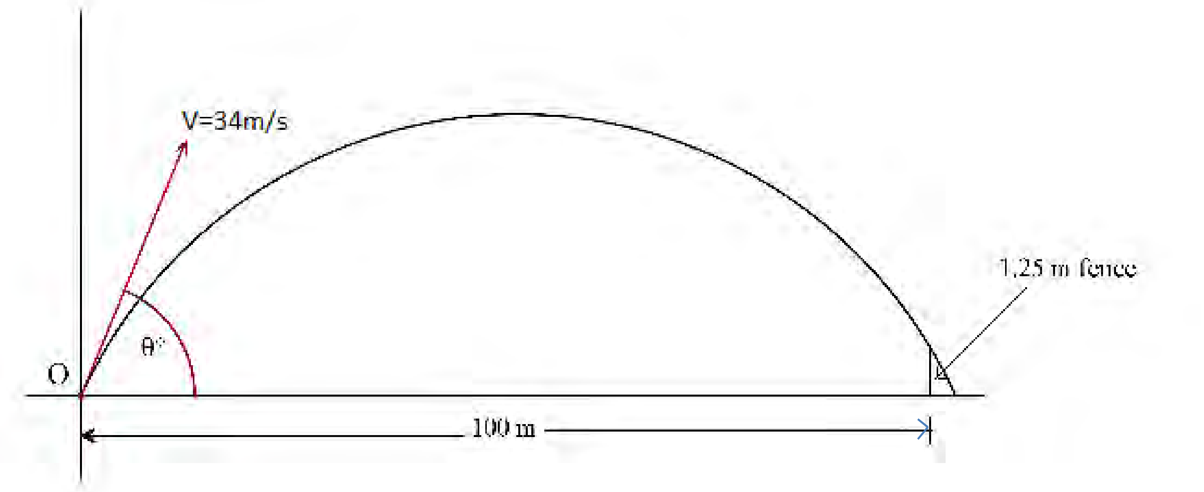
Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The cubic function $f(x) = x^3 + ax^2 + bx + c$ has a relative maximum at $x = \propto$ and a relative minimum at $x = \beta$. **1**

(i) Prove that $\propto + \beta = \frac{-2}{3} a$

(ii) Deduce that the point of inflexion occurs at $x = \frac{\propto + \beta}{2}$. **1**

(b) A ball is hit from the centre (O) of a field with a velocity of V = 34 m/s at an angle θ to the horizontal and towards a 1.25 metre high boundary fence which is 100 metres away.



(i) Derive the equations for horizontal and vertical displacement of the ball in flight. Air resistance may be neglected and acceleration can be taken as $-10\ ms^{-2}$. **2**

(ii) Show the ball just clears the boundary fence when: **2**

$$50000\tan^2\theta - 115600\tan\theta + 51445 = 0$$

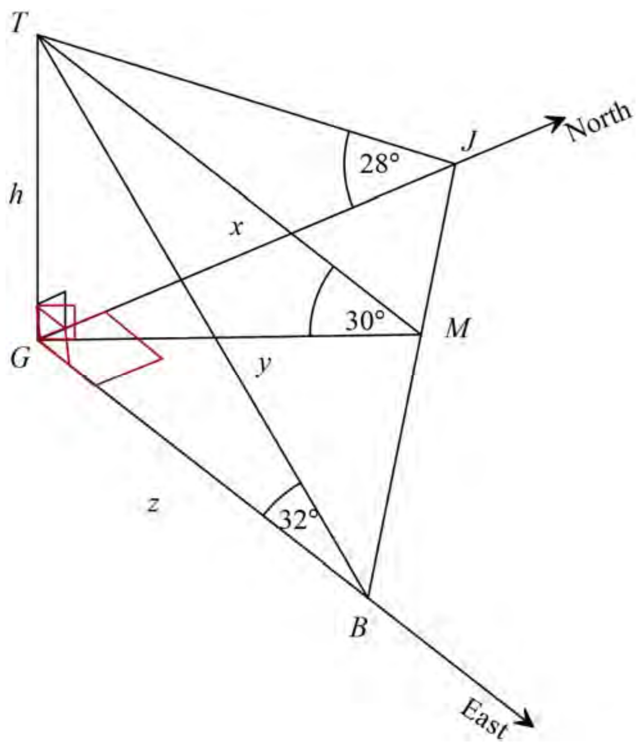
(iii) Between what two values does θ lie, if the ball was to clear the boundary fence? **2**

(iv) During the flight of the ball, a gust of wind blows across the ground and the horizontal velocity of the ball is decreased. **2**
Draw a sketch of what affect this will have on the ball.
Discuss without further calculations the effect this would have on the answer to part (iii) above.

Question 14 continues on page 10

Question 14 (continued)

(c) The Eiffel Tower (GT) is on flat ground in central Paris. Three friends Jordan, Maddy and Bella are observing the tower from a straight road on ground level. Jordan, at point J, is due north of the tower, Bella, at point B, is due east of the tower and Maddy, at point M is on the line of sight between Jordan and Bella. The angles of elevation to the summit of the tower from Jordan, Maddy and Bella are 28^{o}, 30^{o} and 32^{o} respectively. The distances to the base of the tower from Jordan, Maddy and Bella are x, y and z respectively.



(i) Show that the bearing of Maddy from the base of the Eiffel Tower is $4° \, 20' \, T$. **3**

(ii) Write an expression which is independent of h, for the ratio $\dfrac{MB^2}{JM^2}$. **2**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section I –10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple choice answer sheet provided at the back.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

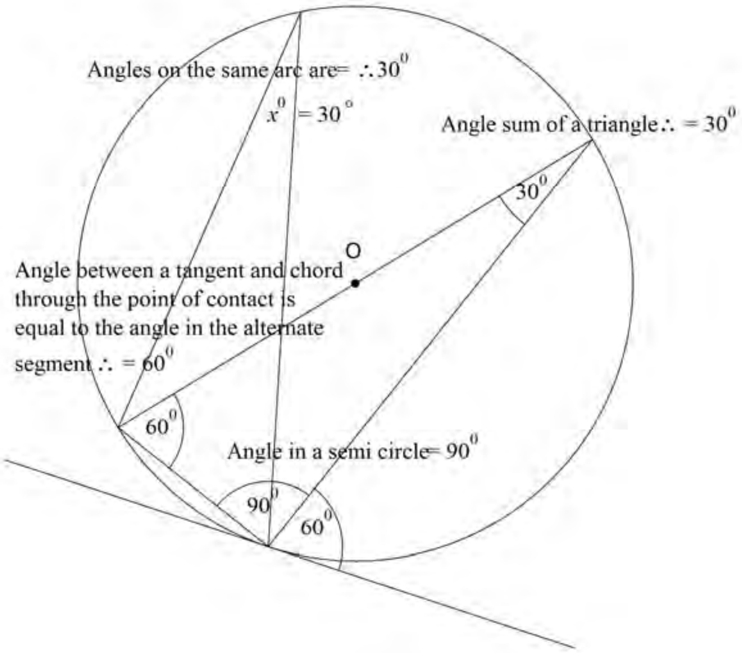
NBHS
HSC TRIAL

2014

Mathematics

Extension 1

SOLUTIONS

Multiple Choice Worked Solutions		
No	**Working**	**Answer**
1	$x = 3\cos 2t + 4\sin 2t$ $\dot{x} = -6\sin 2t + 8\cos 2t$ $\ddot{x} = -12\cos 2t - 16\sin 2t$ $\ddot{x} = -4(3\cos 2t + 4\sin 2t)$ $\ddot{x} = -4x$	B
2		**A**
3	$\sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$	**A**
4	$\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ $= \frac{-\frac{b}{a}}{-\frac{d}{a}}$ $= \frac{-b}{-d}$ $= \frac{-7}{-15}$ $= \frac{7}{15}$	**B**
5	$x = t - 3 \quad y = t^2 + 2$ when $t = -3$ $x = -6$ $y = 11$ $\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = 2t$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $\frac{dy}{dx} = 2t$ when $t = -3$ $\frac{dy}{dx} = -6$ Tangent $y - 11 = -6(x + 6)$ $6x + y + 25 = 0$	**A**

6	Want x such that $x=\sqrt[3]{33}$ $x^3=33$ $x^3-33=0$ $f(x)=x^3-33$ begin with $f(3)=-6$ $f'(x)=3x^2\ \ f'(3)=27$ If $x_1=3$ $x_2=3-\left(-\frac{6}{27}\right)$ $=3\frac{2}{9}$	**C**
7	$\int cos^2x\ \sin x\ dx$ $=\int -\sin x\ \ cos^2x\ dx$ $=-\frac{1}{3}cos^3x+c$	**D**
8	$\int \frac{dx}{\sqrt{1-3x^2}} = \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{\frac{1}{3}-x^2}}$ $\frac{1}{\sqrt{3}}\left[\sin^{-1}\sqrt{3}\ x\right]+C$	**D**
9	$y=e^{1+\ln x}$ $\frac{dy}{dx}=\ e^{1+\ln x}.\frac{1}{x}$ $=\ x^{-1}e^{1+\ln x}$ $=\frac{1}{x}e^{1+\ln x}$	**C**
10	$\int_0^1 x^2\sqrt{1-x^3}\ dx$ Let $u=1-x^3$ $du=-3x^2dx$ when $x=0\ u=1,\ x=1\ u=0$ $\int_1^0 \frac{-u^{\frac{1}{2}}}{3}\,\mathrm{du}$ $=\left[\frac{-2u^{\frac{3}{2}}}{9}\right]_1^0$ $=\frac{2}{9}$	**C**

Trial HSC Examination 2014
Mathematics Course

Name ______________________ Teacher ____________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ● B ● correct C ○ D ○

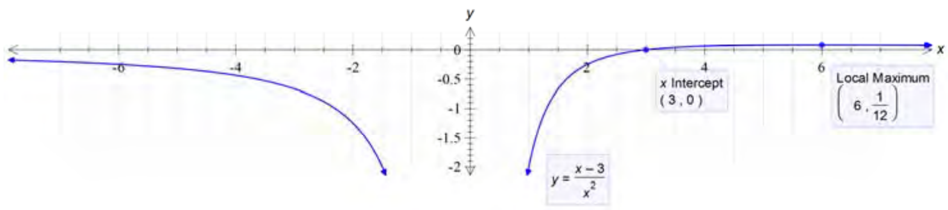
1.	A ○	B ○	C ○	D ●
2.	A ●	B ○	C ○	D ○
3.	A ○	B ○	C ●	D ○
4.	A ●	B ○	C ○	D ○
5.	A ●	B ○	C ○	D ○
6.	A ○	B ○	C ●	D ○
7.	A ○	B ●	C ○	D ○
8.	A ○	B ○	C ○	D ●
9.	A ○	B ●	C ○	D ○
10.	A ○	B ○	C ●	D ○

Question 11		**2014**	
	Solution	Marks	Allocation of marks
(a)	$P(x) = x^3 + ax^2 + bx - 18$ $P(-2) = 0$ $P(1) = -24$ $P(-2) = 0$ $0 = (-2)^3 + a(-2)^2 + b(-2) - 18$ $0 = -8 + 4a - 2b - 18$ $0 = 4a - 2b - 26 \ldots\ldots\ldots\ldots (1)$ $P(1) = -24$ $-24 = 1 + a + b - 18$ $0 = a + b + 7 \ldots\ldots\ldots\ldots (2)$ sub 2 into 1 $4(-7 - b) - 2b = 26$ $-28 - 4b - 2b = 26$ $-28 - 6b = 26$ $-6b = 54$ $b = -9$ $a - 9 = -7$ $a = 2$ $\therefore \quad a = 2$ and $b = -9$	2	1 for finding the 2 equations 1 for solving the simultaneous equations
(b)	$x = \dfrac{mx_2 + nx_1}{m + n}$ $= \dfrac{2 \times 5 + -3 \times 1}{2 - 3}$ $= -7$ $y = \dfrac{my_2 + ny_1}{m + n}$ $= \dfrac{2 \times 2 + -3 \times 4}{2 - 3}$ $= 8$ $\therefore$ pt $(-7, 8)$	1	
(c)	$y = \cos^{-1}(3x + 2)$ Let $u = 3x + 2$ $\dfrac{du}{dx} = 3$ $y = \cos^{-1} u$ $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$	2	

Question 11		2014	
	Solution	Marks	Allocation of marks
	$\frac{dy}{dx} = -\frac{1}{\sqrt{1-u^2}} \times 3$ $= -\frac{3}{\sqrt{1-(3x+2)^2}}$ $= -\frac{3}{\sqrt{-9x^2-12x-3}}$ $y = \cos^{-1}(3x+2)$ is defined $-1 \le 3x+2 \le 1$ $-3 \le 3x \le -1$ $-1 \le x \le -\frac{1}{3}$ $\therefore \cos^{-1}(3x+2)$ is defined for $-1 \le x \le -\frac{1}{3}$		1 correct differentiation 1 for stating defined values
(d)	$V = \pi\int_a^b x^2 dy$ $y = x^3 + 1$ $y - 1 = x^3$ $x = \sqrt[3]{y-1}$ $x^2 = \left(\sqrt[3]{y-1}\right)^2$ $x^2 = \left((y-1)^{\frac{1}{3}}\right)^2$ $x^2 = (y-1)^{\frac{2}{3}}$ $V = \pi\int_0^a (y-1)^{\frac{2}{3}}\, dy$ $= \pi\left[\frac{3(y-1)^{\frac{5}{3}}}{5}\right]_0^a$ $= \pi\left[\frac{3(a-1)^{\frac{5}{3}}}{5}\right] - \left[-\frac{3}{5}\right]$ $= \frac{3\pi}{5}\left[\sqrt[3]{(a-1)^5} + 1\right]$	3	1 for finding x^2 1 for correct integration 1 for correct answer

Question 11		**2014**	
	Solution	Marks	Allocation of marks
e	$y = \dfrac{1}{9 + x^2}$ Let $x = 3\tan\theta$ $\dfrac{dx}{d\theta} = 3\sec^2\theta$ $y = \dfrac{1}{9 + 9\tan^2\theta}$ $= \dfrac{1}{9(1 + \tan^2\theta)}$ $= \dfrac{1}{9\sec^2\theta}$ When $x = \sqrt{3}$ $\sqrt{3} = 3\tan\theta$ $\tan\theta = \dfrac{\sqrt{3}}{3}$ $= \dfrac{1}{\sqrt{3}}$ $\theta = \dfrac{\pi}{6}$ When $x = 0$ $0 = 3\tan\theta$ $\theta = 0$ $\therefore \int_0^{\sqrt{3}} \dfrac{1}{9 + x^2}\,dx$ $= \int_0^{\frac{\pi}{6}} \dfrac{1}{9 + x^2}\dfrac{dx}{d\theta} \times d\theta$ $= \int_0^{\frac{\pi}{6}} \dfrac{1}{9\sec^2\theta} \times 3\sec^2\theta\,d\theta$ $= \int_0^{\frac{\pi}{6}} \dfrac{1}{3}\,d\theta$ $= \left[\dfrac{1}{3}\theta\right]_0^{\frac{\pi}{6}}$ $= \left[\dfrac{\pi}{18} - 0\right]$ $= \dfrac{\pi}{18}\text{ unit}^2$	3	1 for correct substitution 1 for correct substitution and change of end points 1 for correct answer

Question 11		**2014**	
	Solution	Marks	Allocation of marks
(f)	$y = \frac{x-3}{x^2}$ $x^2 \neq 0$ $\therefore\ x \neq 0$ vertical asymptote $\lim_{x \to \infty} \frac{x-3}{x^2}$ $= 0$ $y' = \frac{x^2 - 2x(x-3)}{x^4}$ $= \frac{x^2 - 2x^2 + 6x}{x^4}$ $= \frac{-x^2 + 6x}{x^4}$ $= \frac{-x + 6}{x^3}$ Stat pts $y' = 0$ $0 = -x + 6$ $x = 6$ When $x = 6$ $y = \frac{1}{12}$ $\therefore$ Stat pt at $\left(6, \frac{1}{12}\right)$ $y'' = \frac{-x - 3x^2(-x+6)}{x^6}$ $= \frac{-x^3 + 3x^3 - 18x^2}{x^6}$ $= \frac{2x^3 - 18x^2}{x^6}$ $= \frac{2x - 18}{x^4}$	4	1 for stationary point with testing 1 for point of inflexion with testing 1 for correct shape of the graph 1 for showing the critical features on the graph.

Question 11		**2014**	
	Solution	Marks	Allocation of marks
	Inflexion $y'' = 0$ $0 = 2x - 18$ $x = 9$ When $x = 9$ $y = \frac{2}{27}$ Possible inflexion pt $\left(9\ ,\ \frac{2}{27}\right)$ check concavity Left $y'' < 0$ Right $y'' > 0$ change of concavity $\therefore$ inflexion pt $\left(9\ ,\ \frac{2}{27}\right)$ Classify stat pt $\left(6\ ,\ \frac{1}{12}\right)$ $y'' < 0$ concave down $\therefore$ Maximum pt  		

<table>
<tr><td colspan="2">Question 12</td><td colspan="2">2014</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(a)</td><td>(i)
$3\sin x\cos x$
$= \sin x\cos x + 2\sin x\cos x$
$= \frac{1}{2}\sin 2x + \sin 2x$
$= \sin 2x\left(\frac{1}{2} + 1\right)$
$= \frac{3}{2}\sin 2x$</td><td>1</td><td></td></tr>
<tr><td></td><td>(ii)
$\int_0^{\frac{\pi}{2}} 9\ \sin^2 x\cos^2 x\,dx = \int_0^{\frac{\pi}{2}} (3\sin x\cos x)^2\,dx$
$= \int_0^{\frac{\pi}{2}} \frac{9}{4}\sin^2 2x\,dx$
$= \int_0^{\frac{\pi}{2}} \frac{9}{4} \times \left[\frac{1}{2}(1 - \cos 4x)\right] dx$
$= \int_0^{\frac{\pi}{2}} \frac{9}{8}(1 - \cos 4x)dx$
$= \left[\frac{9}{8}x - \frac{9}{32}\sin 4x\right]_0^{\frac{\pi}{2}}$
$= \left[\frac{9\pi}{16} - \frac{9}{32}\sin 2\pi\right] - \left[0 - \frac{9}{32}\sin 0\right]$
$= \frac{9\pi}{16}$</td><td>2</td><td>1 for using part (i) or other method for changing form

1 for correct integration</td></tr>
<tr><td>(b)</td><td>(i) $\boldsymbol{\sin(\theta + 2\theta) = \sin\theta\cos 2\theta + \cos\theta\sin 2\theta}$

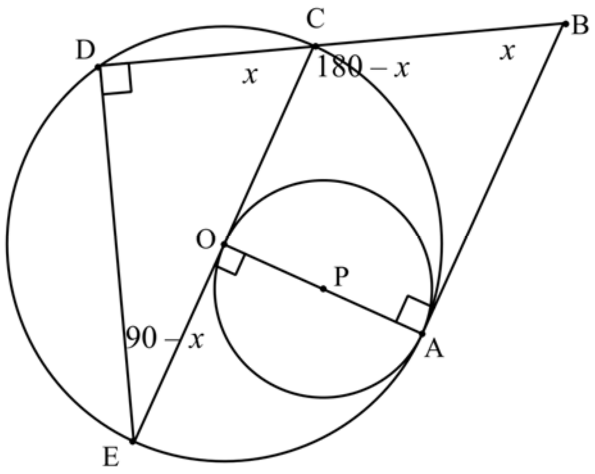
$\boldsymbol{= \sin\theta(\cos^2\theta - \sin^2\theta) + \cos\theta\,.\,2\sin\theta\cos\theta}$
$= \sin\theta\cos^2\theta - \sin^3\theta + 2\sin\theta\cos^2\theta$
$= 3\sin\theta\cos^2\theta - \sin^3\theta$
$= 3\sin\theta\,(1 - \sin^2\theta) - \sin^3\theta$
$= 3\sin\theta - 3\sin^3\theta - \sin^3\theta$
$= 3\sin\theta - 4\sin^3\theta$</td><td>2</td><td>1 mark for first 2 steps: correct expansion of $\boldsymbol{\sin(\theta + 2\theta)}$, $\boldsymbol{\sin 2\theta}$, $\boldsymbol{\cos 2\theta}$

1 mark for rest of simplification</td></tr>
<tr><td></td><td>(ii) $\sin 3\theta = 2\sin\theta$
$\therefore 3\sin\theta - 4\sin^3\theta = 2\sin\theta$
$0 = 4\sin^3\theta - \sin\theta$
$\sin\theta\,(4\sin^2\theta - 1) = 0$
$\therefore \sin\theta = 0$ or $4\sin^2\theta - 1 = 0$= 0

$\theta = 0°, 180°, 360°$ $4\sin^2\theta = 1$
$\sin\theta = \pm\frac{1}{2}$
$\theta = 30°, 150°, 210°, 360°$

$\theta = 0, \pi, 2\pi$ $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$</td><td>3</td><td>1 for correct factorisation

1 mark each for correct values for angle for each factor</td></tr>
</table>

<table>
<tr><td colspan="2">Question 12</td><td colspan="2">2014</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(c)</td><td>

$\angle PAB = 90°$ (Angle between a tangent and radius)
$\angle POC = 90°$ (Angle between a tangent and radius)
$\therefore OC \parallel AB$ (cointerior angles are supplementary)
$\therefore \angle OCB = 180 - x°$ (Cointerior angles on || lines)
$\therefore \angle DCE = x°$ (angles on straight line)
$\angle EDC = 90°$ (angle in a semicircle is a right angle)
$\angle CED = 180° - 90° - x°$ (angle sum ΔCED)
$\therefore \angle CED = 90° - x°$</td><td>3</td><td>Alternative solutions possible, allocate marks as appropriate.

1 for showing parallel lines

1 for external angle $\angle DCE$.

1 for final answer</td></tr>
<tr><td>(d)</td><td>(i)
$V = P + Ae^{-kt}$
$\frac{dV}{dt} = -kAe^{-kt}$
but $Ae^{-kt} = V - P$
$\therefore \frac{dV}{dt} = -k(V - P)$
Or
$\frac{dV}{dt} = -kAe^{-kt}$
$= -k\left(P + Ae^{-kt} - P\right)$
$= -k(V - P)$</td><td>1</td><td>Either method is acceptable</td></tr>
<tr><td></td><td>(ii)</td><td>2</td><td></td></tr>
</table>

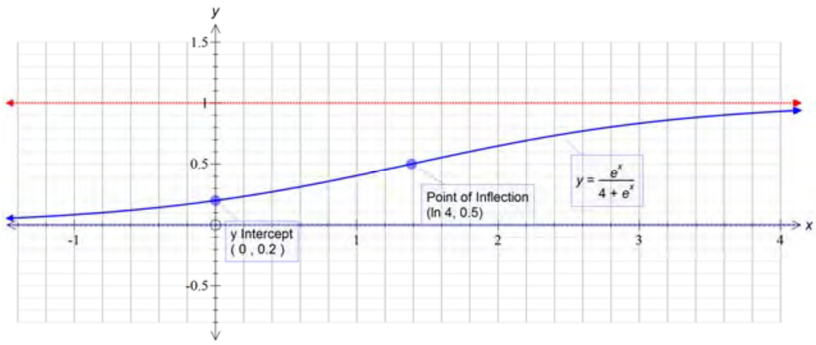
<table>
<tr><td colspan="2">Question 12</td><td colspan="2">2014</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td></td><td>$V = 0 \quad P = 55 \quad t = 0$
$V = 55 + Ae^{-kt}$
Initially $t = 0 \quad v = 0$
$0 = 55 + Ae^{0}$
$A = -55$
$\therefore \quad V = 55 - 55e^{-kt}$

When $t = 10 \quad V = 27$
$27 = 55 - 55e^{-10k}$
$55e^{-10k} = 28$
$e^{-10k} = \frac{28}{55}$
$-10k = \ln\left[\frac{28}{55}\right]$
$k = 0.067512867$</td><td></td><td>1 for A

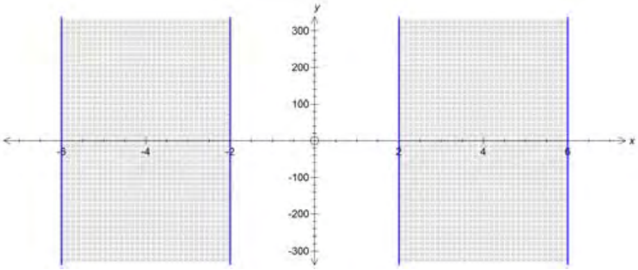
1 for k</td></tr>
<tr><td></td><td>(iii)
When $t = 17$
$V = 55 - 55e^{-0.0675t}$
$V = 55 - 55e^{-0.0675 \times 17}$
$V = 37.5\text{m/s}$</td><td>1</td><td>Ignore any rounding either with k or the answer</td></tr>
<tr><td></td><td>(iv)
$50 = 55 - 55e^{-0.0675 \times t}$
$e^{-0.0675 \times t} = \frac{5}{55}$
$t = \frac{\ln\left[\frac{5}{55}\right]}{-0.0675}$
$t = 35.5$ seconds
So it will take approximately 35.5 seconds.</td><td>1</td><td>1 for amount of time needed</td></tr>
</table>

<table>
<tr><td colspan="2">Question 13</td><td colspan="2">2014</td></tr>
<tr><td></td><td>Solution</td><td>Marks</td><td>Allocation of marks</td></tr>
<tr><td>(a)</td><td>$\cos x + \sin x = r\sin(x + \alpha)$
$r = \sqrt{1^2 + 1^2}$
$r = \sqrt{2}$
$\tan\alpha = 1$
$\alpha = \frac{\pi}{4}$
$\cos x + \sin x = 1$
$\cos x + \sin x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$
$\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = 1$
$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$
$\text{Let}\theta = \left(x + \frac{\pi}{4}\right) \therefore \sin\theta = \frac{1}{\sqrt{2}}$
$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$
$\therefore \left(x + \frac{\pi}{4}\right) = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}$
$x = 0, \frac{\pi}{2}, 2\pi$</td><td>2</td><td>1 for expressing in correct terms

1 for correct solutions</td></tr>
<tr><td>(b)</td><td>(i)
$f'(x) = \frac{4e^x}{\left(4 + e^x\right)^2}$
$u = 4e^x \quad v = \left(4 + e^x\right)^2$
$u' = 4e^x \quad v' = 2e^x\left(4 + e^x\right)$
$f''(x) = \frac{4e^x\left(4 + e^x\right)^2 - 8e^{2x}\left(4 + e^x\right)}{\left(\left(4 + e^x\right)^2\right)^2}$
$f''(x) = \frac{\left(4 + e^x\right)\left(4e^x\left(4 + e^x\right) - 8e^{2x}\right)}{\left(4 + e^x\right)^4}$
$f''(x) = \frac{\left(4 + e^x\right)\left(16e^x + 4e^{2x} - 8e^{2x}\right)}{\left(4 + e^x\right)^4}$
$f''(x) = \frac{4e^x\left(4 - e^x\right)}{\left(4 + e^x\right)^3}$</td><td>2</td><td>1 for finding the second derivative</td></tr>
</table>

Question 13		2014	
	Solution	Marks	Allocation of marks
	For inflexion points $f''(x) = 0$ $4 - e^x = 0$ $e^x = 4$ $x = \ln 4$ When $x = \ln 4$ $y = \dfrac{4}{4+4}$ $y = \dfrac{1}{2}$ $\therefore$ possible inflexion $= \left(\ln 4, \dfrac{1}{2}\right)$ Test $f''(1) > 0$ $f''(2) < 0$ $\therefore f''(x)$ changes sign $\therefore$ Inflexion at $= \left(\ln 4, \dfrac{1}{2}\right)$		1 for the inflexion with test
	(ii) *f(x)* has an inverse because it is an increasing function. ie $f'(x) > 0$ for all x. If you check it graphically with a horizontal line test, it will only cut the function once. Therefore if you reflect the graph in the line y=x it will pass the vertical line test.  	1	1 any valid explanation is acceptable, with or without a graph/sketch

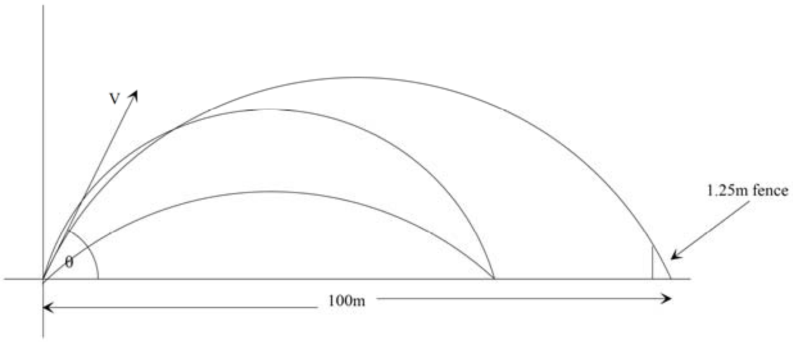
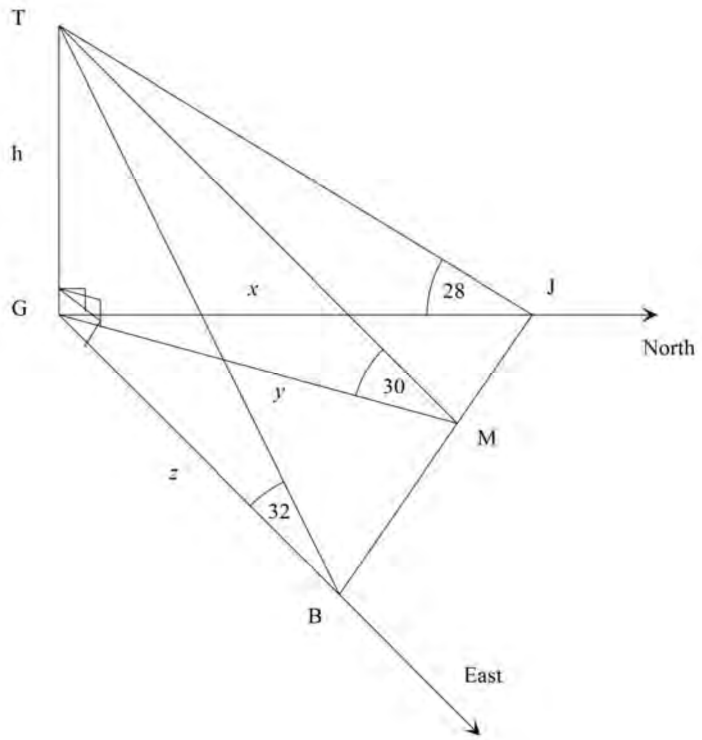
	(iii) $f(x) = \dfrac{e^x}{4+e^x}$ ie $y = \dfrac{e^x}{4+e^x}$ Inverse $x = \dfrac{e^y}{4+e^y}$ $4x + xe^y = e^y$ $e^y(1-x) = 4x$ $e^y = \dfrac{4x}{1-x}$ $y = \ln\left(\dfrac{4x}{1-x}\right)$	2	1 for interchanging x and y 1 for correct rearrangement for y
(c)	(i) $\dfrac{d^2x}{dt^2} = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d}{dt}v = \dfrac{dv}{dt}$ $= \dfrac{dv}{dx}\times\dfrac{dx}{dt}$ $= \dfrac{dv}{dx}\times v$ $= v\dfrac{dv}{dx}$ $= \dfrac{d}{dv}\left(\dfrac{1}{2}v^2\right)\times\dfrac{dv}{dx}$ $= \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	2	Any acceptable proof may be used 1 1
	(ii) Since $\dfrac{d^2x}{dt^2} = -2x\left(x^2-20\right)$ $= 40x - 2x^3$ and $\dfrac{d^2x}{dt^2} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$ $\therefore \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = 40x - 2x^3$ $\therefore \quad \dfrac{1}{2}v^2 = \int 40x - 2x^3\, dx$ $\dfrac{1}{2}v^2 = 20x^2 - \dfrac{x^4}{2} + C$ When $v = 0$ $x = 2$ $\therefore \quad 0 = 20\times 2^2 - \dfrac{2^4}{2} + C$ $\therefore \quad C = -72$	2	1 for correct integration
	$\therefore \dfrac{1}{2}v^2 = 20x^2 - \dfrac{x^4}{2} - 72$ $v^2 = 40x^2 - x^4 - 144$ $= -\left(x^4 - 40x^2 + 144\right)$ $= -\left(x^2-36\right)\left(x^2-4\right)$ $= -(x+6)(x-6)(x+2)(x-2)$		1 for expressing v^2 in terms of x

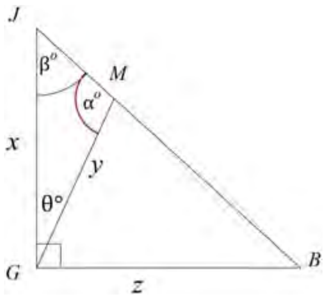
	(iii) $$v^2 \geq 0$$ Now $\therefore\ 40x^2 - x^4 - 144 \geq 0$ And this occurs when $-6 \leq x \leq -2$ or $2 \leq x \leq 6$ Only consider the positive values.  When $x = 2 \quad v = 0$ $\ddot{x} = -4 \times -16 = 64$ in + direction When $x = 6 \quad v = 0$ $\ddot{x} = -12 \times 16 = -192$ in – direction So the particle is at rest at $x = 2$, the acceleration is 64 in + direction which means the particle is moving to the right. Then at $x = 6$, $v = 0$ again and acceleration is -192 in – direction, so the particle is moving back to the left and so on. The particle oscillates between the points x= 2 and x=6, this however is not simple harmonic motion as $\ddot{x} = -2x\left(x^2 - 20\right)$ Is not in the form $\ddot{x} = -n^2x$	1	1 mark can be given for any reasonable attempt at explaining motion of particle not being SHM
(d)	$a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} = \dfrac{a\left(r^n - 1\right)}{r - 1}$ Step 1 Prove true for $n = 1$ $\boldsymbol{LHS} = a$ $RHS = \dfrac{a\left(r^1 - 1\right)}{r - 1}$ $= a$ $\therefore \boldsymbol{LHS} = RHS$	3	1 for step 1 and 2

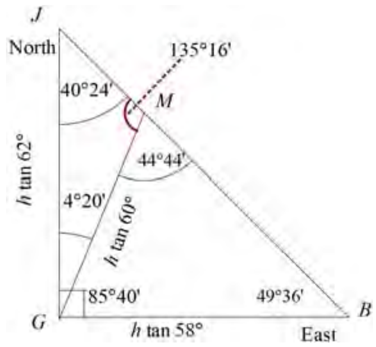
	Step 2 Assume true for $n = k$ $a + ar + ar^2 + ar^3 + \dots ar^{k-1} = \frac{a(r^k - 1)}{r-1}$ Step 3 Prove true for $n = k + 1$ ie Prove that $a + ar + ar^2 + ar^3 + \dots ar^{k-1} + ar^{(k+1)-1} = \frac{a(r^{k+1} - 1)}{r-1}$ $\mathrm{L}HS = a + ar + ar^2 + ar^3 + \dots ar^{k-1} + ar^{(k+1)-1}$ $= \frac{a(r^k - 1)}{r-1} + ar^k$ $= \frac{(ar^k - a + ar^{k+1} - ar^k)}{r-1}$ $= \frac{ar^{k+1} - a}{r-1}$ $= \frac{a(r^{k+1} - 1)}{r-1}$ $= RHS$ Therefore if true for $n = k$, also true for $n = k + 1$. since true for $n = 1$, by induction it is true for all positive integral values of n.		1 mark for Step 3 statement 1 mark for the proof in step 3

Question 14		2014	
	Solution	Marks	Allocation of marks
(a)	(i) $f(x) = x^3 + ax^2 + bx + c$ $f'(x) = 3x^2 + 2ax + b$ For Max/ min, $f'(x) = 3x^2 + 2ax + b$ $3\alpha^2 + 2a\alpha + b = 0$ and $3\beta^2 + 2a\beta + b = 0$ $3\alpha^2 + 2a\alpha + b - (3\beta^2 + 2a\beta + b) = 0$ $3(\alpha - \beta)(\alpha + \beta) + 2a(\alpha - \beta) = 0$ $3(\alpha + \beta) + 2a = 0$ $\alpha + \beta = -\frac{2}{3}a$	1	1 any correct approach
	(ii) For point of inflexion $f''(x) = 0$ $f''(x) = 6x + 2a = 0$ $x = -\frac{a}{3}$ $= -\frac{1}{3} \times \frac{-3}{2}(\alpha + \beta)$ $= \frac{\alpha+\beta}{2}$	1	Correct method
(b)	(i)	2	

Question 14		**2014**	
	Solution	Marks	Allocation of marks
	Horizontal Motion $\dot{x} = V\cos\theta$ $\dot{x} = 34\cos\theta$ $x = 34t\cos\theta + c_1$ $t = 0 \quad x = 0 \quad c_1 = 0$ $\therefore x = 34t\cos\theta$ $t = \dfrac{x}{34\cos\theta}$ Vertical Motion $\ddot{y} = -10$ $\dot{y} = -10t + C_2$ $t = 0 \quad \dot{y} = V\sin\theta$ $\therefore c_2 = 34\sin\theta$ $\therefore \dot{y} = -10t + 34\sin\theta$ $\therefore y = -5t^2 + 34\sin\theta + C_3$ $t = 0 \quad y = 0 \quad C_3 = 0$ $\therefore y = -5t^2 + 34t\sin\theta$		
	(ii) $y = -5t^2 + 34t\sin\theta$ $= -5\left[\dfrac{x}{34\cos\theta}\right]^2 + 34 \times \left[\dfrac{x}{34\cos\theta}\right] \times \sin\theta$ $= -\dfrac{5x^2}{1156}\sec^2\theta + x\tan\theta$ $= -\dfrac{5x^2}{1156}\left(1 + \tan^2\theta\right) + x\tan\theta$ $1156y = -5x^2 - 5x^2\tan^2\theta + 1156x\tan\theta$ When $x = 100$ $y = 1.25$ since the ball just clears the boundary fence. $1445 = -50000 - 50000\tan^2\theta + 1156000\tan\theta$ $\therefore 50000\tan^2\theta - 115600\tan\theta + 51445 = 0$	2	1 for subst in t correctly 1 for correct values and manipulating to the equation given
	(iii)	2	

Question 14		**2014**	
	Solution	Marks	Allocation of marks
	For a six, when $x = 100$ $y > 1.25$ $50000\tan^2\theta - 115600\tan\theta + 51445 < 0$ $\tan\theta = = \dfrac{115600 \pm \sqrt{(115600)^2 - 4 \times 50000 \times 51445}}{2 \times 50000}$ $\tan\theta = 1.7105$ or 0.6015 $\theta = 59°41'$, $31°2'$ $\therefore \theta$ lies between $31°2'$ and $59°41'$		1 for using quadratic formula correctly 1 for the range of angles
	(iv) The wind causes air resistance and both horizontal and vertical velocities are decreased. The ball will not rise as far or cover the same distance. You would then need to increase the velocity you hit the ball and reduce the angle range to still score a six. 	2	1 for sketch 1 for discussing results
(c)	(i) 	3	

Question 14		**2014**	
	Solution	Marks	Allocation of marks
	Let $GT = h$ $GJ = x$ $GM = y$ $GB = z$ $\angle TGJ = \angle TGM = \angle TGB = \angle JGB = 90^\circ$ $\angle GTJ = 62^\circ$ $\angle GTM = 60^\circ$ $\angle GTB = 58^\circ$ $\therefore \quad x = h \tan 62^\circ$ $y = h \tan 60^\circ$ $z = h \tan 58^\circ$  Ground triangle viewed from above		1 for a correct diagram and expressions to find side lengths
	$\therefore \quad \tan\beta = \frac{h\tan 58^\circ}{h\tan 62^\circ}$ $\beta = 40^\circ\, 24'$ $\frac{\sin\alpha}{h\tan 62^\circ} = \frac{\sin 40^\circ 24'}{h\tan 60^\circ}$ $\sin\alpha = \frac{\sin 40^\circ 24' \tan 62^\circ}{\tan 60^\circ}$ $= 0.70367$ $\alpha = \sin^{-1}(0.70367)$ $\alpha = 44^\circ 44'$ or $135^\circ\, 16'$ Now if $\alpha = 44^\circ 44'$ and $\beta = 40^\circ 24'$, then $\theta = 180 - 44^\circ 44' - 40^\circ 24' = 94^\circ 52'$. But $\theta < 90^\circ$, since it lies between north and east. $\therefore \alpha = 135^\circ 16'$ $\therefore \ \theta = 180^\circ - 40^\circ 24' - 135^\circ 16'$ $= 4^\circ\, 20'$ $= 4^\circ$ $\therefore$ Maddy is on a bearing of $004^\circ T$ from the Eiffel Tower		1 for finding the needed angles 1 for correct bearing 1 or 2 marks can be awarded if student was on the right track but has made a small error

Question 14		**2014**	
	Solution	Marks	Allocation of marks
	(ii) Using the angles calculated above we can obtain the diagram below:  Using the cos rule on ΔMBG. $MB^2 = (h\tan 60)^2 + (h\tan 58)^2 - 2 \times (h\tan 60)(h\tan 58)\cos\ 85°40'$ $= h^2\tan^2 60 + h^2\tan^2 58 - 2 \times 2h^2.\tan 60.\ \tan 58.\cos\ 85°40'$ $= h^2\left(\tan^2 60 + \tan^2 58 - 2 \times 2\tan 60.\ \tan 58.\cos\ 85°40'\right)$ Using the cos rule on ΔJMG. $JM^2 = (h\tan 60)^2 + (h\tan 62)^2 - 2 \times (h\tan 60)(h\tan 62)\cos\ 4°20'$ $= h^2\tan^2 60 + h^2\tan^2 62 - 2 \times 2h^2.\tan 60.\ \tan 62.\cos\ 4°20'$ $= h^2\left(\tan^2 60 + \tan^2 62 - 2 \times 2\tan 60.\ \tan 62.\cos\ 4°20'\right)$ $\frac{MB^2}{JM^2} = \frac{h^2\left(\tan^2 60 + \tan^2 58 - 2 \times 2\tan 60.\ \tan 58.\cos\ 85°40'\right)}{h^2\left(\tan^2 60 + \tan^2 62 - 2 \times 2\tan 60.\ \tan 62.\cos\ 4°20'\right)}$ $= \frac{\left(\tan^2 60 + \tan^2 58 - 2 \times 2\tan 60.\ \tan 58.\cos\ 85°40'\right)}{\left(\tan^2 60 + \tan^2 62 - 2 \times 2\tan 60.\ \tan 62.\cos\ 4°20'\right)}$	2	1 mark for obtaining an expression for at least one of MB^2 and/or JM^2. 1 mark for writing the ratio and simplifying out the terms in *h*. No need to evaluate the expression.