NORTH SYDNEY BOYS' HIGH SCHOOL

YEAR 12 Trial Higher School Certificate

1998

MATHEMATICS

3 UNIT

Time allowed - 2 hours (plus 5 minutes reading time)

"This is a Trial Paper only and does not necessarily reflect the content or format of the final HSC Examination paper for this subject"

DIRECTIONS TO CANDIDATES

- Attempt ALL questions on the paper provided.
- Write on ONE SIDE of paper only.
- Start each question on a new page.
- Show all necessary working.
- Marks may not be awarded for careless or untidy work.
- Write in blue or black ink only.
- All questions are of equal value.

Question 1 **Mai**

(a) **(i)** Differentiate $\ln\sqrt{1-x^2}$.

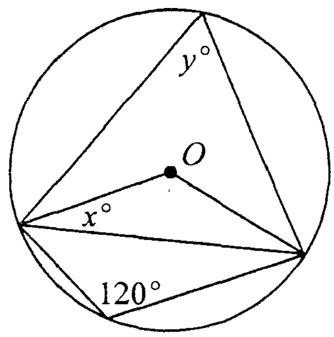
(ii) Find the coordinates of the point P that divides the interval AB externally in the ratio $3:1$, where A and B are the points $(-4,-6)$ and $(6,-1)$ respectively.

(iii) Show that the line $5x-12y-6=0$ is a tangent to the circle $(x+2)^2+(y-3)^2=16$.

(b) Use the table of standard integrals to find $\displaystyle\int\frac{dx}{\sqrt{x^2+16}}$.

(c) Use the substitution $u=1+x$ to evaluate $\displaystyle\int_{-1}^{3}x\sqrt{1+x}\,dx$.

Question 2 (Start a new page)



(a) O is the centre of the circle. Find the values of x and y.

(b) Solve

(i) $|2x-1|\le 5$;

(ii) $\dfrac{x+2}{x}<3$.

(c) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}}\sin^2\theta\,d\theta$.

(i) Show that $\dfrac{d}{dx}\left(\tan^3 x\right)=3\sec^4 x-3\sec^2 x$.

(ii) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{3}}\sec^4 x\,dx$.

Question 3 (Start a new page)

(a) Consider the function $y = 1 - 3\cos 2x$.

(i) Write down the period of the function.

(ii) Write down the amplitude of the function.

(iii) Sketch the curve $y = 1 - 3\cos 2x$ where $-\pi \le x \le \pi$.

(iv) How many solutions exist in the domain $-\pi \le x \le \pi$ to the equation $1 - 3\cos 2x = 5$? Justify your answer.

(b) A bowl of water heated to $100°C$ is placed in a coolroom where the temperature is maintained at $-5°C$. After t minutes, the temperature $T°$ of the water is changing so that $\dfrac{dT}{dt} = -k(T+5)$.

(i) Prove that $T = Ae^{-kt} - 5$ satisfies this equation, and find the value of A.

(ii) After 20 minutes, the temperature of the water has fallen to $40°C$. How long, to the nearest minute will the water need to be in the coolroom before ice begins to form? [ie. the temperature falls to $0°C$]

(c) State the domain and range of the function $y = 2\sin^{-1}\left(\dfrac{x}{3}\right)$.

Hence sketch the curve.

Question 4 (Start a new page)

(a) Solve for n: $2 \times {}^nC_4 = 5 \times {}^nC_2$.

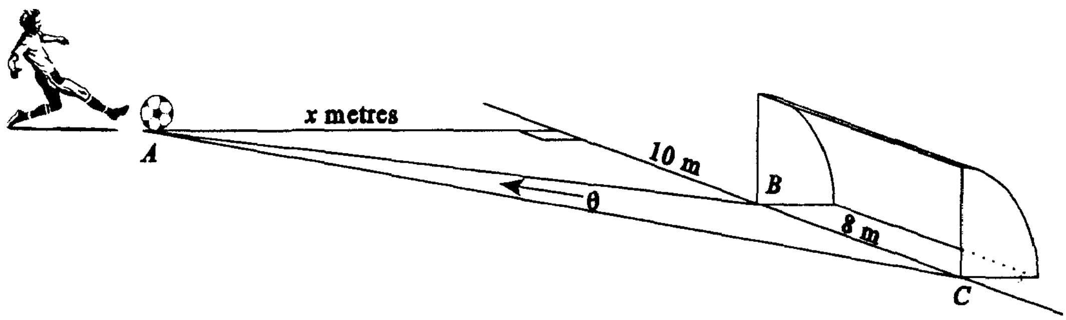
(b) The area between the curve $y = \ln x$, the x-axis, and the lines $x = 2$ and $x = 4$ is rotated about the x-axis. Use Simpson's rule with 3 function values to estimate the volume of the solid so formed. Give your answer correct to two decimal places.

(c) (i) Show that the equation $\ln x + x^2 - 4x = 0$ has a root lying between $x = 3$ and $x = 4$.

(ii) By taking $x = 4$ as a first approximation, use one application of Newton's method to obtain a better approximation for the root.

Question 5 (Start a new page)

(a)



A soccer player A is x metres from a goal line of a soccer field. He takes a shot at the goal BC, with the ball not leaving the ground.

(i) Show that the angle θ within which he must shoot is given by

$$\theta = \tan^{-1}\left(\frac{8x}{180 + x^2}\right)$$

when he is 10 metres to one side of the near goalpost and 18 metres to the same side of the far post.

(ii) Find the value of x that makes this angle a maximum.

(b) A particle moves in a straight line such that its velocity V m/s is given by $v = 2\sqrt{2x - 1}$ when it is x metres from the origin. If $x = \frac{1}{2}$ when $t = 0$, find

(i) the acceleration;

(ii) an expression for x in terms of t;

(c) A monic polynomial of degree 4 has exactly two zeros, $x = 2$ and $x = -2$, and is eve [ie. $P(x) = P(-x)$]. This polynomial has a value of 55 when $x = 3$. Find $P(x)$.

Question 6 (Start a new page)

(a) How many four-digit numbers can be formed from the digits 1, 2, 3, 4

(i) without repetition;

(ii) with repetition ?

(b) The tangent to the parabola $x^2 = 4ay$ at the variable point $P(2ap, ap^2)$ cuts the x-axis at Q and the y-axis at R.

(i) Show that the equation of the tangent at P is $y = px - ap^2$.

(ii) Find the coordinates of Q and R.

(c) Show that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{4}{7}\right)$.

(d) The displacement x metres of a particle from the origin is given by $x = 5\cos\left(3t - \frac{\pi}{6}\right)$, where t is the time elapsed in seconds. Show that $\ddot{x} = -9x$, and describe the motion

Question 7 (Start a new page)

(a) Prove that $\dfrac{\cos A - \sin A}{\cos A + \sin A} = \dfrac{\cos 2A}{1 + \sin 2A}$.

(b) Solve for $0 \le x \le 2\pi$:

$$4\cos x - 3\sin x = -1 .$$

Give your solutions correct to two decimal places.

(c) Use the remainder theorem to find one factor of $P(x) = x(x - 1) - a(a - 1)$.
Hence factorize $P(x)$.

(d) A stone is thrown from the top of a cliff, 70 metres above sea level, with a speed of 65 m/s, at an angle of elevation of $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$ to the horizontal. Find

(i) the distance from the base of the cliff to the point where the stone hits the sea.

(ii) the velocity at the instant of hitting the water.