Question 1 **Marks**

a) Differentiate with respect to x $\quad x^2\tan^{-1}x$ **2**

b) Solve $\dfrac{2x+1}{x+3} < 1$ **2**

c) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \cos x\, e^{\sin x}\, dx$ **3**

d) Prove that if the roots of $ax^2 - 2x + (1-2a) = 0$ are reciprocals, then their sum is 6. **3**

e) Sketch the graph $y = \dfrac{|x|}{x}$ **2**

Question 2

a) Find $\displaystyle\int \frac{x}{\sqrt{1+x}}\, dx$ using the substitution $u = 1 + x$ **2**

b) The lines $y = mx + 2$ and $y = 3x - 1$ intersect forming an angle whose tangent is $\frac{2}{3}$. Find 2 possible values for m. **3**

c) If $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$ and $\beta = \tan^{-1}\left(\frac{1}{3}\right)$, show that $\tan(\alpha + \beta) = 1$ **2**

d) Find the domain and range of $y = 3\cos^{-1}(2x+3)$ **2**

e) i) Show that $y = \sqrt{x} + x - 1$ has a zero between 0 and 1. **1**

ii) Taking 0·4 as your first approximation use **one** application of Newton's Method to find a better approximation. **2**

Question 3 **Marks**

a) Consider the curve $y = \dfrac{x^2}{1 + x^2}$

i) Find the coordinates of any stationary points and determine their nature. There is no need to find points of inflexion. **3**

ii) State the equation of the horizontal asymptote **1**

iii) Sketch the function indicating all important features. **1**

b) A bequest of \$20000 is invested at 6%p.a. to pay an annual prize of \$1500. The prize is removed each year after the interest has been paid.

i) How much is in the account after the first prize is awarded? **1**

ii) Show that the amount remaining after n prizes A_n is given by **2**

$$A_n = 25000 - 5000 \times 1{\cdot}06^n$$

iii) Hence find the number of years the full prize can be awarded. **2**

c) Prove $\dfrac{\sin 2A - \sin A}{\cos 2A - \cos A + 1} \equiv \tan A$ **2**

Question 4

a) Prove by the process of Mathematical Induction **5**

that $3^{2n} - 1$ is divisible by 8 for $n \geq 1$

b) Find the locus of the mid point of PS where P is the point $(2ap, ap^2)$ on the parabola $x^2 = 4ay$ and S is the focus. **3**

c) The area enclosed between the curve $y = \sqrt{2}\cos x$ and the two axes is rotated about the x-axis. Find the volume so formed. **4**

Question 5

a) Express $\sin t - \sqrt{3}\cos t$ in the form $A\sin(t - \alpha)$. **4**

Hence solve $\sin t - \sqrt{3}\cos t = \sqrt{2} \qquad 0 \leq t \leq 2\pi$

Question 5 continued

Marks

b) Show that $x = -1$ is the only real zero of $P(x) = x^3 + 2x^2 + 2x + 1$. **3**

c) A cyclist riding along a straight, flat road passes by three stop signs R,E, and D, spaced 200m apart. From these three signs the respective angles of elevation to the top of a mobile phone tower are 30^0, 45^0 and 45^0. Let h be the height of the tower and T be the base of the tower. **5**

i) Draw a diagram representing the situation.

ii) Find in terms of h, the distances RT, ET, and DT.

iii) Let $\angle TED = \alpha$. Find two different expressions for $\cos\alpha$ in terms of h and hence by eliminating $\cos\alpha$ find the height of the tower.

Question 6

a) A particle moving in simple harmonic motion with centre the origin and amplitude 8m passes the origin at 6m/s.

i) Find x and v as functions of time. **3**

ii) Find the period of the motion. **1**

iii) Find the acceleration at maximum amplitude. **1**

b) Consider the function $f(x) = \dfrac{e^x}{(1+e^x)}$

i) State the domain of $f(x)$ **1**

ii) Show $f'(x) = \dfrac{e^x}{(1+e^x)^2}$ **1**

iii) Hence explain why $f(x)$ is increasing for all x. **1**

iv) Explain why $f(x)$ has an inverse function. **1**

v) Find the inverse function $f^{-1}(x)$. **1**

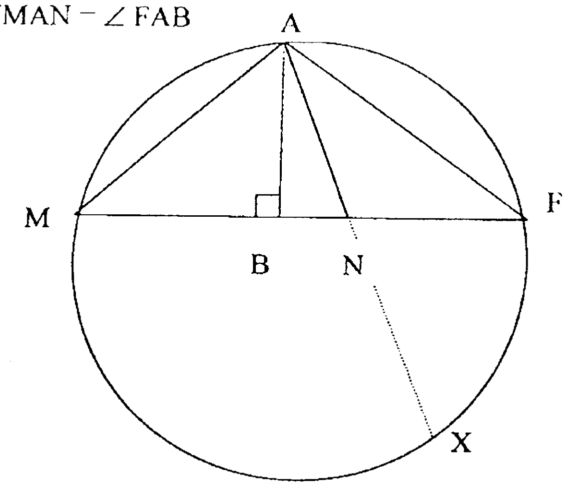
c) Sketch the function $y = \cos^{-1}(x) + \cos^{-1}(-x)$. **2**

Question7

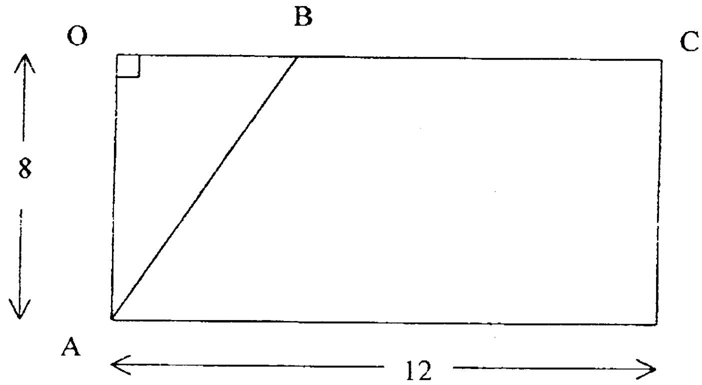
a) Rain is falling into a conical rain gauge at a constant rate of 3π cm^3/h. If the radius r of the cone is one third its height h, find the rate in cm/h at which the height is increasing when $h = 6$cm. **4**

Question 7 continued **Marks**

b) Copy the diagram below onto your answer sheet and then prove that if AN produced to X is a diameter of circle MAF, then $\angle MAN = \angle FAB$ **4**



c) The diagram below represents two roads AO and OC which meet at right angles at O. A hiker decides to walk from A through the bush and meet the road at B, and then continue along the road to C.



His walking speed through the bush is 3km/h and along the road 6km/h.
OA = 8km, OC = 12km and let OB = x km.

i) Show that the time t hours taken for the journey is **2**

given by $t = \dfrac{2\sqrt{x^2 + 64} + 12 - x}{6}$

ii) Find the distance OB such that the time taken for the journey will be minimum. **2**

END OF EXAMINATION