23 Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2005
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

- Attempt all questions

Class Teacher:
(Please tick or highlight)
- O Mr Lowe
- O Mr Rezcallah
- O Mr Trenwith
- O Mr Ee
- O Ms Silverman
- O Mr Weiss

Student Number:________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	Total	Total
Mark	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{84}$	$\frac{}{100}$

Question 1

(a) Differentiate $e^x \cos 2x$. **2**

(b) Find the acute angle between the lines $y = 2x + 3$ and $3x - 2y - 1 = 0$. **2**
Write your answer correct to the nearest degree.

(c) Use the table of standard integrals to evaluate $\displaystyle\int_3^5 \frac{dx}{\sqrt{x^2 - 9}}$. **3**

Write your answer in the form $\ln a$, where a is a constant.

(d) A and B are the points $(5, 1)$ and $(-1, 4)$ respectively. **2**
Find the coordinates of the point P which divides AB externally in the ratio $5 : 2$.

(e) Evaluate $\displaystyle\int_{\frac{1}{3}}^{\frac{2}{3}} 9x(3x - 1)^4\, dx$ using the substitution $u = 3x - 1$. **3**

Question 2 (Start a new page)

(a) Evaluate $\displaystyle\lim_{x \to 0} \frac{\sin 3x}{2x}$ **1**

(b) α, β, γ are the roots of the polynomial equation $2x^3 - 3x + 2 = 0$. Evaluate

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ **2**

(ii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(c) Assuming $x = 2$ is a close approximation to a root of $2 \sin x = x$, use one application of Newton's method to find a better approximation. Give your answer correct to three decimal places. **2**

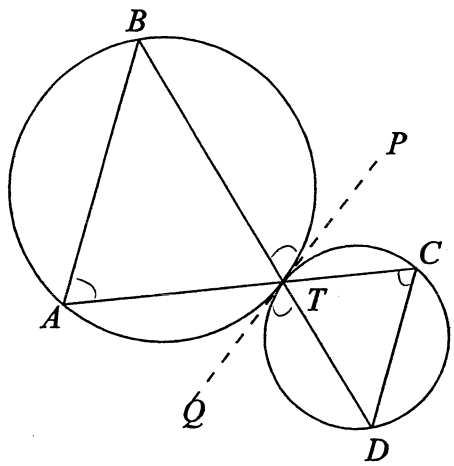
(d) Find

(i) $\displaystyle\int \sin^2 2x\, dx$ **2**

(ii) $\displaystyle\int_0^{\frac{4}{3}} \frac{2\, dx}{16 + 9x^2}$ **3**

Question 3 (Start a new page)

(a)



Two circles touch externally at T. PQ is a tangent to each circle at T.
AB and CD are chords in the respective circles.
ATC and BTD are straight lines.

(i) State why $\angle QTD = \angle TCD$. **1**

(ii) Prove that $AB \parallel CD$. **3**

(b) (i) Write $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x + \alpha)$, where $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve $\cos x - \sqrt{3}\sin x = \sqrt{3}$ for $0 \le x \le 2\pi$. **2**

(c) (i) Write a simplified expression for $2\sin 2A\cos 2A$. **1**

(ii) Prove the identity $\dfrac{\sin 3A}{\sin A} + \dfrac{\cos 3A}{\cos A} = 4\cos 2A$. **3**

[The result in part (b) (i) may be useful]

Question 4 (Start a new page)

(a) Sketch the graph of $y = 2\sin^{-1}(x - 1)$. **2**

(b) A piece of cork moves vertically in Simple Harmonic Motion on the surface of the water as waves pass under it. Its velocity v m/s is given by $v^2 = -x^2 + 7x - 12$, where x is the cork's vertical displacement in metres.

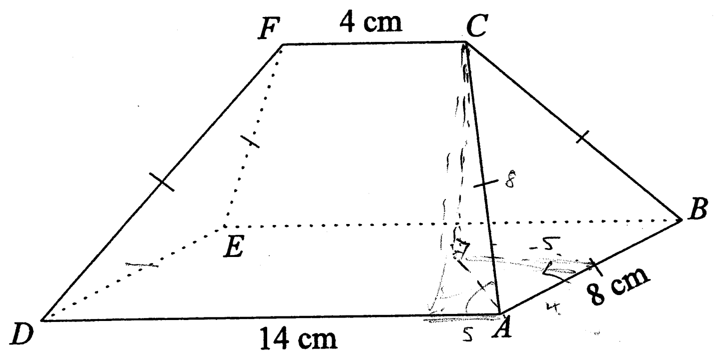
(i) Using $\ddot{x} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$, find the acceleration of the cork in terms of x. **1**

(ii) What is the centre of motion ? **1**

(iii) Find the period of oscillation. **1**

(c) Solve for x : $\dfrac{2}{x-1} < x$ **3**

(d)



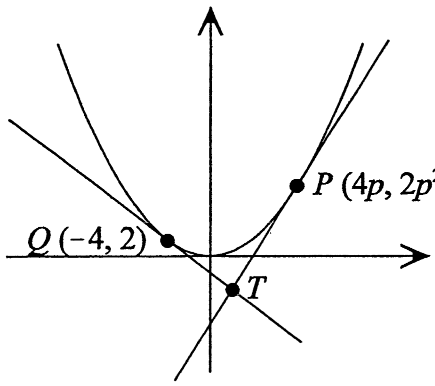
In the solid shown, $ABED$ is a rectangle of length 14 cm and breadth 8 cm. ABC and DEF are congruent equilateral triangles of side length 8 cm. $ACFD$ and $BCFE$ are congruent isosceles trapezia, whose parallel sides are 14 cm and 4 cm, as shown. Find

(i) the angle in the trapezium between AC and AD. 51°19' **2**

(ii) the angle between AC and the base $ABED$. **2**

Question 5 (Start a new page)

(a)



$P\,(4p, 2p^2)$ and $Q\,(-4, 2)$ are two points on the parabola $x^2 = 8y$.
The tangents to the parabola at P and Q intersect at T.

(i) Show that the tangent at P has equation $y = px - 2p^2$. **2**

(ii) Hence, write down the equation of the tangent at Q. **1**

(iii) Show that T has coordinates $(2p - 2,\ -2p)$. **3**

(iv) M is the midpoint of P and T. **3**
Show that the locus of M, as P varies on the parabola, has equation
$9y = (x + 1)(x - 2)$

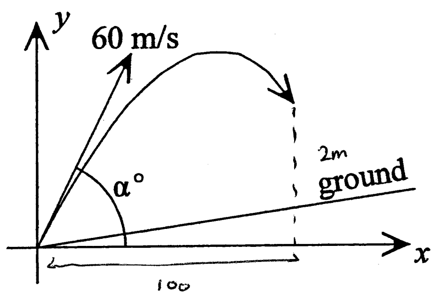
(b) A cup of soup with a temperature of 95°C is placed in a room which has a temperature of 20°C. After 10 minutes, the soup cools to 70°C. The rate of heat loss is proportional to the difference between the soup's temperature and room temperature, that is $\dfrac{dT}{dt} = -k\,(T - 20)$.

(i) Show that $T = 20 + Ae^{-kt}$ is a solution of this differential equation **1**

(ii) Find the temperature of the soup after a further 5 minutes, correct to the nearest degree. **2**

Question 6 (Start a new page)

(a)



At the Battle of Hastings, the Normans fired arrows at the Anglo-Saxons up a hill which had a gradient of 1 in 10. The diagram shows the path of the arrows (assume that the arrows were fired from ground level). All arrows were fired with an initial velocity of 60 m/s. The archers varied the range by varying the angle of projection, α. Assume that the acceleration due to gravity is 10 m/s^2.

(i) Show that the equations for horizontal and vertical displacement of an arrow are respectively $x = 60t\cos\alpha$ and $y = -5t^2 + 60t\sin\alpha$, where t is the time in seconds after firing the arrow. **4**

(ii) Show that the Cartesian equation for the path of an arrow is **2**

$$y = -\frac{1}{720}x^2\left(1 + \tan^2\alpha\right) + x\tan\alpha.$$

(iii) According to legend Harold, King of the Anglo-Saxons, was killed when hit in the eye by a Norman arrow. Assume that Harold was 100 metres horizontally from the Norman archers, and that his eye was 2 metres above the ground. At what angle(s), α, must this arrow have been fired if it hit Harold on the way down. **3**

(b) When the polynomial $P(x)$ is divided by $x - 4$, the remainder is -5. **3**
When $P(x)$ is divided by $x + 1$, the remainder is 5.
Find the remainder when $P(x)$ is divided by $(x - 4)(x + 1)$.

Question 7 (Start a new page)

(a) (i) By using the formula for the sum of an arithmetic series, show that **1**

$$1 + 2 + 3 + \ldots + n + (n+1) = \frac{(n+1)(n+2)}{2}$$

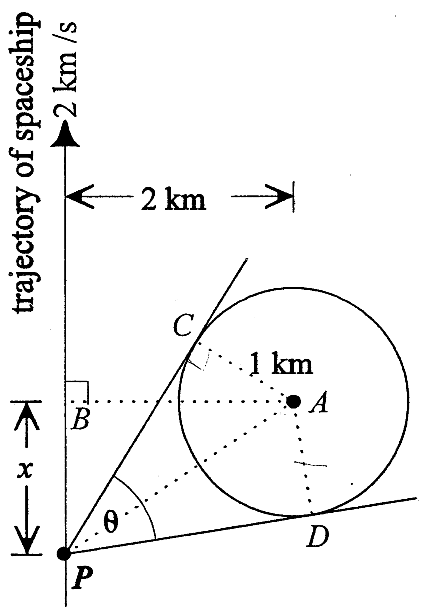
(ii) Use mathematical induction to prove

$$\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots + \frac{1}{1+2+3+\ldots+n} = \frac{2n}{n+1} \quad \text{for } n \geq 1.$$ **4**

The result of part (i) may be useful.

(iii) Hence, write down the limiting sum of a series whose general term is given by $T_n = \dfrac{1}{1+2+3+\ldots+n}$. **1**

(b)



The diagram shows a spaceship flying past an asteroid. The asteroid has a radius of 1 km, and the spaceship is 2 km from the asteroid's centre at its closest approach.

When the spaceship is at the point P, it is x km from its closest approach. At this moment, the asteroid subtends an angle of θ radians at the spaceship.

The spaceship is travelling in a straight line at a constant speed of 2 km/s.

(i) Show that the angle θ is given by $\theta = 2\sin^{-1}\dfrac{1}{\sqrt{4+x^2}}$. **2**

(ii) At what rate, in degrees per second, is the angle θ changing when x is 3 km? **4**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

b) (i) $AP^2 = PB^2 + BA^2$

$AP = \sqrt{4+x^2}$ [1]

$\sin A\hat{P}C = \dfrac{1}{\sqrt{4+x^2}}$

$A\hat{P}C = \sin^{-1}\dfrac{1}{\sqrt{4+x^2}}$

$\theta = 2\sin^{-1}\dfrac{1}{\sqrt{4+x^2}}$ [1] [2]

(ii) $\theta = 2\sin^{-1}(4+x^2)^{-1/2}$

$$\frac{d\theta}{dx} = \frac{2}{\sqrt{1-\frac{1}{4+x^2}}} \cdot -\frac{1}{2}(4+x^2)^{-3/2} \cdot 2x$$ [1] for correct derivative (unsimplified)

$$= \frac{-2x(4+x^2)^{-3/2}}{\sqrt{\frac{4+x^2-1}{4+x^2}}}$$

$$= -2x(4+x^2)^{-3/2} \cdot \frac{(4+x^2)^{1/2}}{(3+x^2)^{1/2}}$$

$$= -\frac{2x}{(4+x^2)\sqrt{3+x^2}}$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dx} \cdot \frac{dx}{dt}$$

$$= \frac{-4x}{(4+x^2)\sqrt{3+x^2}}$$ [1] for $\frac{d\theta}{dt}$

$x=3$, $\dfrac{d\theta}{dt} = \dfrac{-12}{13\sqrt{12}}$ rad/s [1] for correct substitution

$= 15.3°/\text{sec}$ [1] for conversion

1

23

Question 1

(a) $$\frac{d}{dx}\left(e^x \cos 2x\right) = \cos 2x \cdot e^x + e^x \cdot (-2\sin 2x)$$ [2]

$$= e^x \cos 2x - 2e^x \sin 2x$$

$$= e^x(\cos 2x - 2\sin 2x)$$

[1] for correctly using product rule

[1] for correctly differentiating e^x and $\cos 2x$ (but only if product rule correct)

(b) $y = 2x+3 \Rightarrow m_1 = 2$

$3x - 2y - 1 = 0 \Rightarrow y = \frac{3}{2}x - \frac{1}{2} \Rightarrow m_2 = \frac{3}{2}$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{3}{2} - 2}{1 + \frac{3}{2}\cdot 2}\right|$$ [1] for correct substitution into correct formula.

$$= \frac{1}{8}$$

$\theta = \tan^{-1}\frac{1}{8}$

$= \_\_\_\_$ [1] for correct answer (ignore rounding) [2]

(c) $$\int_3^5 \frac{dx}{\sqrt{x^2-9}} = \left[\ln\left(x + \sqrt{x^2-9}\right)\right]_3^5$$ [1]

$= \ln(5+4) - \ln(3+0)$ [1]

$= \ln 9 - \ln 3$

$= \ln 3$ [1] [3]

)

$m:n = 5:-2$

$x = \frac{nx_1 + mx_2}{m+n}$

$= \frac{(-2)(5) + 5(-1)}{5-2}$

$= -5$

$y = \frac{ny_1 + my_2}{m+n}$

$= \frac{(-2)(1) + 5(4)}{5-2}$

$= 6$

$\therefore P(-5, 6)$

2) $\int_{1/3}^{2/3} 9x(3x-1)^4\,dx$

$u = 3x - 1$

$du = 3\,dx$

$x = \frac{1}{3},\ u = 0$

$x = \frac{2}{3},\ u = 1$

$= \int_{1/3}^{2/3} 3x \cdot (3x-1)^4 \cdot 3\,dx$

$= \int_0^1 (u+1) \cdot u^4 \cdot du$ [2] – [1] for correct integral – [1] for correct limits

$= \int_0^1 (u^5 + u^4)\,du$

$= \left[\frac{u^6}{6} + \frac{u^5}{5}\right]_0^1$

$= \left(\frac{1}{6} + \frac{1}{5}\right) - 0$

$= \frac{11}{30}$ [1] for final answer

[3]

Minus 1 for mixing x & u in same integral, or for not changing limits in the first line where the variable was changed

Question 7

(a) (i) $S_n = \frac{n}{2}(a+l)$

$\therefore 1+2+3+\ldots+n+(n+1) = \frac{n+1}{2}\left[1+(n+1)\right]$

$= \frac{(n+1)(n+2)}{2}$ [1]

(ii) Prove: $\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots + \frac{1}{1+2+3+\ldots+n} = \frac{2n}{n+1}$

Test $n=1$: LHS $= \frac{1}{1} = 1$

RHS $= \frac{2(1)}{1+1} = 1$

$\therefore$ True for $n=1$

Assume true for $n=k$:

ie $\frac{1}{1} + \frac{1}{1+2} + \ldots + \frac{1}{1+2+\ldots+k} = \frac{2k}{k+1}$ [1]

Prove true for $n=k+1$:

ie. $\frac{1}{1} + \frac{1}{1+2} + \ldots + \frac{1}{1+2+\ldots+k} + \frac{1}{1+2+\ldots+k+k+1} = \frac{2(k+1)}{k+2}$

LHS $= \frac{1}{1} + \frac{1}{1+2} + \ldots + \frac{1}{1+2+\ldots+k} + \frac{1}{1+2+\ldots+k+k+1}$

$= \frac{2k}{k+1} + \frac{1}{1+2+\ldots+k+k+1}$ by assumption [1]

$= \frac{2k}{k+1} + \frac{2}{(k+1)(k+2)}$ by part (i)

$= \frac{2k(k+2)+2}{(k+1)(k+2)}$

$= \frac{2(k^2+2k+1)}{(k+1)(k+2)}$

$= \frac{2(k+1)^2}{(k+1)(k+2)}$

$= \frac{2(k+1)}{k+2}$ [1]

$=$ RHS

must indicate reliance on assumption.

$\therefore$ true for $n=k+1$ when true for $n=k$

$\therefore$ by MI, true for all +ve integers n. [1]

[4]

(iii) $S_\infty = \lim_{n\to\infty}\left(\frac{2k}{k+1}\right)$

$= 2$ [1]

(b) $P(x) = (x-4)(x+1)Q(x) + (ax+b)$

$P(4) = -5 \quad : \quad -5 = 4a + b$

$P(-1) = 5 \quad : \quad 5 = -a + b$ [1]

Solve simult: $a = -2$ (1) for the solving.

$b = 3$

$\therefore$ remainder $= \underline{-2x+3}$ [1] [3]

1 mark for stating the correct remainder polynomial.

P.S. Most students got 1 mark since only 1 answer for $y=2$, $x=100$. However, to get the marks the solution should be as follows.

For $y = 2$, $x = 100$.

$-\frac{100^2}{720}(1+\tan^2\alpha) + 100\tan\alpha = 2$

$-\frac{125}{9}(1+\tan^2\alpha) + 100\tan\alpha = 2$

$-125(1+\tan^2\alpha) + 900\tan\alpha = 18$

$0 = 125\tan^2\alpha - 900\tan\alpha + 143$

$\tan\alpha = \frac{900 \pm \sqrt{900^2 - 4(143)(125)}}{2(125)}$

$= \frac{900 \pm \sqrt{738500}}{250}$

$\tan\alpha = 7.037$. $\quad\quad \tan\alpha = 0.162559$.

$\alpha = 81°54'$. $\quad\quad \alpha = 9°13'$.

$t = \frac{x}{\ldots} = \frac{5}{3\cos\alpha}$ $\quad \dot{y} = -10t + 60\sin\alpha = -\frac{50}{3\cos\alpha} + 6\sin\alpha$. both $81°54'$

both answers $81°54'$, $9°13'$

Question 2

(a) $\lim_{x \to 0} \frac{\sin 3x}{2x} = \frac{3}{2}$ [1] answer only

(b) (i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma}$ [1] for correct algebra

$= \frac{-3/2}{-1}$ [1] for both correct substitutions.

$= \frac{3}{2}$ [2]

(ii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$ [1]

$= 0^2 - 2(-3/2)$ [1] as above

$= 3$ [2]

c) let $f(x) = 2\sin x - x$

$f'(x) = 2\cos x - 1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2 - \frac{2\sin 2 - 2}{2\cos 2 - 1}$ [1]

$= 1.901$ (3d.p) [1] [2]

Minus 1 for not using radian mode on calculator

No marks for wrong $f(x)$

d) (i) $\cos 4x = 1 - 2\sin^2 2x$

$\therefore \sin^2 2x = \frac{1}{2}(1 - \cos 4x)$ [1]

$\int \sin^2 2x \, dx = \frac{1}{2}\int (1 - \cos 4x)\, dx$

$= \frac{1}{2}\left(x - \frac{1}{4}\sin 4x\right) + c$ [1]

$= \frac{x}{2} - \frac{1}{8}\sin 4x + c$ [2]

No marks if double angle formulas have not been used

Don't penalize if no 'c'.

(ii) $\int_0^{4/3} \frac{2\,dx}{16 + 9x^2} = \frac{2}{9}\int_0^{4/3} \frac{dx}{\frac{16}{9} + x^2}$

$= \frac{2}{9} \cdot \frac{3}{4}\left[\tan^{-1}\frac{3x}{4}\right]_0^{4/3}$

$= \frac{1}{6}\left(\frac{\pi}{4} - 0\right)$

$= \frac{\pi}{24}$

[1] for $\tan^{-1}\frac{3x}{4}$

[1] for correct coefficient

[1] for final answer.

[3]

Question 3

a) (i) Alternate segment theorem [1]

(ii) $T\hat{C}D = B\hat{T}D$ (alt. seg. thm)

$= B\hat{T}P$ (vert opp $\angle$'s) [1]

$= B\hat{A}T$ (alt. seg. thm) [1]

$\therefore AB \parallel CD$ (alternate angles equal) [1] [3]

(b) (i) $\cos x - \sqrt{3}\sin x = R\cos(x+\alpha)$

$= R(\cos x\cos\alpha - \sin x\sin\alpha)$

$= (R\cos\alpha)\cos x - (R\sin\alpha)\sin x$

$\therefore R\cos\alpha = 1$ — ①

$R\sin\alpha = \sqrt{3}$ — ②

$①^2 + ②^2: \quad R^2 = 4 \Rightarrow R = 2$

$② \div ①: \quad \tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$ (1st quad)

$\therefore \cos x - \sqrt{3}\sin x = 2\cos\left(x + \frac{\pi}{3}\right)$ [2]

(ii) $\cos x - \sqrt{3}\sin x = \sqrt{3}$

$2\cos\left(x+\frac{\pi}{3}\right) = \sqrt{3}$

$\cos\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$x + \frac{\pi}{3} = \frac{\pi}{6}, \frac{11\pi}{6}$ (1)

$x = -\frac{\pi}{6}, \frac{3\pi}{2}$

$x = \frac{3\pi}{2}, \frac{11\pi}{6}$ [1] [2]

Question 6

i) (i) $\ddot{x} = 0$ $\qquad \ddot{y} = -10$

$\dot{x} = c$ $\qquad \dot{y} = -10t + c$

$= 60\cos\alpha$ [1] $\qquad = -10t + 60\sin\alpha$ [1]

$x = 60t\cos\alpha + c$ $\qquad y = -5t^2 + 60t\sin\alpha + c$

$= 60t\cos\alpha$ [1] $\qquad = -5t^2 + 60t\sin\alpha$ [1] [4]

(ii) $x = 60t\cos\alpha \Rightarrow t = \frac{x}{60\cos\alpha}$

$y = -5t^2 + 60t\sin\alpha$

$= -\frac{5x^2}{3600\cos^2\alpha} + 60\cdot\frac{x}{60\cos\alpha}\cdot\sin\alpha$ [1]

$= -\frac{1}{720}x^2\sec^2\alpha + x\tan\alpha$

$= -\frac{1}{720}x^2(1+\tan^2\alpha) + x\tan\alpha$ [1] [2]

(iii) height of Harold above ~~ground level~~ point of projection $= \frac{1}{10}\times 100 + 2$

$= 12$ metres

1 for correct $y=12$ into Equation.

$\therefore x = 100, y = 12 \Rightarrow 12 = \frac{-1}{720}\times 100^2(1+\tan^2\alpha) + 100\tan\alpha$ [1]

$12 = -\frac{125}{9}(1+\tan^2\alpha) + 100\tan\alpha$

$108 = -125 - 125\tan^2\alpha + 900\tan\alpha$

$125\tan^2\alpha - 900\tan\alpha + 233 = 0$

$\tan\alpha = \frac{900 \pm \sqrt{900^2 - 4\times125\times233}}{250}$

1 for clearly solving the Q. E

$= 6.93107, \; 0.26894$

$\alpha = 15°4', \; 81°47'$ [1] 1 for both answers

0.26... 1.4...

By Testing: Check: $t = \frac{100}{60\cos\alpha} = \frac{5}{3\cos\alpha}$

both answers work!

$\therefore \dot{y} = -\frac{50}{3\cos\alpha} + 60\sin\alpha$

$\alpha = 15°4', \; \dot{y} = -1.66$ m/s

$\alpha = 81°47', \; \dot{y} = -57.23$ m/s

... for both 15°4', 81°47' [3]

(b) (i) $T = 20 + Ae^{-kt}$

$\frac{dT}{dt} = -kAe^{-kt}$

$= -k(T - 20)$ — [1]

(ii) $T = 20 + 75e^{-kt}$

$70 = 20 + 75e^{-10k}$

$e^{-10k} = \frac{2}{3}$

$k = -\frac{1}{10}\ln\frac{2}{3} \doteqdot 0.04055$ [1]

$t = 15$, $T = 20 + 75e^{-15k}$

$= 61°C$ [1] [2]

(c) (i) $2\sin 2A\cos 2A = \sin 4A$ [1]

(iii) $LHS = \frac{\sin 3A}{\sin A} + \frac{\cos 3A}{\cos A}$

$= \frac{\sin 3A\cos A + \cos 3A\sin A}{\sin A\cos A}$ [1]

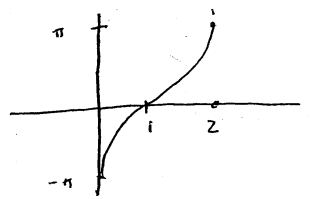
$= \frac{\sin(3A + A)}{\sin A\cos A}$ [1]

$= \frac{2\cancel{\sin 2A}\cos 2A}{\frac{1}{2}\cancel{\sin 2A}}$ from part (i)

$= 4\cos 2A$ [1]

$= RHS$ [3]

Question 4

i) 

[2] wrong shape : 0
deduct one mark for ~~any~~ each of the following incorrect: domain, range, orientation.

b) (i) $\ddot{x} = \frac{d}{dx}\left(\frac{-x^2+7x-12}{2}\right)$

$= -x + \frac{7}{2}$ [1]

(ii) $\ddot{x} = -1(x - \frac{7}{2})$

$\therefore$ centre is $x = \frac{7}{2}$ [1]

(iii) $n^2 = 1 \Rightarrow n = 1$

$\therefore$ period $= \frac{2\pi}{1}$

$= 2\pi$ sec. [1]

c) $\frac{2}{x-1} < x$

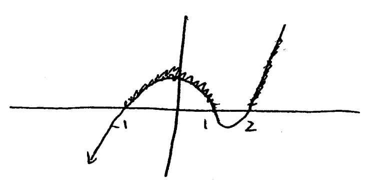
$2(x-1) < x(x-1)^2$ [1]

$x(x-1)^2 - 2(x-1) > 0$

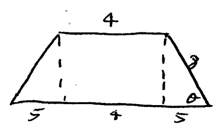
$(x-1)[x(x-1) - 2] > 0$

$(x-1)(x^2 - x - 2) > 0$

$(x-1)(x-2)(x+1) > 0$



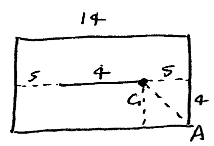
$-1 < x < 1$ or $x > 2$ (2) [3]

d) (i) 

$\cos\theta = \frac{5}{8}$

$\theta = 51°19'$ [2] 1 for diagram showing the 5.

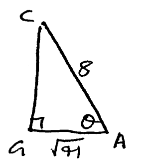
(ii) Let G be point on base vertically below C.



$GA^2 = 5^2 + 4^2$

$GA = \sqrt{41}$

[1]



$\cos\theta = \frac{\sqrt{41}}{8}$

$\theta = 36°\ 50'$

[1]

[2]

Question 5

a) (i) $x = 4p$ $\quad y = 2p^2$

$\frac{dx}{dp} = 4$ $\quad \frac{dy}{dx} = 4p$ OR $y = \frac{x^2}{8}$, $\frac{dy}{dx} = \frac{x}{4}$, $x = 4p \Rightarrow m_T = p$

$\frac{dy}{dx} = \frac{4p}{4} = p$ [1]

$y - 2p^2 = p(x - 4p)$

$y - 2p^2 = px - 4p^2$

$y = px - 2p^2$ [1] [2]

(ii) at Q, $p = -1$

$\therefore$ tangent: $y = -x - 2$ [1] If they do it the long way, give them the mark.

(iii) $y = px - 2p^2$

$y = -x - 2$

$\therefore px - 2p^2 = -x - 2$ [1]

$px + x = 2p^2 - 2$

$x(p+1) = 2(p-1)(p+1)$

$x = 2(p-1)$

$= 2p - 2$ [1]

$y = -(2p-2) - 2$

$= -2p + 2 - 2$

$= -2p$ [1]

$\therefore T(2p-2, -2p)$ [3]

(iv) $P(4p, 2p^2)$, ~~M~~ $T(2p-2, -2p)$

$\therefore M\left(\frac{4p+2p-2}{2}, \frac{2p^2-2p}{2}\right) \Rightarrow M(3p-1, p^2-p)$ [1]

ie. $x = 3p - 1 \Rightarrow p = \frac{x+1}{3}$ [1]

$y = p^2 - p$

$= \left(\frac{x+1}{3}\right)^2 - \frac{x+1}{3}$

$= \frac{(x+1)^2 - 3(x+1)}{9}$

$9y = (x+1)[x+1-3]$

$9y = (x+1)(x-2)$ [1] [3]