Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2019 HSC ASSESSMENT TASK 3 (TRIAL HSC)

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Mr Ireland
- O Dr Jomaa
- O Ms Lee
- O Mr Lin
- O Ms Ziaziaris

Student Number:

(To be used by the exam markers only.)

Question No	1-10	11	12	13	14	Total	Total
Mark	/10	/15	/15	/15	/15	/70	/100

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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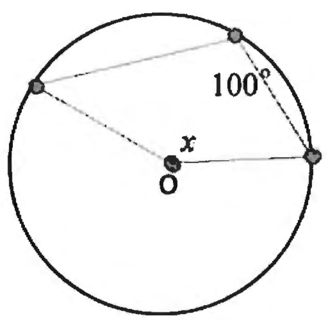
1. The acute angle between the lines with equations $y = 3x - 2$ and $2x - 3y + 1 = 0$ is

   A. 39°17'
   B. 37°52'
   C. 105°15'
   D. 74°45'

2. Find the value of x



A. 80°

B. 160°

C. 100°

D. 200°

3. $\int \sin x \cos x dx =$

A. $\frac{1}{2}\sin 2x + C$

B. $-\cos x \sin x + C$

C. $\frac{1}{2}\sin^2 x + C$

D. $\frac{1}{2}\cos^2 x + C$

4. Differentiate $x^3 \log(\sqrt{x})$

A. $\frac{3x^2 \log x}{2} + \frac{x^2}{2}$

B. $\frac{1}{2}x^2$

C. $\frac{3x^2 \log x - x^2}{(\log x)^2}$

D. $\frac{5}{2}x^{\frac{3}{2}}$

5. A particle moving in simple harmonic motion has equation $x = 2\sin\frac{1}{2}t$ where x is the displacement of the particle after t seconds.
Find the speed of the particle when $x = \sqrt{3}$.

A. $\frac{1}{4}$

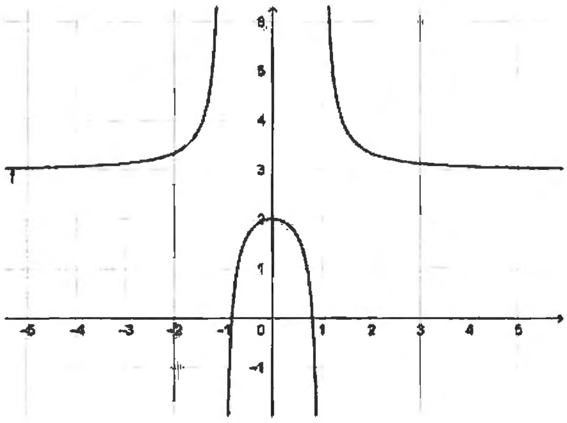
B. 1

C. $\frac{1}{2}$

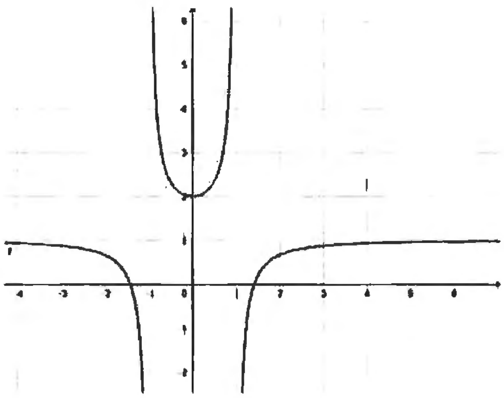
D. 2

6. The graph of $y = 2 - \dfrac{x^2}{1-x^2}$

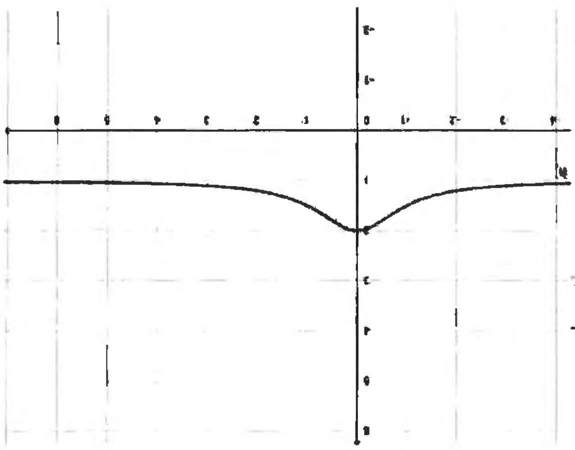
A.



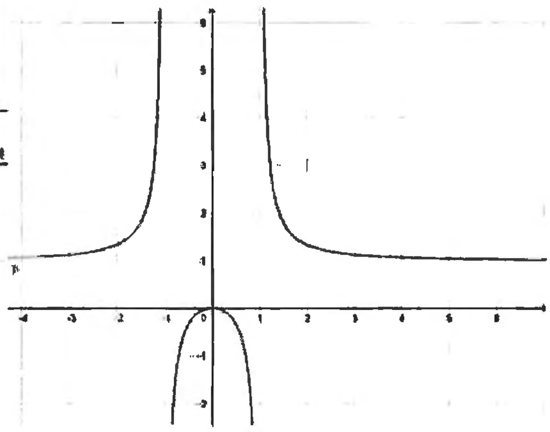
B.



C.



D.



7. The general solution of $\sin 2\theta = \sin\theta$ is

A. $2n\pi \pm \dfrac{\pi}{3}$

B. $2n\pi \pm \dfrac{\pi}{6}$

C. $n\pi$ or $2n\pi \pm \dfrac{\pi}{3}$

D. $n\pi$ or $2n\pi \pm \dfrac{\pi}{6}$

8. Evaluate $\lim_{x\to 0}\frac{\sin\frac{x}{2}}{3x}$

A. 6

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{6}$

9. If $y=\log_{1/a}(\frac{1}{N})$, $a>0, N>0$ then which one of the following is true.

A. $y=\log_a(-N)$

B. $y=\log_a(N)$

C. $y=\log_{-a}(\frac{1}{N})$

D. $y=\frac{\log_{1/a} 1}{\log_{1/a} N}$

10. The polynomial $P(x)=x^3+px^2+qx+r$ has real roots $\sqrt{m}, -\sqrt{m}$ and n. Find the value of pq.

A. 0

B. r

C. $-mn$

D. $-r$

Section II

60 marks

Attempt questions 11-14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) A, B are the points (-4,6) and (1,-7) respectively. Find the coordinates of the point P dividing AB externally in the ratio 2:3. 2

(b) Solve $\frac{3x+4}{x-5} \geq 2$ 3

(c) Find the domain and range of $y = 2\sin^{-1}\frac{x}{3}$ 2

(d) Differentiate $\sqrt{\sec x}$ 2

(e) $\int_4^{16} \frac{1}{\sqrt{x}(4+x)}dx$ using $u = \sqrt{x}$. Leave answer in exact form. 3

(f) $\int_0^{\frac{\pi}{2}} \cos^2 3x dx$ Leave answer in exact form. 3

Question 12 continues on the next page..

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find the values of a for which $(x-a)$ is a factor of $x^2-(a+2)x+6$ 1

(b) (i) Express $7\cos\theta-\sin\theta$ in the form $R\cos(\theta+\alpha)$

where $R>0$ and $0^\circ \le \alpha \le 360^\circ$. 2

(ii) Hence, solve $7\cos\theta-\sin\theta=5$ for $0^\circ \le \theta \le 360^\circ$ 2

(c) If x_1 is an approximation to the value $\sqrt[4]{m}$, show that a closer approximation is $\dfrac{3x_1^4+m}{4x_1^3}$, using Newton's Method. 2

(d) The acceleration of a particle moving in a straight line is given by

$$\frac{d^2x}{dt^2}=2x-3$$

where x is the displacement, in metres, from the origin O and t is the time in seconds. Initially the particle is at rest at $x=4$.

(i) If the velocity of the particle is v m/s, show that $v^2=2(x^2-3x-4)$. 2

(ii) Show that the particle does not pass through the origin. 1

(iii) Determine the position of the particle when $v=10$. Justify your answer. 2

(e) Use the principle of mathematical induction to prove if n is a positive integer, that

$n(n+3)$ is divisible by 2 3

Question 13 continues on the next page...

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) $\int \frac{dx}{\sqrt{9-4x^2}}$ 1

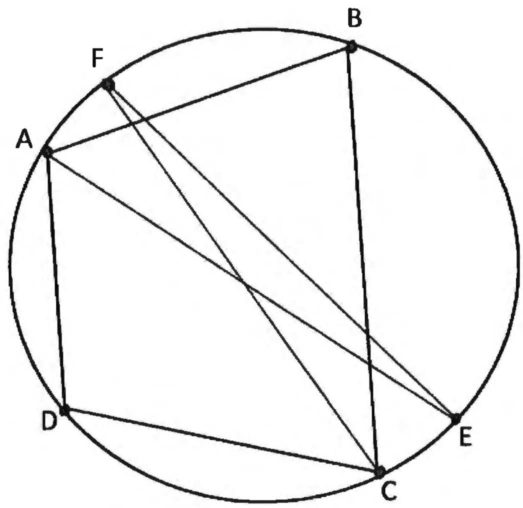
(b) In a certain University course, the rate of students dropping out of the course was proportional to the number N, still attending, in excess of 400. This rate can be expressed by the equation $\frac{dN}{dt} = k(N-400)$, where t is the number of weeks and k is constant.

(i)Show that $N = 400 + Ae^{kt}$ is a solution of this differential equation. 1

(ii) If 1000 students started the course and 10% had dropped out after 12 weeks, estimate the number of weeks it will take for half the students to remain. 2

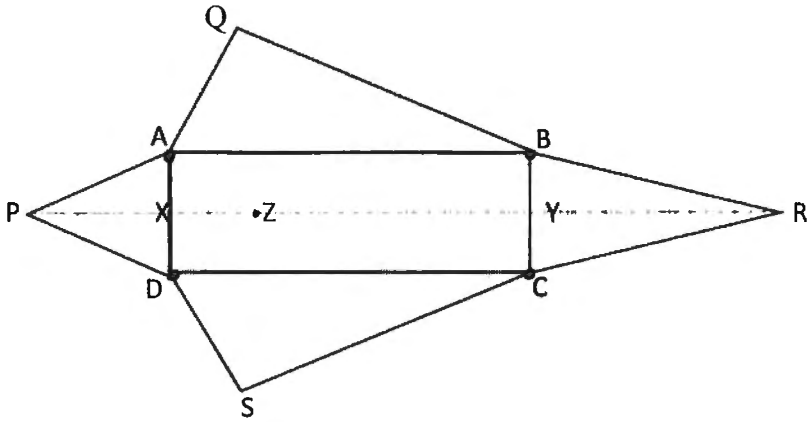
(c) ABCD is a cyclic quadrilateral. The bisectors of the angles at A and C meet the circle again at E and F respectively.

3



Question 13 continues on the next page..

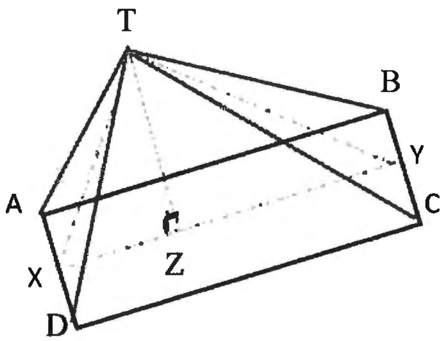
(d)



The figure shows the net of an oblique pyramid with a rectangular base. In this figure, PXZYR is a straight line, $PX = 15cm$, $RY = 20cm$, $AB = 25cm$, and $BC = 10cm$.

Further, $AP = PD$ and $BR = RC$.

When the net is folded, points P, Q, R and S all meet at the apex T, which lies vertically above the point Z in the horizontal base, as shown below.



(i)Show that ΔTXY is right angled. 1

(ii) Hence show that T is 12cm above the base. 2

(iii) Hence find the angle that the face DCT makes with the base. 1

(e)(i) Sketch the parabola whose parametric equations are $x = t$ and $y = t^2$.

On your diagram mark the points P and Q which correspond to $t = -1$ and $t = 2$ respectively. 2

(ii) Show that the tangents to the parabola at P and Q intersect at $R(\frac{1}{2}, -2)$. 1

(iii) Let $T(t, t^2)$ be the point **on the parabola** between P and Q such that the tangent at T meets QR at the midpoint of QR. Let the midpoint be M. Show that the tangent at T is parallel to PQ. 1

Question 14 continues on the next page...

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A particle moves such that $x = \dfrac{3(\cos 2t - \sin 2t)}{\sqrt{2}}$ where x represents displacement in metres and t represents time in seconds.

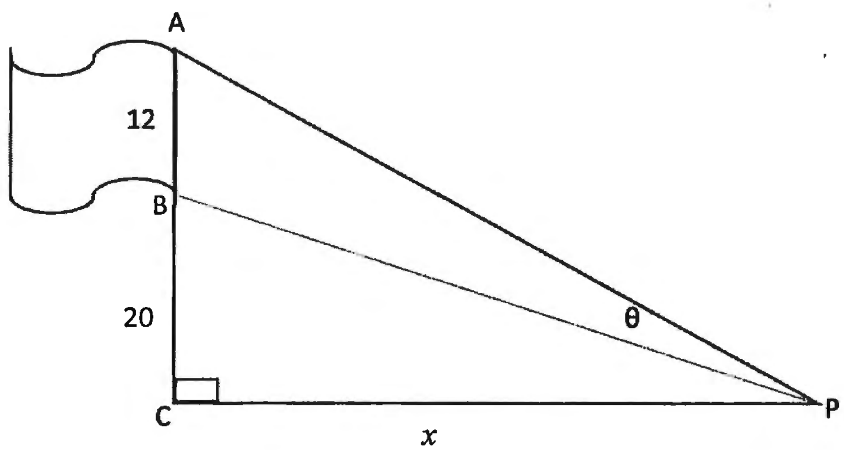
(i)Show that the particle is moving in simple harmonic motion. 2

(ii) Find the period of the motion. 1

(iii) Show that $x = 0$ when $t = \dfrac{\pi}{8}$. 1

(iv) Find the amplitude of the motion. 2

(b) A flagpole 12 m high stands on the top of a building 20 m high. The flagpole subtends an angle of θ at a point P on the ground and the distance from P to the building is x metres.

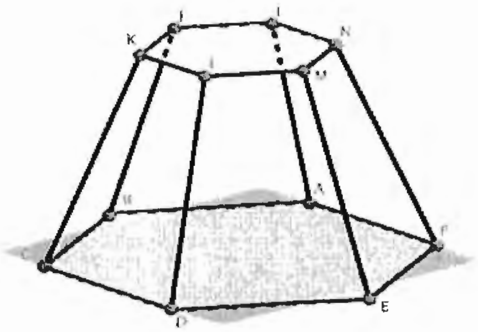


(i) Show that $\theta = \tan^{-1}(\dfrac{12x}{x^2 + 640})$ 2

(ii) Find x that makes θ a maximum. 3

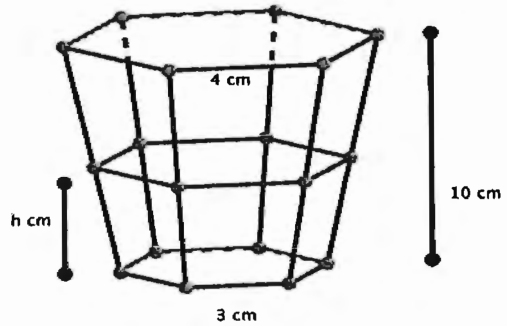
Question 14 continues on the next page...→

(c) A frustum of height H is made by cutting off a hexagonal pyramid of base IJKLMN with side length b from a regular hexagonal pyramid of base ABCDEF with side length a. It is given that the volume of the frustum is $\frac{\sqrt{3}}{2}(a^2 + ab + b^2)$H



An empty glass in the form of an inverted frustum described above with the height 10 cm, the side of the base and the top are 3cm and 4 cm respectively. Water is being poured into the glass. Let h be the depth of the water inside the glass at time t seconds where

$0 \leq h \leq 10$.



i. Show that the volume $V cm^3$ of water inside the glass at time t is given by 2

$$V = \frac{\sqrt{3}}{2000}(27000h + 900h^2 + 10h^3)$$

ii. If the volume of the water in the glass is increasing at the rate $7cm^3/s$, find the rate of increase of depth of water at the instant when h=5cm. 2

END OF EXAMINATION

Yr 12 Solutions Ext 1 Task 3 2019

M/C

1. B
2. B
3. C
4. A
5. C
6. A
7. C
8. D
9. B
10. B.

11. a) $\overset{x_1,\,y_1}{(-4,6)}$ $\overset{x_2\,y_2}{(1,-7)}$ $m=-2$

$n=3$.

$$P(x,y)=\left(\frac{nx_1+mx_2}{m+n},\ \frac{ny_1+my_2}{m+n}\right)$$

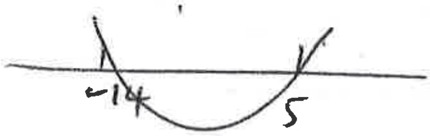
$$=\frac{3(-4)-2(1)}{1},\ \frac{3(6)-2(-7)}{1}$$

$$=(-14,\ 32)$$ 2

b) $(3x+4)(x-5)-2(x-5)^2 \geqslant 0.$

$(x-5)[3x+4-2(x-5)] \geqslant 0$

$(x-5)[x+14] \geqslant 0$



$x \leq -14$ OR $x>5$ $x \neq 5$. 3

c) $y = 2\sin^{-1}\frac{x}{3}$

D: $-1 \le \frac{x}{3} \le 1$

$\boxed{-3 \le x \le 3}$ 1

R: $-\frac{\pi}{2} \le \frac{y}{2} \le \frac{\pi}{2}$

$\boxed{-\pi \le y \le \pi}$ 1

d) Let $y = (\sec x)^{\frac{1}{2}}$

$y = \frac{1}{(\cos x)^{\frac{1}{2}}}$

$y = (\cos x)^{-1/2}$

$\frac{dy}{dx} = -\frac{1}{2}(\cos x)^{-3/2} \cdot -\sin x$ OK. 2

$= \frac{\sin x}{2(\cos x)^{3/2}}$

$= \frac{\sin x}{2\sqrt{\cos^3 x}} = \left(\frac{1}{2}\tan x\sqrt{\sec x}\right)$

e) $\int_4^{16} \frac{1}{\sqrt{x}(4+x)}\,dx$

$u = \sqrt{x}$

$u = x^{\frac{1}{2}}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$du = \frac{1}{2\sqrt{x}}\,dx$

$x = 4,\ u = 2$

$x = 16,\ u = 4$

$= 2\int_4^{16} \frac{1}{2\sqrt{x}(4+x)}\,dx$

$= 2\int_2^4 \frac{du}{4+u^2}$

$= 2 \times \frac{1}{2}\left[\tan^{-1}\frac{u}{2}\right]_2^4$

$= \tan^{-1}2 - \tan^{-1}1$

$= \tan^{-1}2 - \pi/4$ 3

f) $\int_0^{\pi/2} \cos^2 3x \, dx$.

$\cos 6x = 2\cos^2 3x - 1$

$\frac{\cos 6x + 1}{2} = \cos^2 3x$

$\int_0^{\pi/2} \frac{\cos 6x + 1}{2} \, dx$

$\frac{1}{2}\left[\frac{\sin 6x}{6} + x\right]_0^{\pi/2}$.

$= \frac{1}{2}\left[\frac{\sin 3\pi}{6} + \frac{\pi}{2} - 0\right]$

$= \frac{1}{2}\left[0 + \frac{\pi}{2}\right]$

$= \frac{\pi}{4}$

3.

(12) a) Let $P(x) = x^2 - (a+2)x + 6$

$P(a) = 0$

$0 = a^2 - (a+2)a + 6$

$0 = a^2 - a^2 - 2a + 6$

$2a = 6$

$a = 3$

b) (i) $7\cos\theta - \sin\theta = R\cos(\theta + a)$

$= R\cos\theta\cos a - R\sin\theta\sin a$

$\therefore R\cos a = 7$

$R\sin a = 1$

$R^2 = 50$

$R = 5\sqrt{2}$

$\tan a = \frac{1}{7}$

$a = \tan^{-1}\frac{1}{7} = 8°8'$

$\therefore 7\cos\theta - \sin\theta = 5\sqrt{2}\cos(\theta + 8°8')$

(ii) $5\sqrt{2}\cos(\theta + 8°8') = 5$

$\cos(\theta + 8°8') = \frac{1}{\sqrt{2}}$

$\theta + 8°8' = 45°\ 315°$

$\theta = 36°52',\ 306°52'$

c) Let $x = \sqrt[4]{m}$

$\therefore x^4 = m$

$x^4 - m = 0$

Let $f(x) = x^4 - m$

$f'(x) = 4x^3$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = x_1 - \frac{x_1^4 - m}{4x_1^3}$

$x_2 = \frac{4x_1^4 - x_1^4 + m}{4x_1^3}$

$x_2 = \frac{3x_1^4 + m}{4x_1^3}$

d) $\frac{d^2x}{dt^2} = 2x-3$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x}$

$\frac{1}{2}v^2 = \int(2x-3)\,dx$

$\frac{1}{2}v^2 = x^2-3x+C$

$v=0, x=4$

$0 = 16-12+C$

$\therefore C = -4$

$\frac{1}{2}v^2 = x^2-3x-4$

$v^2 = 2(x^2-3x-4)$

(ii) Let $x=0$

$v^2 = 2(0-0-4)$

$v^2 = -8$

which is not poss.

$\therefore$ Particle does not pass through origin.

(iii) $10^2 = 2(x^2-3x-4)$

$50 = x^2-3x-4$

$0 = x^2-3x-54$

$0 = (x-9)(x+6)$

$\therefore x=9$ or $x=-6$.

Since particle starts at $x=4$ from rest and does not pass through origin, then only solution is $x=9$.

(iv) Step 1. Prove true for $n=1$

$1(1+3) = 4$ which is div. by 2

$\therefore$ True for $n=1$

Step 2: Assume true for $n=k$

ie. $k(k+3) = 2M$ for a pos integer M

ie. $k^2+3k = 2M$

Step3 : Prove true for $n=k+1$

ie. $(k+1)(k+4)$ is div by 2

$(k+1)(k+4) = k^2+5k+4$

$= k^2+3k+2k+4$

$= 2m+2k+4$ from Step 2 assumption

$=2(m+k+2)$

$\therefore$ True for $n=k+1$

Step4 : By principle of mathematical induction true for all positive integers n.

13. a)

$$\int \frac{dx}{\sqrt{9-4x^2}}$$

$$\int \frac{dx}{\sqrt{4\left(\frac{9}{4}-x^2\right)}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\frac{9}{4}-x^2}}$$

$$= \frac{1}{2}\sin^{-1}\frac{2x}{3} + C$$

1

b) $\frac{dN}{dt} \propto N$

$$\frac{dN}{dt} = k(N-400)$$

i) $N = 400 + Ae^{kt}$

$$\frac{dN}{dt} = k.Ae^{kt}$$

$$= k(N-400)$$ since $N = 400 + Ae^{kt}$

$Ae^{kt} = N - 400$

(ii) $t=0,\ N=1000$

$t=12,\ N=900.$

$t=?\ N=500.$

$$N = 400 + Ae^{kt}.$$

$$1000 = 400 + Ae^{0}$$

$$A = 600$$

$$\therefore N = 400 + 600e^{kt}$$

$$900 = 400 + 600e^{12k}.$$

$$\frac{500}{600} = e^{12k}.$$

$$\frac{1}{12}\ln\frac{5}{6} = k$$

$\therefore N = 400 + 600e^{\frac{1}{12}\ln 5/6\, t}$

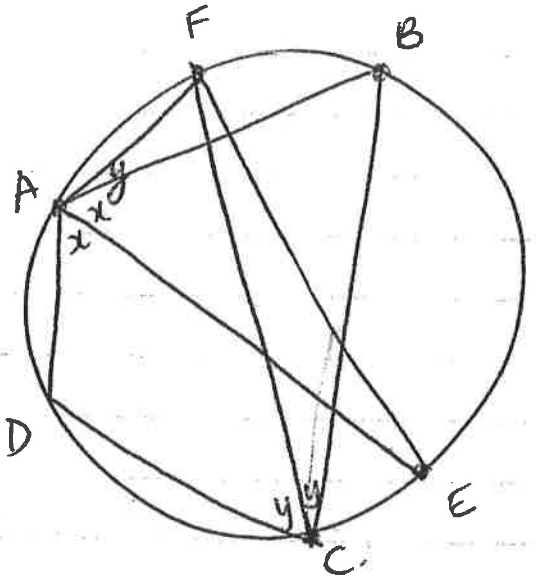
$500 = 400 + 600e^{\frac{1}{12}\ln 5/6\, t}$.

$\frac{100}{600} = e^{1/12 \ln 5/6\, t}$.

$\ln \frac{1}{6} = \frac{t}{12}\ln\frac{5}{6}$

$\frac{12\ln\frac{1}{6}}{\ln 5/6} = t \qquad t = 117.9$ weeks   2

(b)



Let $\angle DAE = \angle BAE = x$

and $\angle FCD = \angle FCB = y$.

$2x + 2y = 180$ (opposite angles of cyclic quad are supplementary)

$\therefore x + y = 90$.

Join AF

$\angle BCF = \angle FAB = y$. (angles in same segment standing on same arc are equal)

$\therefore \angle FAE = x + y$

$= 90°$ $\therefore$ FE is diameter (angle in a semicircle is $90°$).

d) i) If rightangled then from converse of Pyth Thm.

$XY^2 = XT^2 + TY^2$

$LHS = 25^2$

$= 625$

$RHS = 15^2 + 20^2$

$= 225 + 400$

$= 625$

$LHS = RHS$

1

(ii) $\tan\theta = \frac{20}{15} = \frac{4}{3}$

$\theta = \tan^{-1}\frac{4}{3}$

$\therefore \sin\theta = \frac{h}{15}$

$h = 15\sin\left(\tan^{-1}\frac{4}{3}\right)$

$h = 12$

OR using similar Δ's.

2

(iii) Let the angle be θ

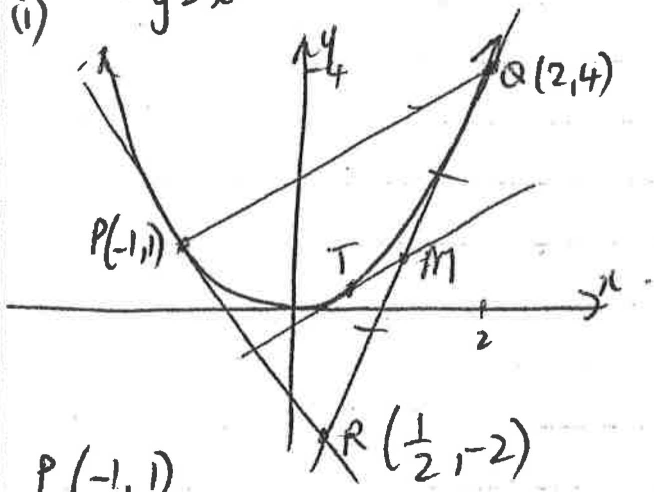
$\tan\theta = \frac{12}{5}$

$\theta = \tan^{-1}\frac{12}{5}$

$= 67^\circ 23'$

e) $x = t,\ y = t^2$

(i) $\therefore y = x^2$



$P(-1, 1)$

$Q(2, 4)$

(ii) $y = x^2$

$\frac{dy}{dx} = 2x$

At $x = -1$

$\frac{dy}{dx} = -2.$

At $x = 2$

$\frac{dy}{dx} = 4$

m of PR : $y - 1 = -2(x+1)$

$y = -2x - 1$

m of QR : $y - 4 = 4(x-2)$

$y - 4 = 4x - 8$

$y = 4x - 4.$

$\therefore y = -2x - 1$

$y = 4x - 4$

2

$\therefore 4x-4=-2x-1$

$6x=3$

$x=\frac{1}{2}$

$\therefore y=-2$

$\angle R\left(\frac{1}{2},-2\right)$

2.

(iii) $\frac{dy}{dx}=2x$

At $T(t,t^2)$

$\frac{dy}{dx}=2t$

Eqn of tang. at T.

$y=2tx-t^2$

Let M = Midpt QR

$=\left(\frac{2+\frac{1}{2}}{2},\frac{4-2}{2}\right)$

$=\left(\frac{5}{4},1\right)$

Sub M into eqn of tang. at T.

$1=2t\left(\frac{5}{4}\right)-t^2$

$1=\frac{5t}{2}-t^2$

$2=5t-2t^2$

$2t^2-5t+2=0$

$2t \quad -1$
$t \quad -2$

$(2t-1)(t-2)=0$

$\therefore t=\frac{1}{2}, t=2$

But Q : $t=2$

$\therefore T : t=\frac{1}{2}$

But m of tangent

$= 2t$

$= 1$

m of $PQ = 4-1/2+1 = 1$

$\therefore$ tangent at T is parallel to PQ.

2

(14)

a)(i) $x = \dfrac{3(\cos 2t - \sin 2t)}{\sqrt{2}}$

$\dot{x} = \dfrac{3}{\sqrt{2}}(-2\sin 2t - 2\cos 2t)$

$\ddot{x} = \dfrac{3}{\sqrt{2}}(-4\cos 2t + 4\sin 2t)$

$= -4 \cdot \dfrac{3}{\sqrt{2}}(\cos 2t - \sin 2t)$

$\ddot{x} = -4x$ which takes the form $\ddot{x} = -n^2 x$ where $n = 2$ 2

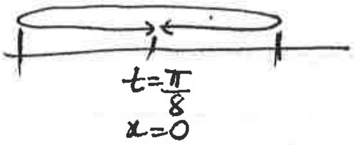
(ii) $T = \dfrac{2\pi}{n} = \dfrac{2\pi}{2} = \pi$ seconds 1

(iii) Sub $t = \dfrac{\pi}{8}$ into x

$x = \dfrac{3\left(\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)}{\sqrt{2}}$

$x = \dfrac{3}{\sqrt{2}}\left(\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}\right)$ 1

$x = 0$

(iv) 

$\frac{\pi}{4}$ seconds to go from centre to endpt. as period $= \pi$.

Amplitude occurs at

$t = \dfrac{\pi}{8} + \dfrac{\pi}{4} = \dfrac{3\pi}{8}$

(since $x=0$ is centre of motion)

Sub $t = \dfrac{3\pi}{8}$ into x

$x = \dfrac{3}{\sqrt{2}}\left(\cos\dfrac{3\pi}{4} - \sin\dfrac{3\pi}{4}\right)$

$= \dfrac{3}{\sqrt{2}}\left(-\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}\right)$ 2

$= \dfrac{3}{\sqrt{2}} \times \dfrac{-2}{\sqrt{2}} = -3$ $\therefore$ Amplitude $= 3$

OK let $v=0$

$$\frac{3}{\sqrt{2}}\left(-2\sin 2t - 2\cos 2t\right) = 0$$

$$-2\sin 2t = 2\cos 2t.$$

$$\tan 2t = -1.$$

$$2t = \pi - \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$$

$$2t = \frac{3\pi}{4},\ \frac{7\pi}{4}$$

$$t = \frac{3\pi}{8},\ \frac{7\pi}{8}$$

Sub $t = \frac{3\pi}{8}$

$$x = \frac{3\left(\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4}\right)}{\sqrt{2}}$$

$$x = \frac{3}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$x = \frac{3}{\sqrt{2}} \cdot \frac{-2}{\sqrt{2}}$$

$x = -3$ Amplitude $= 3$

(b) (i) In ΔPBC

Let $\angle BPC = \alpha$

$$\tan\alpha = \frac{20}{x}.$$

~~tan~~ In ΔAPB

$$\tan(\alpha+\theta) = \frac{32}{x}$$

$$\frac{\tan\alpha + \tan\theta}{1-\tan\alpha\tan\theta} = \frac{32}{x}$$

$$\frac{\frac{20}{x} + \tan\theta}{1 - \frac{20}{x}.\tan\theta} = \frac{32}{x}$$

$$\frac{\frac{20 + x\tan\theta}{x}}{\frac{x-20\tan\theta}{x}} = \frac{32}{x}$$

$$x(20+x\tan\theta) = 32(x-20\tan\theta)$$

$$20x + x^2\tan\theta = 32x - 640\tan\theta$$

$$x^2\tan\theta - 12x + 640\tan\theta = 0.$$

$$\tan\theta(x^2+640) = 12x.$$

$$\tan\theta = \frac{12x}{x^2+640}$$

$$\theta = \tan^{-1}\left(\frac{12x}{x^2+640}\right)$$

(ii)
$$\frac{d\theta}{dx} = \frac{1}{1+\frac{(12x)^2}{(x^2+640)^2}} \times \frac{(x^2+640).12 - 12x(2x)}{(x^2+640)^2}$$

$$= \frac{\cancel{(x^2+640)^2}}{(x^2+640)^2+144x^2} \times \frac{12(x^2+640-2x^2)}{\cancel{(x^2+640)^2}}$$

$$= \frac{12(640-x^2)}{(x^2+640)^2+144x^2}$$

Let $\dfrac{d\theta}{dx} = 0$

$$12(640 - x^2) = 0.$$

$$x^2 = 640$$

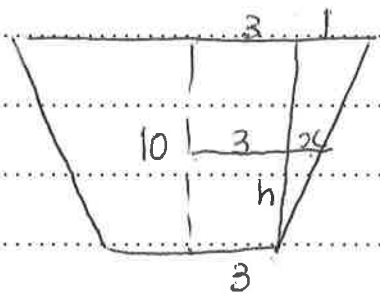
$$x = 8\sqrt{10} \quad x \geqslant 0$$

<u>Test $x = 8\sqrt{10}$</u>

x	25	$8\sqrt{10}$	26
$\dfrac{d\theta}{dx}$	12×15	0	12×-36
	$+$		$-$

Sign change $\nearrow\searrow$ $\therefore$ max at $x = 8\sqrt{10}$

(i)



Using similar triangles

$$\frac{x}{1} = \frac{h}{10}$$

$$x = \frac{h}{10}$$

$H = h$

$a = 3 + \frac{h}{10}$

$b = 3$

$$V = \frac{\sqrt{3}}{2}\left(\left(3 + \frac{h}{10}\right)^2 + 3\left(3 + \frac{h}{10}\right) + 9\right) \cdot h$$

$$= \frac{h\sqrt{3}}{2}\left(9 + \frac{6h}{10} + \frac{h^2}{100} + 9 + \frac{3h}{10} + 9\right)$$

$$= \frac{h\sqrt{3}}{2}\left(\frac{900 + 60h + h^2 + 900 + 30h + 900}{100}\right)$$

$$= \frac{h\sqrt{3}}{2}\left(\frac{2700 + 90h + h^2}{100}\right)$$

$$= \frac{\sqrt{3}}{2}\left(\frac{2700h + 90h^2 + h^3}{100}\right) \times \frac{10}{10}$$

$$V = \frac{\sqrt{3}}{2000}\left(27000h + 900h^2 + 10h^3\right)$$

(ii) $\frac{dV}{dt} = 7$

$$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$$

$$\frac{dV}{dh} = \frac{\sqrt{3}}{2000}\left(27000 + 1800h + 30h^2\right)$$

$$\therefore \frac{dh}{dt} = 7 \times \frac{2000}{\sqrt{3}\left(27000 + 1800h + 30h^2\right)}$$

When $h = 5$

$$\frac{dh}{dt} = 7 \times \frac{2000}{\sqrt{3}\left(27000 + 1800 \times 5 + 30 \times 25\right)}$$

$$= \frac{14000}{\sqrt{3}(36750)}$$

$$= \frac{8}{\sqrt{3}\,21} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{8\sqrt{3}}{63} \text{ cm/s}$$

$$= 0.2199 \text{ cm/s}$$