



2006
TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

Attempt Questions 1 – 7
All questions are of equal value

NAME: _____ **TEACHER:** _____

NUMBER: _____

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

QUESTION 1. (12 marks) Use a *separate* writing booklet **Marks**

- a) Evaluate $\lim_{x \rightarrow 0} \frac{x}{3 \tan 3x}$. **2**
- b) When the polynomial $P(x)$ is divided by $x^2 - 1$ the remainder is $3x - 1$.
What is the remainder when $P(x)$ is divided by $x - 1$? **2**
- c) Solve the inequality $\frac{x^2 - 4}{x} > 0$. **2**
- d) Using the substitution $u = 2 + x^2$, find $\int x\sqrt{2 + x^2} dx$. **2**
- e) Divide the interval PQ internally in the ratio $4 : 9$,
where P is the point $(2, 3)$ and Q is $(5, -7)$. **2**
- f) Differentiate $e^{2x} \cos x$. **2**

QUESTION 2. (12 marks) Use a *separate* writing booklet

- a) Differentiate $\sin^{-1}(5x)$. **2**
- b) Find:
- i) $\int \frac{2}{1 + 9x^2} dx$ **2**
- ii) $\int 5 \cos^2 x dx$ **2**
- c) If α, β, γ are the roots of the equation $x^3 - x^2 + 4x - 1 = 0$
find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. **2**
- d) Consider the function $f(x) = \frac{1}{2} \cos^{-1}(1 - 3x)$.
- i) State the domain and range of $f(x)$. **2**
- ii) Hence, or otherwise, sketch the graph of $y = f(x)$. **2**

QUESTION 3. (12 marks) Use a *separate* writing booklet **Marks**

- a) The only information given about a certain graph is that **2**
 $f(2) = 3$, $f'(2) = 1$ and $f''(2) = -2$

Describe in as much detail as possible, the graph of $f(x)$ near $x = 2$

- b) A formula for the rate of change in population of a colony of bacteria **4**
is given by $P = 3200 + 400 e^{kt}$.

If the population doubles after 20 hours, how long would it take to triple the original population?

- c) i) Show that the equation $5x^4 - 4x^5 - 0.9 = 0$ has a root **1**
between $x = 0$ and $x = 1$.

- ii) Starting with the approximation $x = 1$ attempt to find an **2**
improved value for this root using Newton's Method.
Explain why this attempt fails.

- d) i) Express $\sqrt{3} \cos x - \sin x$ in the form $R \cos(x + \alpha)$ where **2**
 $0 < \alpha < \frac{\pi}{2}$ and $R > 0$.

- ii) Hence, or otherwise, solve $\sqrt{3} \cos x - \sin x = 1$ for $0 \leq x \leq \frac{\pi}{2}$. **1**

QUESTION 4. (12 marks) Use a *separate* writing booklet

- a) Prove $\tan^{-1} \frac{2}{3} + \cos^{-1} \frac{2}{\sqrt{5}} = \tan^{-1} \frac{7}{4}$. **2**

- b) Evaluate $\int_{\frac{1}{2}}^{\frac{e}{2}} \frac{\ln 2x}{x} dx$, using the substitution $u = \ln 2x$. **3**

- c) i) Sketch the curve $y = x + \frac{4}{x}$ showing clearly all the **3**
stationary points and asymptotes.

- ii) Hence, or otherwise, find the values of k such that **1**
 $x + \frac{4}{x} = k$ has no real roots.

- d) Use the method of mathematical induction to prove that, for all **3**
positive integers n ,
 $1 + 2 + 4 + \dots + 2^{n-1} = 2^n - 1$

QUESTION 5. (12 marks) Use a *separate* writing booklet **Marks**

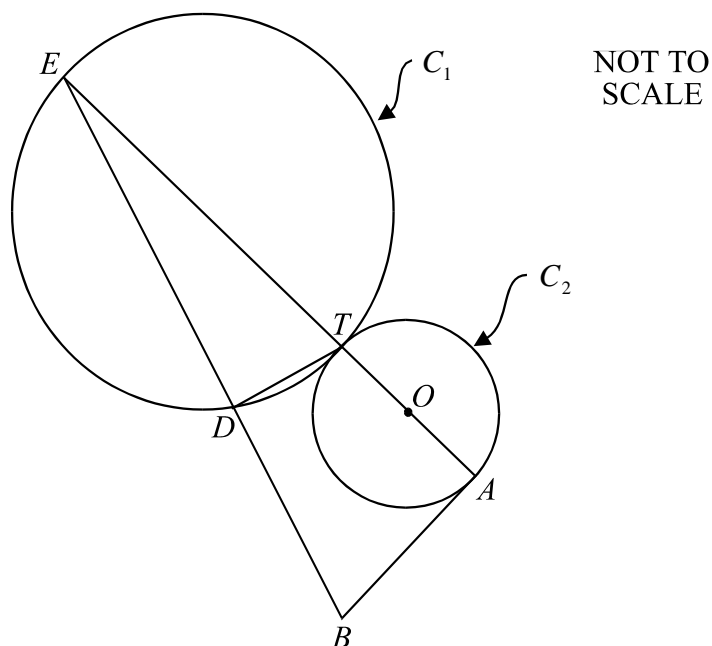
- a) Given that $0 < x < \frac{\pi}{4}$, prove that **2**

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{\cos x + \sin x}{\cos x - \sin x}$$

- b) i) Show that the graphs of $y = 2x - 1$ and $y = x^3$ intersect at $x = 1$. **1**
 ii) Find the size of the acute angle between the graphs at $x = 1$. **2**

- c) A polynomial is given by $p(x) = x^3 + ax^2 + bx - 18$. **3**
 Find values for a and b if $(x + 2)$ is a factor of $p(x)$ and if -24 is the remainder when $p(x)$ is divided by $(x - 1)$.

d)



Two circles C_1 and C_2 touch at T .
 The line AE passes through O , the centre of C_2 , and through T .
 The point A lies on C_2 and E lies on C_1 .
 The line AB is a tangent to C_2 at A , D lies on C_1 and BE passes through D .
 The radius of C_1 is R and the radius of C_2 is r .

- i) Explain why $\angle EDT = 90^\circ$. **2**

- ii) If $DE = 2r$, show that $EB = \frac{2R(R + r)}{r}$. **2**

QUESTION 6. (12 marks) Use a *separate* writing booklet

Marks

- a) The volume, V , of a sphere of radius r mm is increasing at a constant rate of 200 mm^3 per second.

$$\left(V = \frac{4}{3}\pi r^3 \quad ; \quad S = 4\pi r^2 \right)$$

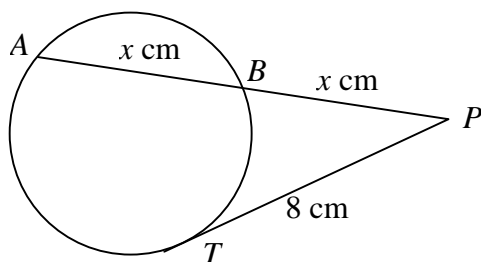
- i) Find $\frac{dr}{dt}$ in terms of r . **2**
- ii) Determine the rate of increase of the surface area, S , of the sphere when the radius is 50 mm **2**
- b) In the diagram below, A , B and T are points on the circumference of the circle. P is an external point. The tangent PT is drawn 8 centimetres long, and B is the midpoint of secant AP .

Let AB be x centimetres.

3

Find the value of x giving reasons.

NOT TO SCALE



- c) i) Show that the equation of the normal at $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ is given by $x + py = ap^3 + 2ap$. **2**
- ii) The normal intersects with the y -axis at point Q . Find the co-ordinates of Q and hence find the co-ordinates of R where R is the midpoint of PQ . **2**
- iii) Hence find the Cartesian equation of the locus of R . **1**

QUESTION 7. (12 marks) Use a *separate* writing booklet **Marks**

- a) Given that $x = \cos t + t \sin t$
 $y = \sin t - t \cos t$
- i) Show that $\frac{dx}{dt} = t \cos t$ **1**
- ii) Hence, or otherwise, find $\frac{dy}{dx}$ in terms of t **2**
- b) At the North Sydney Tennis Competition, Jemma served a ball from a height of 1.8 metres above the ground. The ball was hit in a horizontal direction with an initial velocity of 35 m/s. Assume that the equations of motion for the ball in flight are $y = -5t^2 + 1.8$ and $x = 35t$ where the acceleration due to gravity is taken at 10 m/s^2
- i) How long does it take for the ball to hit the ground? **2**
- ii) How far will the ball travel horizontally before bouncing? **1**
- iii) The net is 0.95 metres high and is 14 metres away from where Jemma hit the ball. Will the ball clear the net? **2**
Explain your answer.
- c) A is the top of a vertical radio mast AB standing on level ground. **4**
 C and D are points on the ground level such that C is due east of B and D is 500 metres due north of C .
The angles of elevation of A from C and D are respectively, $10^\circ 13'$ and $7^\circ 18'$.
Calculate the height of the mast to the nearest metre.
Include a diagram with your answer.

End of paper

Year 12 2006 Trial HSC
Extension 1

Question 1

$$a) \lim_{x \rightarrow 0} \frac{1}{3} \times \frac{3x}{\tan 3x} \times \frac{1}{3} = \frac{1}{9}$$

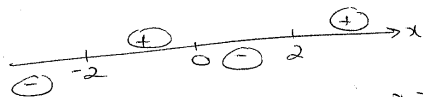
$$b) P(x) = (x^2 - 1)Q(x) + (3x - 1)$$

$$P(1) = 2$$

\therefore remainder when $P(x)$ is divided by $(x-1)$ is 2

$$c) \frac{x^2(x^2 - 4)}{x} > 0 \times x^2$$

$$x(x-2)(x+2) > 0$$



$$\therefore -2 < x < 0 \text{ OR } x > 2$$

$$d) u = 2 + x^2$$

$$\frac{du}{dx} = 2x$$

$$\int x \sqrt{2+x^2} dx = \frac{1}{2} \int u^{\frac{1}{2}} du$$

$$= \frac{1}{2} u^{\frac{3}{2}} \times \frac{2}{3} + C$$

$$= \frac{1}{3} u^{\frac{3}{2}} + C$$

$$= \frac{1}{3} (2+x^2)^{\frac{3}{2}} + C$$

$$= \frac{1}{3} (2+x^2) \sqrt{2+x^2} + C$$

Question 1 cont

$$e) x = \frac{4 \times 5 + 9 \times 2}{4 + 9}$$

$$= \frac{38}{13}$$

$$y = \frac{4 \times -7 + 9 \times 3}{4 + 9}$$

$$= -\frac{1}{13}$$

\therefore point is $(\frac{38}{13}, -\frac{1}{13})$

$$f) \frac{d}{dx} (e^{2x} \cos x) = 2e^{2x} \cos x - e^{2x} \sin x$$

$$= e^{2x} (2 \cos x - \sin x)$$

Question 2

$$a) \frac{d}{dx} (\sin^{-1} 5x) = \frac{1}{\sqrt{1-(5x)^2}} \times 5$$

$$= \frac{5}{\sqrt{1-25x^2}}$$

$$b) i) \int \frac{2}{1+9x^2} dx = 2 \tan^{-1} 3x \times \frac{1}{3} + C$$

$$= \frac{2}{3} \tan^{-1} 3x + C$$

$$ii) \int 5 \cos^2 dx = \frac{5}{2} \int (1 + \cos 2x) dx$$

$$= \frac{5}{2} (x + \frac{\sin 2x}{2}) + C$$

$$= \frac{5}{4} (2x + \sin 2x) + C$$

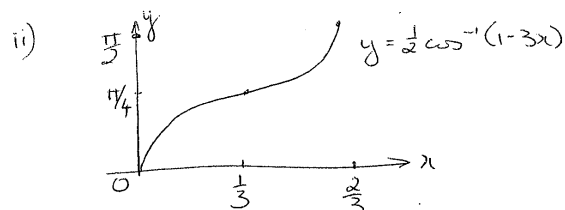
Question 2 cont

$$\begin{aligned} c) (\alpha+1)(\beta+1)(\gamma+1) &= (\alpha+1)(\beta\gamma + \beta + \gamma + 1) \\ &= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \beta\gamma + \alpha + \beta + \gamma + 1 \\ &= 1 + 4 + 1 + 1 \\ &= 7 \end{aligned}$$

$$d) f(x) = \frac{1}{2} \cos^{-1}(1-3x)$$

i) Domain: $-1 \leq 1-3x \leq 1$
 $-2 \leq -3x \leq 0$
 $2 \geq 3x \geq 0$
 $0 \leq x \leq \frac{2}{3}$

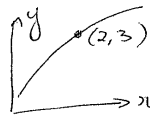
Range: $0 \leq \cos^{-1}(1-3x) \leq \pi$
 $0 \leq f(x) \leq \frac{\pi}{2}$



Question 3

a) at $x=2$, the value of $f(x)$ is 3
 gradient of curve is 1
 \therefore curve is increasing
 second derivative is 2
 \therefore curve is concave downwards

at $(2,3)$ the curve is increasing & concave downwards



Question 3 cont

$$b) P = 3200 + 400e^{kt}$$

when $t=0$, $P = 3200 + 400 = 3600$

when $t=20$, $P = 7200$

$$7200 = 3200 + 400e^{20k}$$

$$4000 = 400e^{20k}$$

$$e^{20k} = 10$$

$$k = \frac{\ln 10}{20}$$

$$10800 = 3200 + 400e^{\frac{t \ln 10}{20}}$$

$$7600 = 400e^{\frac{t \ln 10}{20}}$$

$$e^{\frac{t \ln 10}{20}} = 19$$

$$t = \frac{\ln 19 \times 20}{\ln 10}$$

$$\approx 25.575 \dots$$

Population triples after approximately 25.6 hrs

c) i) $f(x) = 5x^4 - 4x^5 - 0.9$ $f'(x) = 20x^3 - 20x^4$

$$f(0) = -0.9 < 0 \quad f'(1) = 0$$

$$f(1) = 0.1 > 0$$

sign change ($f(x)$ is continuous)
 \therefore root exists between $x=0$ & $x=1$

ii) $x = 1 - \frac{f(1)}{f'(1)}$
 $= 1 - \frac{0.1}{0}$

this attempt fails since $f'(1) = 0$
 re stationary point at $x=1$

Question 3 cont

$$d) i) \sqrt{3} \cos x - \sin x = R \cos(x+\alpha)$$

$$= R \cos x \cos \alpha - R \sin x \sin \alpha$$

$$R \sin \alpha = 1$$

$$R \cos \alpha = \sqrt{3}$$

$$\tan \alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \frac{\pi}{6}$$

$$R^2 = 1+3$$

$$= 4$$

$$R = 2$$

$$\therefore \sqrt{3} \cos x - \sin x = 2 \cos(x + \frac{\pi}{6})$$

$$ii) 2 \cos(x + \frac{\pi}{6}) = 1$$

$$\cos(x + \frac{\pi}{6}) = \frac{1}{2}$$

$$x + \frac{\pi}{6} = \frac{\pi}{3}$$

$$x = \frac{\pi}{6}$$

Question 4

$$a) \text{ Let } \tan^{-1} \frac{2}{3} = x \Rightarrow \tan x = \frac{2}{3}$$

$$\cos^{-1} \frac{2}{\sqrt{5}} = y \Rightarrow \cos y = \frac{2}{\sqrt{5}} \quad \begin{array}{c} \sqrt{5} \\ \hline y \quad 2 \end{array}$$

$$\tan(\tan^{-1} \frac{2}{3} + \cos^{-1} \frac{2}{\sqrt{5}}) = \tan(x+y)$$

$$= \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$= \frac{\frac{2}{3} + \frac{1}{2}}{1 - \frac{2}{3} \times \frac{1}{2}}$$

$$= \frac{\frac{7}{6}}{\frac{1}{3}} = \frac{7}{2}$$

$$= \frac{7}{2}$$

$$= \frac{7}{4}$$

$$\therefore \tan^{-1} \frac{2}{3} + \cos^{-1} \frac{2}{\sqrt{5}} = \tan^{-1} \left(\frac{7}{4} \right)$$

Question 4 cont

$$b) \int_{\frac{1}{2}}^{\frac{e}{2}} \frac{\ln 2x}{x} dx$$

$$u = \ln 2x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\text{when } x = \frac{1}{2}, u = \ln 1 = 0$$

$$\text{when } x = \frac{e}{2}, u = \ln e = 1$$

$$= \int_0^1 u du$$

$$= \left[\frac{u^2}{2} \right]_0^1$$

$$= \frac{1}{2} - 0$$

$$= \frac{1}{2}$$

$$c) y = x + \frac{4}{x} \quad x \neq 0$$

vertical asymptote at $x = 0$

$\cos x \rightarrow \infty, y \rightarrow x$

$\therefore y = x$ is an asymptote.

$$\frac{dy}{dx} = 1 - 4x^{-2}$$

$$1 - \frac{4}{x^2} = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

stationary points at $(2, 4)$ and $(-2, -4)$

$$\frac{d^2y}{dx^2} = 8x^{-3}$$

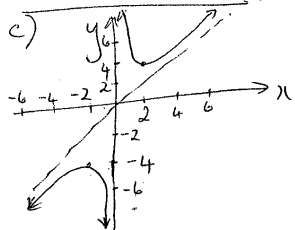
$$= \frac{8}{x^3}$$

at $x = 2, \frac{d^2y}{dx^2} = 1 > 0$ minimum turning point

at $x = -2, \frac{d^2y}{dx^2} = -1 < 0$ maximum turning point

$$\frac{8}{x^3} \neq 0 \therefore \text{no points of inflexion}$$

Question 4 cont



ii) $4 + \frac{4}{x} = k$ has no real roots
if $-4 < k < 4$

d) $1 + 2 + 4 + \dots + 2^{n-1} = 2^n - 1$

let $n=1$ LHS = 1
RHS = $2^1 - 1 = 1$

\therefore result is true for $n=1$

Assume true for $n=k$

i.e. $1 + 2 + 4 + \dots + 2^{k-1} = 2^k - 1$

consider $n=k+1$

LHS = $1 + 2 + 4 + \dots + 2^{k-1} + 2^k$

$= (2^k - 1) + 2^k$

$= 2 \times 2^k - 1$

$= 2^{k+1} - 1$

\therefore true for $n=k+1$ if true for $n=k$

Since the result is true for $n=1$, it is true for $n=1+1=2$ + hence for all positive integers n .

Question 5

a) $\tan\left(\frac{\pi}{4} + x\right) = \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \times \tan x}$
 $= \frac{1 + \tan x}{1 - \tan x}$
 $= 1 + \frac{\sin x}{\cos x}$
 $= \frac{\cos x + \sin x}{\cos x - \sin x}$

b) i) $y = 2x - 1$ when $x=1, y=1$
 $y = x^3$ when $x=1, y=1$
 \therefore graphs intersect at $x=1$

ii) $y = 2x - 1$ $y = x^3$
 $\frac{dy}{dx} = 2$ $\frac{dy}{dx} = 3x^2$
 $m_1 = 2$ $= 3$ when $x=1$
 $m_2 = 3$

$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_2 m_1} \right|$

$= \left| \frac{3 - 2}{1 + 3 \times 2} \right|$

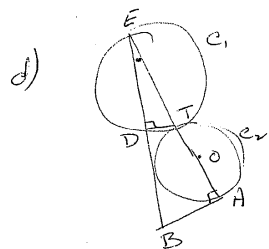
$= \frac{1}{7}$

$\theta = 8^\circ 8'$ (to nearest minute)

the acute angle between the curves is $8^\circ 8'$.

Question 5 cont

$$c) \begin{aligned} p(x) &= x^3 + ax^2 + bx - 18 \\ p(-2) &= -8 + 4a - 2b - 18 = 0 & 4a - 2b &= 26 \\ p(1) &= 1 + a + b - 18 = 24 & 2a - b &= 13 \\ & & a + b &= -7 \\ & \therefore 3a &= 6 \\ & a &= 2 \\ & b &= -9 \end{aligned}$$



- i) AE passes thru centre of C_2 & point of contact of C_1 & C_2 (given)
 \therefore AE passes thru centre of C_1
 $\therefore \hat{EDT} = 90^\circ$ (angle in a semi circle is a right angle)
- ii) $\hat{EAB} = 90^\circ$ (angle between tangents & radius at point of contact)

\hat{E} is common
 $\therefore \triangle EAB \sim \triangle EDT$ (equiangular)
 $\therefore \frac{EB}{ET} = \frac{EA}{ED}$ (ratio of sides of similar triangles)

$$\frac{EB}{2R} = \frac{2R + 2r}{2r}$$

$$EB = \frac{2R(2R + 2r)}{2r}$$

$$= \frac{2R(R + r)}{r}$$

Question 6

a) i) $\frac{dV}{dt} = 200$ $\frac{dV}{dr} = 4\pi r^2$

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

$$= 200 \times \frac{1}{4\pi r^2}$$

$$= \frac{50}{\pi r^2}$$

ii) $\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$

$$= 8\pi r \times \frac{50}{\pi r^2}$$

$$= \frac{400}{r}$$

when $r = 50$, $\frac{dS}{dt} = 8 \text{ mm}^2 \text{ per second}$

b) $AP \cdot PB = PT^2$
 $2x \cdot x = 8^2$
 $2x^2 = 64$
 $x = \sqrt{32}$
 $= 4\sqrt{2}$ ($x > 0$)

c) i) $y = \frac{x^2}{4a}$

$$\frac{dy}{dx} = \frac{x}{2a}$$

when $x = 2ap$, $\frac{dy}{dx} = p \therefore$ slope of normal $= -\frac{1}{p}$

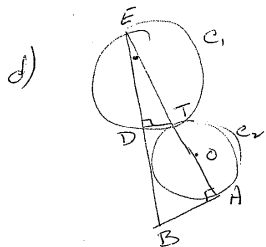
$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

$$py - ap^3 = -x + 2ap$$

$$x + py = ap^3 + 2ap$$

Question 5 cont

$$\begin{aligned}
 c) \quad p(x) &= x^3 + ax^2 + bx - 18 \\
 p(-2) &= -8 + 4a - 2b - 18 = 0 & 4a - 2b &= 26 \\
 p(1) &= 1 + a + b - 18 = 24 & 2a - b &= 13 \\
 & & a + b &= -7 \\
 & \therefore 3a &= 6 \\
 & \quad a &= 2 \\
 & \quad b &= -9
 \end{aligned}$$



- i) AE passes thru centre of C_2 & point of contact of C_1 & C_2 (given)
 \therefore AE passes thru centre of C_1
 $\therefore \hat{EDT} = 90^\circ$ (angle in a semi circle is a right angle)
- ii) $\hat{EAB} = 90^\circ$ (angle between tangents & radius at point of contact)

\hat{E} is common
 $\therefore \triangle EAB \sim \triangle EDT$ (equiangular)

$\therefore \frac{EB}{ET} = \frac{EA}{ED}$ (ratio of sides of similar triangles)

$$\begin{aligned}
 \frac{EB}{2R} &= \frac{2R + 2r}{2r} \\
 EB &= \frac{2R(2R + 2r)}{2r} \\
 &= \frac{2R(R + r)}{r}
 \end{aligned}$$

Question 6

a) i) $\frac{dV}{dt} = 200$ $\frac{dV}{dr} = 4\pi r^2$

$$\begin{aligned}
 \frac{dr}{dt} &= \frac{dr}{dV} \times \frac{dV}{dt} \\
 &= 200 \times \frac{1}{4\pi r^2} \\
 &= \frac{50}{\pi r^2}
 \end{aligned}$$

ii) $\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$

$$\begin{aligned}
 &= 8\pi r \times \frac{50}{\pi r^2} \\
 &= \frac{400}{r}
 \end{aligned}$$

when $r = 50$, $\frac{dS}{dt} = 8 \text{ mm}^2 \text{ per second}$

b) $AP \cdot PB = PT^2$

$$\begin{aligned}
 2x \cdot x &= 8^2 \\
 2x^2 &= 64 \\
 x &= \sqrt{32} \\
 &= 4\sqrt{2} \quad (x > 0)
 \end{aligned}$$

c) i) $y = \frac{x^2}{4a}$

$$\frac{dy}{dx} = \frac{x}{2a}$$

when $x = 2ap$, $\frac{dy}{dx} = p \therefore$ slope of normal $= -\frac{1}{p}$

$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

$$py - ap^3 = -x + 2ap$$

$$x + py = ap^3 + 2ap$$

Question 6 cont

c) ii) $py = ap^3 + 2ap$

$y = ap^2 + 2a$

$Q(0, ap^2 + 2a)$

$P(2ap, ap^2)$

R $\frac{x}{2} = \frac{0 + 2ap}{2}$
 $= ap$

$y = \frac{ap^2 + ap^2 + 2a}{2}$
 $= \frac{2ap^2 + 2a}{2}$
 $= ap^2 + a$

$R(ap, ap^2 + a)$

iii) $x = ap \Rightarrow p = \frac{x}{a}$

$y = ap^2 + a$
 $= a \cdot \frac{x^2}{a^2} + a$

$= \frac{x^2}{a} + a$

$ay = x^2 + a^2$
 $x^2 = a(y - a)$

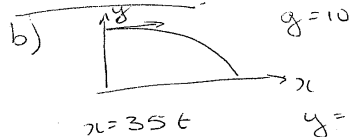
Question 7

a) i) $x = \cos t + t \sin t$
 $\frac{dx}{dt} = -\sin t + \sin t + t \cos t$
 $= t \cos t$

ii) $y = \sin t - t \cos t$
 $\frac{dy}{dt} = \cos t - \cos t + t \sin t$
 $= t \sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$
 $= t \sin t \times \frac{1}{t \cos t}$
 $= \tan t$

Question 7 cont



i) $y = 0 \Rightarrow -5t^2 + 1.8 = 0$
 $t^2 = 0.36$
 $t = 0.6 \text{ (} t > 0 \text{)}$

\therefore ball hits the ground after 0.6 seconds

ii) when $t = 0.6 \Rightarrow x = 35 \times 0.6$
 $= 21$

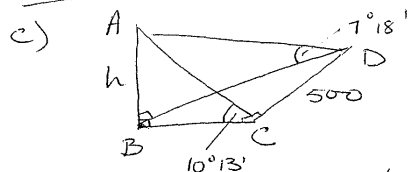
\therefore ball travels 21 metres before bouncing

iii) $x = 14 \Rightarrow 35t = 14$
 $t = \frac{14}{35}$
 $= 0.4$

$y = -5 \times 0.4^2 + 1.8$
 $= 1$
 $1 > 0.95$

since ball is at a height of 1m when $x = 14$, the ball will clear the net.

Question 7 cont



let height of AB be h

$$\frac{h}{BD} = \tan 7^{\circ}18' \Rightarrow BD = \frac{h}{\tan 7^{\circ}18'}$$

$$\frac{h}{BC} = \tan 10^{\circ}13' \Rightarrow BC = \frac{h}{\tan 10^{\circ}13'}$$

$$BC^2 + 500^2 = BD^2$$

$$\frac{h^2}{\tan^2 10^{\circ}13'} + 500^2 = \frac{h^2}{\tan^2 7^{\circ}18'}$$

$$\frac{h^2}{\tan^2 7^{\circ}18'} - \frac{h^2}{\tan^2 10^{\circ}13'} = 500^2$$

$$h^2 \left(\frac{1}{\tan^2 7^{\circ}18'} - \frac{1}{\tan^2 10^{\circ}13'} \right) = 500^2$$

$$h^2 = 500^2 \div \left(\frac{1}{\tan^2 7^{\circ}18'} - \frac{1}{\tan^2 10^{\circ}13'} \right)$$

$$h = 91.057 \dots$$

height of tower is 91 metres.



	Algebra, co-ord geom, parameters	Calculus	Trig	Circle geom	Polynomials	Inverse functions	Applications of calculus	Total
1a			2					
1b					2			
1c	2							
1d		2						
1e	2							
1f		2						12
2a						2		
2b	4							
2c					2			
2d						4		12
3a	2							
3b		4						
3c					3			
3d			3					12
4a						3		
4b							3	
4c		3						
4d	3							12
5a			2					
5b	3							
5c					3			
5d				4				12
6a							4	
6b				3				
6c	5							12
7a	3							
7b							5	
7c			4					12
Totals	24	11	11	7	10	9	12	84