

NORTH SYDNEY GIRLS HIGH SCHOOL



2013
TRIAL HSC EXAMINATION

Mathematics Extension 1

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this booklet
- Show all necessary working in questions 11 – 14

Total Marks – 70

Section 1 10 marks

- Attempt Questions 1 -10
- Allow about 15 minutes for this section.

Section 2 60 Marks

- Attempt Questions 11 - 14

NAME: _____

TEACHER: _____

NUMBER: _____

QUESTION	MARK
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
TOTAL	/70

Section I**Objective response questions****Total marks – 10****Attempt Questions 1–10**Answer each question on the multiple choice answer sheet provided.

1. The polynomial $P(x) = x^4 - kx^3 - 2x + 33$ has $(x-3)$ as a factor.

What is the value of k ?

- (A) -5
- (B) -4
- (C) 4
- (D) 5

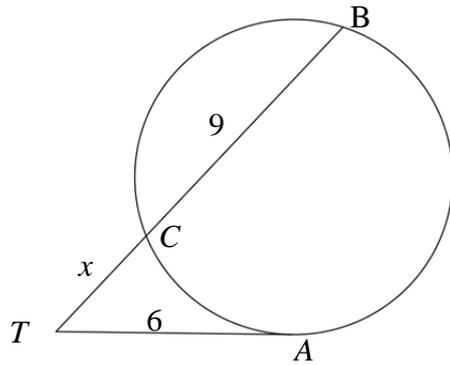
2. Which is the correct condition for $y = mx + b$ to be a tangent to $x^2 = 4ay$?

- (A) $am - b = 0$
- (B) $am^2 - b = 0$
- (C) $am + b = 0$
- (D) $am^2 + b = 0$

3. What is the correct expression for $\int \frac{dx}{9+4x^2}$?

- (A) $\frac{1}{4} \tan^{-1} \left(\frac{2x}{3} \right) + c$
- (B) $\frac{1}{3} \tan^{-1} \left(\frac{2x}{3} \right) + c$
- (C) $\frac{1}{6} \tan^{-1} \left(\frac{2x}{3} \right) + c$
- (D) $\frac{2}{3} \tan^{-1} \left(\frac{2x}{3} \right) + c$

4. Line TA is a tangent to the circle at A . TB is a secant cutting the circle at B and C .



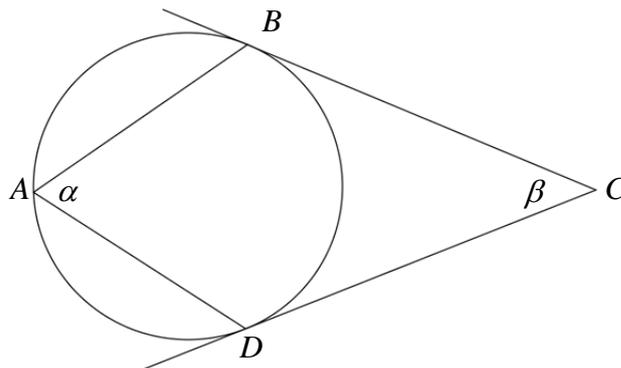
$$TA = 6 \text{ cm}$$

$$BC = 9 \text{ cm}$$

$$TC = x \text{ cm}$$

What is the value of x ?

- (A) 2
 - (B) 3
 - (C) 4
 - (D) 12
5. In the diagram below, BC and DC are tangents. Which statement is correct?



- (A) $\alpha + \beta = 180^\circ$
 - (B) $2\alpha + \beta = 180^\circ$
 - (C) $\alpha + 2\beta = 180^\circ$
 - (C) $2\alpha - \beta = 180^\circ$
6. If $t = \tan \frac{\theta}{2}$, which expression is equivalent to $4 \sin \theta + 3 \cos \theta + 5$?
- (A) $\frac{2(t+2)^2}{1+t^2}$
 - (B) $\frac{2(t+2)^2}{1-t^2}$
 - (C) $\frac{(t+4)^2}{1-t^2}$
 - (D) $\frac{(t+4)^2}{1+t^2}$

7. A particle moves under simple harmonic motion such that its position x metres after t seconds is given by $x = 8 \sin\left(\frac{t}{4} - \frac{\pi}{2}\right)$. Which of the following statements is FALSE?
- (A) The maximum speed of the particle is 2 m/s.
 - (B) The maximum acceleration of the particle is 0.5 m/s^2 .
 - (C) The particle takes 2π seconds to travel between its extremities.
 - (D) The particle is initially to the left of the origin.

8. Consider the function $f(x) = \sqrt{4-x}$. Which of the following is the correct inverse function, $f^{-1}(x)$?

- (A) $f^{-1}(x) = 4 - x^2$
- (B) $f^{-1}(x) = 4 - x^2, x \geq 0$
- (C) $f^{-1}(x) = 4 - x^2, x \leq 0$
- (D) $f^{-1}(x) = 4 - x^2, x \leq 4$

9. Which of the following is an expression for $\int_0^{\frac{1}{2}} x\sqrt{1-2x} \, dx$ using the substitution $u = (1-2x)$?

- (A) $\frac{1}{4} \int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}}) du$
- (B) $\int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}}) du$
- (C) $\frac{1}{4} \int_0^1 (u^{\frac{3}{2}} - u^{\frac{1}{2}}) du$
- (D) $\int_0^1 (u^{\frac{3}{2}} - u^{\frac{1}{2}}) du$

10. Consider the function f such that $f(x) = a \cos^{-1}(x-b)$ given that f has domain $2 \leq x \leq 4$ and range $0 \leq y \leq 6\pi$. What are the values of a and b ?

- (A) $a = 6, b = -3$
- (B) $a = 12, b = 3$
- (C) $a = 12, b = -3$
- (D) $a = 6, b = 3$

Section II**Free response questions****Total marks – 60****Attempt Questions 11–14**Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11. (15 marks) Use a SEPARATE writing booklet.

- (a) (i) Express $\cos 2x$ in terms of $\sin^2 x$. **1**
- (ii) Hence evaluate $\lim_{x \rightarrow 0} \left(\frac{\cos 2x - 1}{x \sin x} \right)$ **1**
- (b) Find all solutions to the equation $\sin 2\theta + \cos \theta = 0$ for $0 \leq \theta \leq 2\pi$. **3**
- (c) Find the size of the acute angle between the lines $x + y - 2 = 0$ and $2x - y = 0$.
Give your answer to the nearest degree. **2**
- (d) Find $\frac{d}{dx} \left[\cos^{-1} \left(\frac{1}{x} \right) \right]$ **2**
- (e) The equation $x^3 - mx + 2 = 0$ has two equal roots.
- (i) Write down expressions for the sum of the roots and the product of the roots. **1**
- (ii) Hence find the value of m . **2**

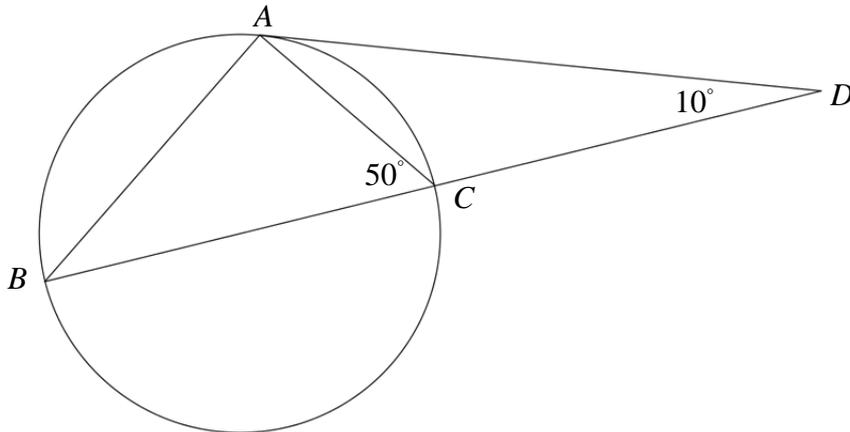
Question 11 continues on page 6

Question 11 (*continued*)

(f) ABC is a triangle inscribed in a circle.

The tangent at A meets BC produced at D .

$$\angle ACB = 50^\circ, \angle CDA = 10^\circ$$



Prove that BC is a diameter of the circle.

3

Question 12. (15 marks) Use a SEPARATE writing booklet.

(a) At a time t minutes after an oven is switched on, its temperature T° is given by

$$T = 250 - 220e^{-0.1t}.$$

- (i) State both the initial and the limiting value of the oven's temperature. 2
- (ii) Find the time taken for the oven's temperature to reach 180° . 2
- (iii) Find the rate at which the temperature is increasing when the temperature reaches 180° . 2

(b) $T(2t, t^2)$ is a point on the parabola $x^2 = 4y$ with focus S . The point P divides ST internally in the ratio 1:2.

- (i) Write down the coordinates of P in terms of t . 2
- (ii) Hence show that as T moves on the parabola $x^2 = 4y$, the locus of P is the parabola $9x^2 = 12y - 8$. 2

(c) (i) Show that $\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$. 1

(ii) Hence evaluate $\int \cos 3x \sin x \, dx$ 2

(d) A particle moves such that its velocity v (metres/second), in terms of its displacement x , is given by $v = 2 + \sin x$.

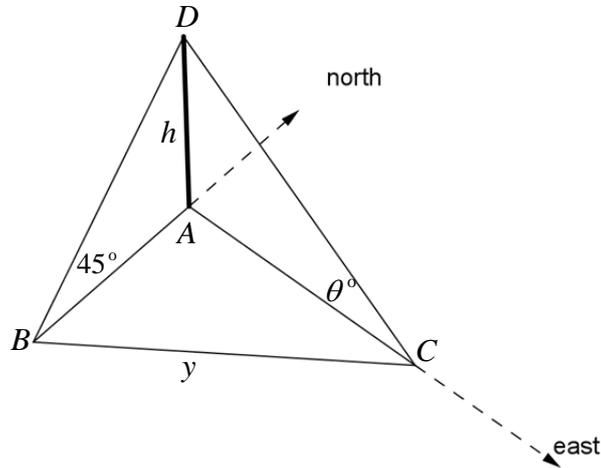
Find the acceleration of the particle at the origin. 2

Question 13. (15 marks) Use a SEPARATE writing booklet.

- (a) Use the substitution $u = \ln x$ to determine $\int \frac{\sqrt{\ln x + \ln x}}{x} dx$ **3**
- (b) Solve the inequality $\frac{2x-5}{x-4} \geq x$. **3**
- (c) Use the method of mathematical induction to prove that $5^n - (-1)^n$ is divisible by 6 where n is a positive integer. **3**
- (d) A particle moves in simple harmonic motion such that its displacement x centimetres after t seconds is given by $x = 5 + 2\sqrt{3} \cos 3t + 2 \sin 3t$.
- (i) Express x in the form $x = 5 + R \cos(3t - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **2**
- (ii) Write down
- (α) the period of the motion **1**
- (β) the range of possible values for x **1**
- (iii) How much time elapses from when the particle first starts until it is first at the point closest to the origin? **2**

Question 14. (15 marks) Use a SEPARATE writing booklet.

(a)



AD is a tower with a height of h metres. A , B and C are 3 points on level ground. B is due south of A and C is due east of A . From B , the angle of elevation of D is 45° . From C , the angle of elevation of D is θ° . Let the length $BC = y$.

Show that $h = y \sin \theta$.

3

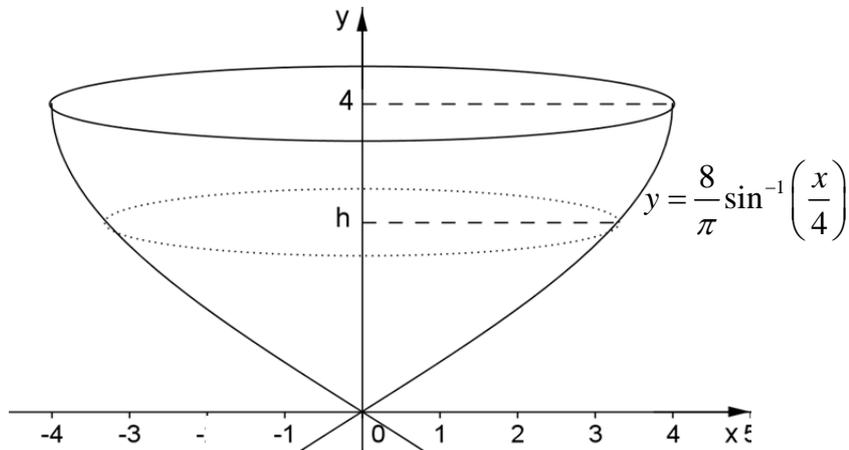
(b) Consider the function $f(x) = e^{2x} + e^x$.

- (i) Explain why $y = f(x)$ has an inverse function $y = f^{-1}(x)$ for all x . **1**
- (ii) Draw a neat sketch of $y = f(x)$ and $y = f^{-1}(x)$ showing all intercepts and asymptotes. **2**
- (iii) Find the equation of the inverse function in terms of x . **3**
- (iv) Hence or otherwise solve $e^{2x} + e^x = 6$. **1**

Question 14 continues on page 10

Question 14 (continued)

- (c) The area bounded by $y = \frac{8}{\pi} \sin^{-1}\left(\frac{x}{4}\right)$, $y = 4$ and the y -axis is rotated about the y -axis to form a storage tank for oil.



- (i) Show that the volume of the oil in cubic metres when the depth is h metres is given by $V = 8\pi h - 32 \sin\left(\frac{\pi h}{4}\right)$ **3**
- (ii) The tank is being filled at a constant rate of $\pi \text{ m}^3/\text{minute}$. Find the rate the depth is increasing when the depth is 2 m. **2**

END OF PAPER

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STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2013 NSGHS EXTENSION 1
TRIAL SOLUTIONS

Section I (answers)

1. C
2. D
3. C
4. B
5. B
6. A
7. C
8. B
9. A
10. D

Section I (solutions)

$$P(x) = x^4 - kx^3 - 2x + 33$$

$$P(3) = 3^4 - k \times 3^3 - 2 \times 3 + 33 = 0$$

$$181 - 27k - 6 + 33 = 0$$

$$27k = 108$$

$$k = 4$$

i.e. C

2.

$$y = mx + b$$

$$x^2 = 4ay$$

$$\therefore x^2 = 4a(mx + b)$$

$$x^2 - 4amx - 4ab = 0$$

$$\Delta = 16a^2m^2 + 16ab = 0$$

$$16a(am^2 + b) = 0$$

$$\therefore am^2 + b = 0$$

i.e. D

3.

$$\int \frac{dx}{9 + 4x^2}$$

$$= \frac{1}{4} \int \frac{dx}{\left(\frac{3}{2}\right)^2 + x^2}$$

$$= \frac{1}{4} \times \frac{2}{3} \tan^{-1} \left(\frac{3x}{2} \right)$$

$$= \frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right)$$

i.e. C

4.

$$x(x+9) = 6^2$$

$$x^2 + 9x - 36 = 0$$

$$(x+12)(x-3) = 0$$

$$x = -12(n.a.), 3$$

$$\therefore x = 3$$

i.e. B

5.

$$\angle BOD = 180 - \beta$$

(cyclic quad *BODC*)

$$\angle BOD = 2 \times \angle BAD$$

(angle at centre)

$$180 - \beta = 2\alpha$$

$$2\alpha + \beta = 180$$

i.e. B

6.

$$4 \sin \theta + 3 \cos \theta + 5$$

$$= 4 \times \frac{2t}{1+t^2} + 3 \times \frac{1-t^2}{1+t^2} + 5$$

$$= \frac{8t + 3 - 3t^2 + 5 + 5t^2}{1+t^2}$$

$$= \frac{2t^2 + 8t + 8}{1+t^2}$$

$$= \frac{2(t+2)^2}{1+t^2}$$

i.e. A

7.

$$x = 8 \sin \left(\frac{t}{4} - \frac{\pi}{2} \right)$$

$$\dot{x} = 2 \cos \left(\frac{t}{4} - \frac{\pi}{2} \right)$$

A true

$$\ddot{x} = -0.5 \sin \left(\frac{t}{4} - \frac{\pi}{2} \right)$$

B true

$$t = 0, x = 8 \sin \left(-\frac{\pi}{2} \right) < 0$$

D true

$$\text{period} = 2\pi \div \frac{1}{4} = 8\pi$$

$$\text{time} = 4\pi$$

C false

i.e. C

8.

$$f(x) = \sqrt{4-x},$$

$$x = \sqrt{4-y}, \text{ range } y \geq 0$$

$$x^2 = 4-y$$

$$y = 4-x^2, \text{ domain } x \geq 0$$

i.e. B

9.

$$I = \int_0^{\frac{1}{2}} x \sqrt{1-2x} \, dx$$

$$u = 1-2x$$

$$\therefore du = -2dx$$

$$x = 0 \rightarrow u = 1$$

$$x = \frac{1}{2} \rightarrow u = 0$$

$$\therefore I = \int_1^0 \frac{1-u}{2} \sqrt{u} \times -\frac{du}{2}$$

$$= \frac{1}{4} \int_0^1 (1-u) \sqrt{u} \, du$$

$$= \frac{1}{4} \int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}}) \, du$$

i.e. A

10.

$$f(x) = a \cos^{-1}(x-b)$$

$$\text{domain } -1 \leq x-b \leq 1$$

$$b-1 \leq x \leq 1+b$$

$$b-1 = 2$$

$$\therefore b = 3$$

$$0 \leq \cos^{-1}(x-b) \leq \pi$$

$$0 \leq a \cos^{-1}(x-b) \leq 6\pi$$

$$\therefore a = 6$$

i.e. D

SECTION II

Question 11

(a)

(i) $\cos 2x = 1 - 2\sin^2 x$

(ii)

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{\cos 2x - 1}{x \sin x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{1 - 2\sin^2 x - 1}{x \sin x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{-2\sin^2 x}{x \sin x} \right) \\ &= -2 \times \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \\ &= -2 \end{aligned}$$

(b)

$$\begin{aligned} \sin 2\theta + \cos \theta &= 0 \\ 2\sin \theta \cos \theta + \cos \theta &= 0 \\ \cos \theta (2\sin \theta + 1) &= 0 \\ \cos \theta = 0, \sin \theta &= -\frac{1}{2} \\ \therefore \theta &= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \end{aligned}$$

(c)

$$x + y - 2 = 0, m_1 = -1$$

$$2x - y = 0, m_2 = 2$$

$$\begin{aligned} \tan \theta &= \left| \frac{-1-2}{1+(-1) \times 2} \right| \\ &= \left| \frac{-3}{-1} \right| \\ &= 3 \\ \therefore \theta &= 71.56^\circ \\ &= 72^\circ \text{ (nearest degree)} \end{aligned}$$

(d)

$$\begin{aligned} & \frac{d}{dx} \left[\cos^{-1} \left(\frac{1}{x} \right) \right] \\ &= \frac{-1}{\sqrt{1 - \left(\frac{1}{x} \right)^2}} \times -x^{-2} \\ &= \frac{1}{x^2 \sqrt{1 - \left(\frac{1}{x} \right)^2}} \\ &= \frac{1}{\sqrt{x^4 - x^2}} = \frac{1}{x\sqrt{x^2 - 1}} \end{aligned}$$

(e)

(i)

$$\begin{aligned} x^3 - mx + 2 &= 0 \\ \therefore \text{roots } \alpha, \alpha, \beta \\ \therefore 2\alpha + \beta &= 0, \alpha^2 \beta = -2 \end{aligned}$$

(ii)

$$\begin{aligned} \beta &= -2\alpha \\ \therefore \alpha^2 \times -2\alpha &= -2 \\ \therefore \alpha^3 &= 1 \\ \therefore \alpha &= 1, \beta = -2 \\ -m &= \alpha^2 + 2\alpha\beta \\ \therefore m &= -(1^2 + 2 \times 1 \times -2) \\ \text{i.e. } m &= 3 \end{aligned}$$

(f)

$$\begin{aligned} \angle ACD &= 180 - 50 = 130 \\ &\text{(straight angle)} \\ \angle CAD &= 180 - 130 - 10 = 40 \\ &\text{(angle sum } \triangle CAD) \\ \angle ABC &= \angle CAD = 40 \\ &\text{(angle in alternate segment)} \\ \angle BAC &= 180 - 40 - 50 = 90 \\ &\text{(angle sum } \triangle ABC) \\ \therefore BC &\text{ a diameter} \\ &\text{(angle in semicircle)} \end{aligned}$$

Question 12

(a)

$$T = 250 - 220e^{-0.1t}$$

(i)

$$\begin{aligned} \text{Initially, } T &= 250 - 220e^0 \\ &= 30^\circ \\ \text{as } t \rightarrow \infty, T &\rightarrow 250 - \frac{220}{e^\infty} \\ \therefore T &\rightarrow 250^\circ \end{aligned}$$

(ii)

$$\begin{aligned} 180 &= 250 - 220e^{-0.1t} \\ 220e^{-0.1t} &= 70 \\ -0.1t &= \ln \left(\frac{70}{220} \right) \\ t &= 10 \ln \left(\frac{22}{7} \right) \\ &= 11.45 \text{ minutes} \end{aligned}$$

(iii)

$$\begin{aligned} T &= 250 - 220e^{-0.1t} \\ \frac{dT}{dt} &= -220 \times -0.1e^{-0.1t} \\ &= 0.1 \times 220e^{-0.1t} \\ &= 0.1(250 - T) \\ \text{when } T &= 180 \\ \frac{dT}{dt} &= 0.1(250 - 180) \\ &= 7^\circ / \text{minute} \end{aligned}$$

(b) (i)

$$\begin{aligned} S(0,1) \quad T(2t, t^2) \\ P \left(\frac{1 \times 2t + 0}{3}, \frac{1 \times t^2 + 2 \times 1}{3} \right) \\ P \left(\frac{2t}{3}, \frac{t^2 + 2}{3} \right) \end{aligned}$$

(ii)

$$x = \frac{2t}{3}$$

$$\therefore t = \frac{3x}{2}$$

$$y = \frac{t^2 + 2}{3}$$

$$3y = \left(\frac{3x}{2}\right)^2 + 2$$

$$3y - 2 = \frac{9x^2}{4}$$

$$9x^2 = 12y - 8$$

(c) (i)

$$\begin{aligned} & \sin(A+B) - \sin(A-B) \\ &= \sin A \cos B + \cos A \sin B \\ & - (\sin A \cos B - \cos A \sin B) \\ &= 2 \cos A \sin B \end{aligned}$$

(ii)

$$\begin{aligned} & \int \cos 3x \sin x \, dx \\ &= \frac{1}{2} \int \sin(3x+x) - \sin(3x-x) \, dx \\ &= \frac{1}{2} \int \sin(4x) - \sin(2x) \, dx \\ &= \frac{1}{2} \left(-\frac{1}{4} \cos 4x + \frac{1}{2} \cos 2x \right) + c \\ &= \frac{1}{4} \cos 2x - \frac{1}{8} \cos 4x + c \end{aligned}$$

(d)

$$v = 2 + \sin x$$

$$\ddot{x} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$= \frac{1}{2} \times \frac{d}{dx} (2 + \sin x)^2$$

$$= \frac{1}{2} \times 2 \times (2 + \sin x) \times \cos x$$

$$= \frac{1}{2} \times 2 \times (2 + \sin 0) \times \cos 0 \quad \text{initially}$$

$$= \frac{1}{2} \times 2 \times 2 \times 1$$

$$= 2$$

Question 13.

(a)

$$\int \frac{\sqrt{\ln x + \ln x}}{x} dx$$

$$u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\therefore \int \frac{\sqrt{\ln x + \ln x}}{x} dx$$

$$\int (\sqrt{u} + u) \frac{dx}{x}$$

$$= \int (\sqrt{u} + u) du$$

$$= \frac{2}{3} u^{\frac{3}{2}} + \frac{1}{2} u^2 + c$$

$$= \frac{2}{3} (\ln x)^{\frac{3}{2}} + \frac{1}{2} (\ln x)^2 + c$$

(b)

$$\frac{2x-5}{x-4} \geq x$$

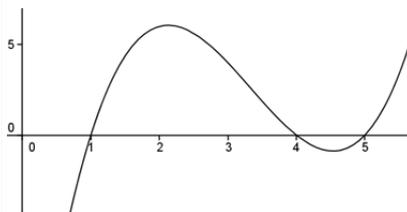
$$\frac{(2x-5)(x-4)^2}{(x-4)} \geq x(x-4)^2$$

$$(2x-5)(x-4) \geq x(x-4)^2$$

$$x(x-4)^2 - (2x-5)(x-4) \leq 0$$

$$(x-4)(x^2 - 4x - 2x + 5) \leq 0$$

$$(x-4)(x-1)(x-5) \leq 0$$



$$x \leq 1 \text{ or } 4 < x \leq 5$$

(c)

to prove $5^n - (-1)^n$ divisible by 6

$$n=1. \quad 5^n - (-1)^n = 5^1 - (-1)^1$$

$$= 5 + 1 = 6$$

\therefore statement true for $n=1$

\therefore can assume statement true for $n=k$

$$\text{i.e. } 5^k - (-1)^k = 6M, M \text{ integer}$$

$$\{5^k = 6M + (-1)^k\}$$

to show true for $n=k+1$

$$\text{i.e. } 5^{k+1} - (-1)^{k+1} \text{ is divisible by 6}$$

$$5^{k+1} - (-1)^{k+1} = 5 \times 5^k - (-1)(-1)^k$$

$$= 5 \times 5^k + (-1)^k$$

$$= 5[6M + (-1)^k] + (-1)^k$$

$$= 30M + 5(-1)^k + (-1)^k$$

$$= 30M + 6(-1)^k$$

$$= 6[5M + (-1)^k]$$

i.e. factor of 6

\therefore true for next term

\therefore true for all $n \geq 1$ by induction

(d) (i)

$$x = 5 + 2\sqrt{3} \cos 3t + 2 \sin 3t$$

$$\equiv 5 + R \cos(3t - \alpha)$$

$$\text{i.e. } 2\sqrt{3} \cos 3t + 2 \sin 3t \equiv R \cos 3t \cos \alpha + R \sin 3t \sin \alpha$$

$$R \cos \alpha \equiv 2\sqrt{3}$$

$$R \sin \alpha \equiv 2$$

$$\therefore \tan \alpha = \frac{1}{\sqrt{3}}$$

$$\therefore \alpha = \frac{\pi}{6}$$

$$R^2 (\cos^2 \alpha + \sin^2 \alpha) = \sqrt{12}^2 + 2^2$$

$$\therefore R = 4$$

$$x = 5 + 4 \cos\left(3t - \frac{\pi}{6}\right)$$

(ii)

$$(\alpha) \text{ period} = \frac{2\pi}{n} = \frac{2\pi}{3}$$

$$(\beta) 1 \leq x \leq 9$$

(iii)

$$x = 1$$

$$1 = 5 + 4 \cos\left(3t - \frac{\pi}{6}\right)$$

$$\cos\left(3t - \frac{\pi}{6}\right) = -1$$

$$3t - \frac{\pi}{6} = \pi$$

$$3t = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$\therefore t = \frac{7\pi}{18} \text{ seconds}$$

Question 14.

(a)

$$\frac{h}{AB} = \tan 45$$

$$\therefore AB = h$$

$$\frac{h}{AC} = \tan \theta$$

$$\therefore AC = h \cot \theta$$

$$y^2 = AB^2 + AC^2 \text{ (Pythag.)}$$

$$= h^2 + h^2 \cot^2 \theta$$

$$= h^2 (1 + \cot^2 \theta)$$

$$= h^2 \operatorname{cosec}^2 \theta$$

$$\therefore y = h \operatorname{cosec} \theta$$

$$y = \frac{h}{\sin \theta}$$

$$\therefore h = y \sin \theta$$

(b) (i)

$$f(x) = e^{2x} + e^x$$

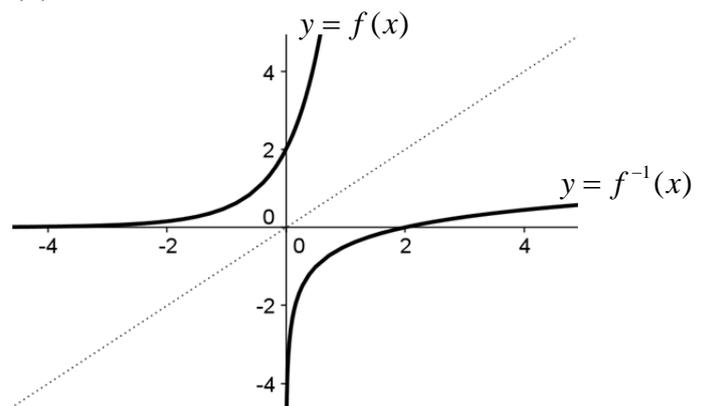
$$f'(x) = 2e^{2x} + e^x$$

$$f''(x) = 4e^{2x} + e^x$$

Always increasing as $f'(x)$ always positive

\therefore inverse for all x

(ii)



(iii)

$$\text{let } y = e^{2x} + e^x$$

for inverse,

$$x = e^{2y} + e^y$$

$$e^{2y} + e^y + \frac{1}{4} = x + \frac{1}{4}$$

$$\left(e^y + \frac{1}{2}\right)^2 = \frac{4x+1}{4}$$

$$e^y + \frac{1}{2} = \pm \frac{\sqrt{4x+1}}{2}$$

$$e^y = \frac{-1 \pm \sqrt{4x+1}}{2}$$

$$y = \ln\left(\frac{-1 \pm \sqrt{4x+1}}{2}\right)$$

but $-1 \pm \sqrt{4x+1}$ must be positive

$$\therefore f^{-1}(x) = \ln\left(\frac{\sqrt{4x+1}-1}{2}\right)$$

(iv)

$$e^{2x} + e^x = 6$$

$$f(x) = 6$$

$$\therefore x = f^{-1}(6)$$

$$= \ln\left(\frac{\sqrt{4 \times 6 + 1} - 1}{2}\right)$$

$$= \ln 2$$

(c) (i)

$$y = \frac{8}{\pi} \sin^{-1}\left(\frac{x}{4}\right)$$

$$\sin^{-1}\left(\frac{x}{4}\right) = \frac{\pi y}{8}$$

$$\frac{x}{4} = \sin\left(\frac{\pi y}{8}\right)$$

$$x = 4 \sin\left(\frac{\pi y}{8}\right)$$

$$V = \pi \int_0^h x^2 dy$$

$$= \pi \int_0^h \left(4 \sin\left(\frac{\pi y}{8}\right)\right)^2 dy$$

$$= 16\pi \int_0^h \sin^2\left(\frac{\pi y}{8}\right) dy$$

$$= 16\pi \int_0^h \frac{1}{2} (1 - \cos\left(\frac{\pi y}{4}\right)) dy$$

$$= 8\pi \left[y - \frac{4}{\pi} \sin\left(\frac{\pi y}{4}\right) \right]_0^h$$

$$= 8\pi \left[h - \frac{4}{\pi} \sin\left(\frac{\pi h}{4}\right) \right]$$

$$= 8\pi h - 32 \sin\left(\frac{\pi h}{4}\right)$$

(ii)

$$\frac{dV}{dt} = \pi$$

$$V = 8\pi h - 32 \sin\left(\frac{\pi h}{4}\right)$$

$$\frac{dV}{dh} = 8\pi - 8\pi \cos\left(\frac{\pi h}{4}\right)$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$\pi = \left(8\pi - 32 \cos\left(\frac{\pi h}{4}\right)\right) \times \frac{dh}{dt}$$

$$\pi = \left(8\pi - 32 \cos\left(\frac{\pi \times 2}{4}\right)\right) \times \frac{dh}{dt}$$

$$\pi = 8\pi \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{8} \text{ cm / minute}$$