Student Name:____________________ Teacher Name:____________________

Penrith High School

Mathematics Department

TRIAL HIGHER SCHOOL CERTIFICATE
2008

Year 12 Mathematics Extension 1

Time Allowed: 2 HOURS plus 5 minutes reading time

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.
- Only approved calculators may be used.

Office Use Only

Question	1	2	3	4	5	6	7	8	TOTAL	%
Mark	/9	/8	/10	/10	/10	/9	/10	/10	/76	

Question(1)

a) Solve the inequality $\frac{2t+1}{t-2} > 1$.†

b) A and B are the points $(-2, -1)$ and $(1, 5)$ respectively. Find the co-ordinates of the point P which divides AB externally in the ratio $5 : 3$.†

c) Prove by mathematical induction that

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$ †

Question(2)

a)

Evaluate $\int_0^3 \frac{x}{\sqrt{1+x}}\,dx$ using the substitution $x = u^2 - 1$.

b) Consider the function $f(x) = \frac{x}{x^2 - 4}$.

i. Find the natural domain of the function.

ii. Show that the function is decreasing throughout its natural domain.

iii. Sketch the graph of the function showing clearly the coordinates of any points of intersection with the x axis or the y axis and the equations of any asymptotes.

Question(3)

a) The function $h(x)$ is given by $h(x) = \cos 3x$. The graph $y = h(x)$ for $0 \le x \le \frac{\pi}{6}$ is rotated about the x-axis. Find the volume of the solid generated. ¤

b) From a point A the bearings of two points B and C are 065°T and 105°T respectively.
From a point D, 5km due east of A, the bearings of B and C are 030°and 117°T.
If the distance between B and C is d km,

i. Draw a diagram of this information

ii. Show that $AC = \frac{5\sin 27°}{\sin 12°}$

iii. By considering ΔABC show that

$$d^2 = 25\{(\frac{\sin 60°}{\sin 35°})^2 + (\frac{\sin 27°}{\sin 12°})^2 - \frac{2\sin 60° \sin 27° \cos 40°}{\sin 35° \sin 12°}\}$$

c) Find all angles θ where $-\pi \le \theta \le \pi$ for which $\sin 2\theta = \cos \theta$¤

Question(4)

a) Find $\lim\limits_{x \to 0} \frac{\sin 5x}{2x}$.

b) i. Show that: $\cos\theta - \sqrt{3}\sin\theta = 2\cos(\theta + \frac{\pi}{3})$.

ii. Hence solve the equation $\cos\theta - \sqrt{3}\sin\theta = 1$ for θ in the interval $0 \le \theta \le 2\pi$.

c) By making the substitution $t = \tan\frac{\theta}{2}$ show that $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$.

d)

Tangents are drawn to the curve $y = e^x$ at the points where $x = 0$ and $x = 1$.

Find i. the gradients of each of these tangents

ii. the acute angle between these tangents correct to the nearest degree.†

Question(5)

a) i. Show that the equation $2x^3 + x - 8 = 0$ has a root that lies between $x=1$ and $x=2$.

ii. Taking $1{\cdot}5$ as a first approximation to this root, use Newton's Method to obtain a second approximation

b) Consider the polynomial $P(x) = 2x^3 - 3x^2 - 11x + 6$

i. Show that 3 is a zero of $P(x)$.

ii. Express $P(x)$ as a product of 3 linear factors.

iii. Sketch the graph of $y=P(x)$ and solve the inequality $P(x) \le 0$.

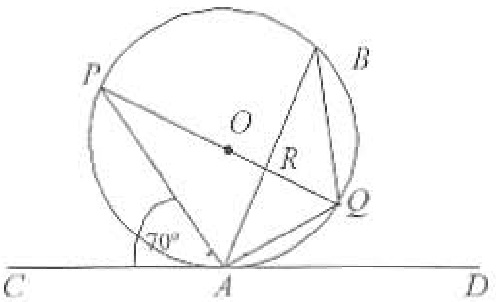
c) The equation $x^3 + 2x^2 - 4x - 12 = 0$ has roots α, β and γ. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

Question(6)

a) Differentiate: $\tan^{-1} 7x$.

b) Find the exact value of $\int_{-2}^{2} \frac{1}{\sqrt{16 - x^2}}\,dx$.

c)

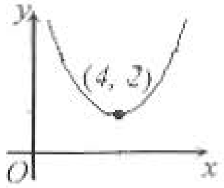


In the figure PQ is a diameter of the circle centre O. CD is a tangent contacting the circle at A.
$\angle CAP = 70^o$

i. Copy the diagram

ii. Find, giving reasons, the size of $\angle ABQ$

d) The graph of $y = (x-4)^2 + 2$ is shown in the diagram.



i. Find the largest positive domain for which the graph defines a function $f(x)$ which has an inverse.

ii. Find this inverse function and state its domain.

Question(7)

a) i. Show that $\frac{d}{dx}(\frac{1}{2}v^2) = \frac{dv}{dt}$.

ii. A particle moves in a straight line with velocity v cms^{-1} so that

$$v^2 = -9x^2 + 18x + 27$$

where x centimetres is its displacement from a fixed point O.

α. Prove that the motion is simple harmonic

β. Find the centre of motion, the period and the extreme points of the motion

b) i. At any time t the rate of cooling of the temperature T of a body, when the surrounding temperature is P, is given by the equation $\frac{dT}{dt} = -k(T - P)$, for some constant k. Show that the solution $T = P + Ae^{-kt}$, for some constant A, satisfies this equation.

ii. A heated body is immersed in a water bath kept at a constant 25°C and cools from 180°C to 120°C in 12 minutes. After how many minutes from the start of cooling does the body cool to 90°?

Question(8)

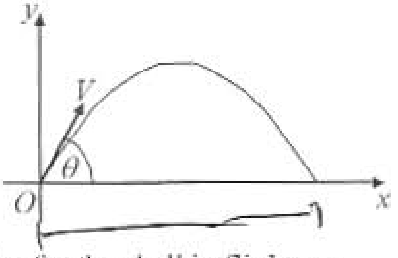
a). $P(2ap, ap^2)$ is a variable point on the parabola $x^2 = 4ay$ and S is the focus. The interval joining P and S is produced to Q so that $PS = SQ$. Find

a. Find the co-ordinates of Q in terms of p

b. Find the Cartesian equation of the locus of Q.

c. Describe the locus in words

b) A gun fires shells with velocity $V = 200$ metres per second at an elevation of θ degrees, $0^\circ \le \theta \le 90^\circ$



i. Show the equations of motion for the shell in flight are
$x = 200\cos\theta t$ and $y = -5t^2 + 200\sin\theta t$ (Air resistance is to be neglected and the acceleration due to gravity is taken as 10 ms^{-2}.)

ii. Show that the range of the shell is $4000\sin 2\theta$ metres

iii. Between what values must θ lie for the range of the shell to be greater than 3000metres?

[End Of Qns]

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ if\ n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a \neq 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$