Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the length of the vector projection of $\begin{bmatrix}3\\1\end{bmatrix}$ onto $\begin{bmatrix}2\\-3\end{bmatrix}$?

A. $\frac{3\sqrt{13}}{13}$ units

B. $\frac{3\sqrt{10}}{10}$ units

C. $\frac{3\sqrt{10}}{13}$ units

D. $\frac{3\sqrt{13}}{10}$ units

2 Which of the following statement does *not* describe a Bernoulli random variable?

A. Selecting a faulty phone in a random quality control check.

B. Guessing the correct answer to a true/false question.

C. The number of tails when tossing 2 coins.

D. A missile hitting a target.

3 The polynomial $P(x) = x^4 + ax^3 - 3x^2 + bx - 2 = 0$ has roots -1 and 2, one of which is a triple root. Find the values of a and b.

A. $a = -1$ and $b = 2$

B. $a = 2$ and $b = -5$

C. $a = 1$ and $b = 3$

D. $a = 1$ and $b = -5$

4 What is the derivative of $\tan^{-1} 4x$?

A. $\dfrac{4}{\sqrt{16 - x^2}}$

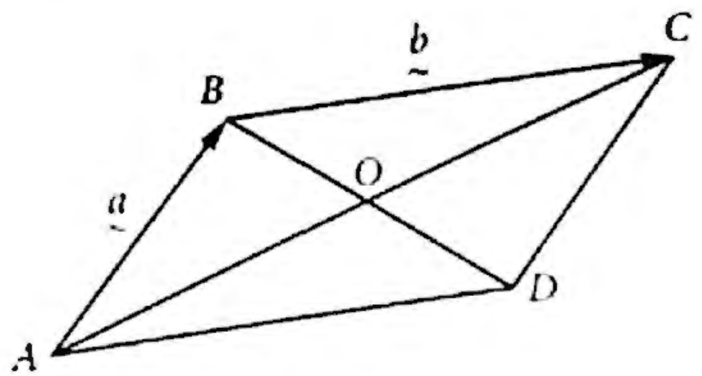
B. $\dfrac{4}{\sqrt{x^2 - 16}}$

C. $\dfrac{4}{1 - 16x^2}$

D. $\dfrac{4}{1 + 16x^2}$

5 In the parallelogram ABCD shown, the point of intersection of the diagonals is at O, where O is the midpoint of both $\overrightarrow{AC}$ and $\overrightarrow{BD}$.

The vector $\overrightarrow{OC}$ is equal to



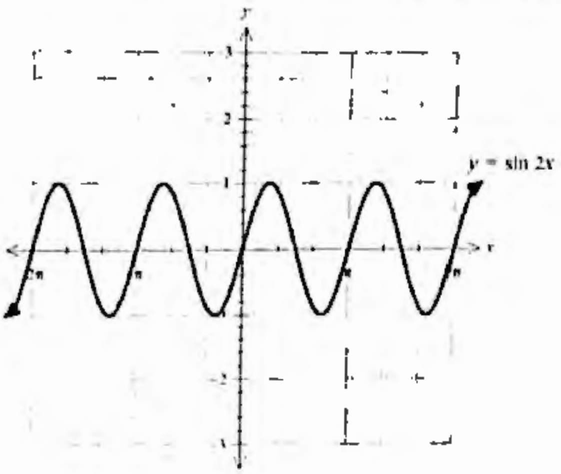
A. $\frac{1}{2}(\overrightarrow{AB} - \overrightarrow{BC})$

B. $\frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC})$

C. $(\frac{1}{2}\overrightarrow{AB} - \overrightarrow{BC})$

D. $\frac{1}{2}(\overrightarrow{BC} - \overrightarrow{AB})$

6 The function $y = \sin(2x)$ is shown in the diagram below.



If this function is transformed using steps I, II and III as below:

I : Reflected in x-axis

II: Vertically translated 1 unit down

III: Dilated horizontally by a scale factor of 2.

Which one of the following equations would represent the transformed function?

A. $y = -(\sin x + 1)$

B. $y = 2(\sin(2x) - 1)$

C. $y = -(\sin(4x)) + 2$

D. $y = -2\sin(2x) - 1$

7 The continuous random variable, X, has the following probability density function:

$$f(x) = \begin{cases} ax^2(4-x), & 1 \le x \le 4 \\ 0 & otherwise \end{cases}$$

What could be the value of a?

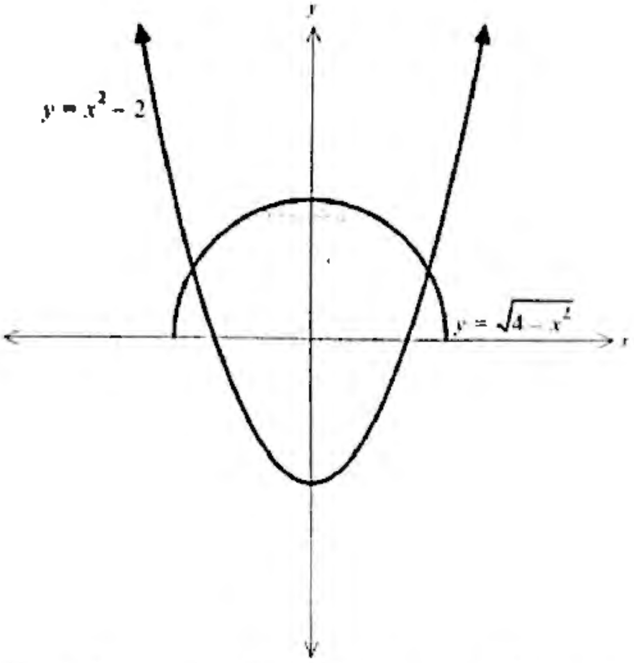
A. $\frac{81}{4}$

B. $\frac{4}{81}$

C. 0

D. 1

8 The area enclosed by the curves $y = \sqrt{4 - x^2}$ and $y = x^2 - 2$ is rotated about x – axis.



Which one of the expressions could be used to calculate the volume?

A. $$V = \pi \int_0^{\sqrt{3}} \left(\sqrt{4 - x^2} - (x^2 - 2)\right)^2 dx$$

B. $$V = \pi \int_0^{\sqrt{3}} (3x^2 - x^4) dx$$

C. $$V = 2\pi \int_0^{\sqrt{3}} (3x^2 - x^4) dx$$

D. $$V = 2\pi \int_0^{\sqrt{3}} \left(\sqrt{4 - x^2} - (x^2 - 2)\right)^2 dx$$

9 A large tank initially holds 2500L of water in which 100kg of salt is dissolved. A solution containing 4kg of salt per litre flows into the tank at a rate of 10L per minute. The mixture is stirred continuously and flows out of the tank through a hole at a rate of 14L per minute.

A differential equation for **Q**, the number of kilograms of salt in the tank after t minutes, is given by

A. $\dfrac{dQ}{dt} = 40 - \dfrac{7Q}{2(625 - t)}$

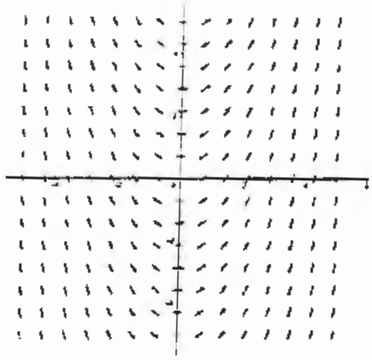
B. $\dfrac{dQ}{dt} = 40 + \dfrac{7Q}{2(625 - t)}$

C. $\dfrac{dQ}{dt} = 40 - \dfrac{7Q}{2(625 + t)}$

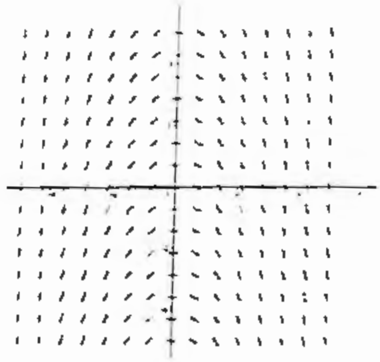
D. $\dfrac{dQ}{dt} = 40 + \dfrac{7Q}{2(625 + t)}$

10 Which of the following slope field best represents the differential equation $y' = x + y$

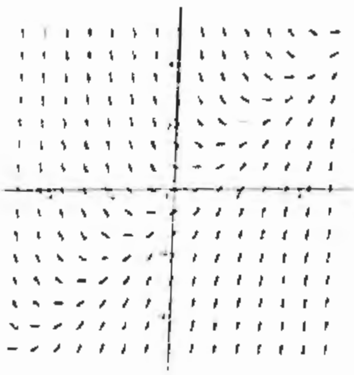
A.



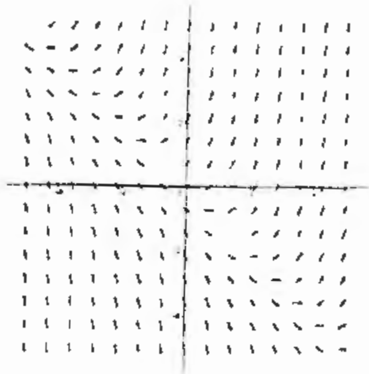
B.



C.



D



END OF SECTION I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question on a new writing sheet. Extra writing sheets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\int_{-1}^{0} 2x\sqrt{1+x}\,dx$ using the substitution $u = 1 + x$. *3*

(b) Find the coefficient of x^4 in the expansion of $(2x^2 - \frac{1}{x})^{11}$. *3*

(c) (i) Solve the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ *1*

(ii) Find the solution of this equation that satisfies the initial condition $y(0) = 2$. *1*

(d) Consider the function $y = cos^{-1}(\sin x)$.

(i) Find $\frac{dy}{dx}$. *1*

(ii) Clearly explain why $\frac{dy}{dx}$ can be written as 1 or -1. *2*

(iii) State the domain and range for this function. *2*

(iv) Hence, neatly sketch the function over the domain $0 \leq x \leq 2\pi$. *2*

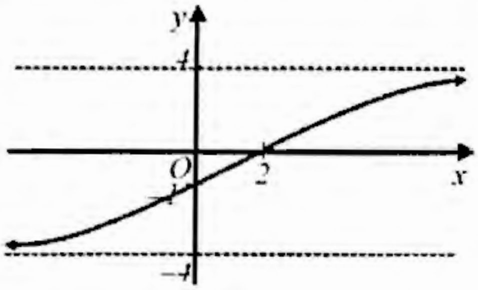
End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Express $\sqrt{3}\cos x - \sin x$ in the form $A\,cos(x + \alpha)$ where α is an acute angle. *2*

(ii) Hence, or otherwise, solve the equation $\sqrt{3}\cos x - \sin x = 1$ for $0 \le x \le 2\pi$ *2*

(b) The graph of $y = f(x)$ is illustrated. The lines $y = \pm 4$ are horizontal asymptotes.



Without using Calculus, sketch each of the graphs below. In each case, clearly label any maxima or minima, intercepts and the equations of any asymptotes.

(i) $y = f(x + 2)$ *1*

(ii) $y = |f(x)|$ *1*

(iii) $y = \sqrt{f(x)}$ *2*

(c) Find the values of k for which the non-zero vectors $\tilde{a} = k^2 i + 2j$ and $\tilde{b} = 3i - (2 + 2k)j$ are perpendicular. *3*

(d) Use the Principle of Mathematical Induction to show that if x is a positive integer then $(1 + x)^n - 1$ is divisible by x for all positive integers $n \ge 1$. *4*

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The equation $4x^3 - 27x + k = 0$ has a double root. Find the possible value(s) of k. *3*

(b) Use vectors to prove that the sum of the squares of the lengths of the two diagonals of a parallelogram is equal to the sum of the squares of the lengths of the four sides. *3*

(c) A company manufactures lithium batteries using a mixture of digital and traditional techniques. Data shows the probability that a random case will fail quality control is 8%. An inspector selects a random batch of 50 cases from the warehouse.

Let X be the binomial random variable of the number of cases that do not pass the inspection.

(i) What is the mean, variance and standard deviation for this distribution, correct to 2 decimal places? *3*

(ii) Find the probability that the number of cases that fail to pass within one standard deviation of the mean. *2*

(iii) A new company standard insists that the number of failures in a batch must be no more that one standard deviation above the mean. Batches that fail to meet this standard are rejected. *2*

What is the probability of a batch being rejected?

(iv) Due to the new regulations and the number of rejected batches, the company improves its manufacturing process so that the new experimental probability of failure is reduced to 4%. Find the new probability that a batch will be accepted. *2*

End of Question 13

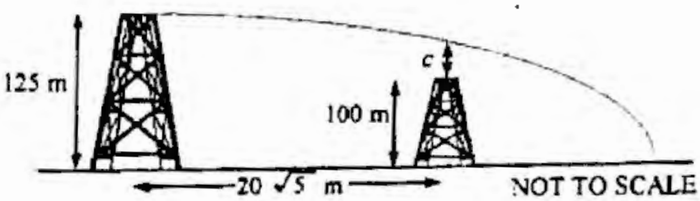
Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Express $\sin 2\theta$ and $\cos 2\theta$ in terms of $t = \tan\theta$. *2*

(ii) Hence, or otherwise, prove that, for $0 < \theta < \frac{\pi}{2}$, *2*

$$\frac{1+\sin 2\theta - \cos 2\theta}{1+\sin 2\theta + \cos 2\theta} = \tan\theta$$

(b) A projectile is shot horizontally from the top of a 125 metre tower with a velocity of V metres per second. The projectile clears a building of height 100 metres by a distance of c metres, as shown in the diagram below. The two towers are $20\sqrt{5}$ metres apart. Air resistance is ignored and use $g = 10 m/s^2$.



(i) Derive the six equations of motion. *2*

(ii) Show that the displacement vector of the projectile at any time t is given by $s(t) = Vt\,i + (125 - 5t^2)j$. *1*

(ii) Show that $V = \frac{100}{\sqrt{25-c}}$. *2*

(iii) Prove that the minimum speed of projection for the projectile to just clear the 100 m tower is 20 m/s *1*

(iv) Hence, find how far past the 100 metre tower the projectile will strike the ground. *2*

(v) Determine the velocity and the angle of projection when it strikes the ground. *3*

End of Question 14

End of paper

Ext 1 Solutions

Monday, 31 August 2020 8:39 AM

1. $\left|\text{Proj}_{\underset{\sim}{b}}\underset{\sim}{a}\right| = \left|\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\right|\left|(\underset{\sim}{b})\right|$

$$= \frac{6-3}{4+9}\begin{pmatrix}2\\-3\end{pmatrix}$$

$$= \frac{3}{13}\left|\sqrt{13}\right|$$

$$= \frac{3\sqrt{13}}{13} \quad \boxed{A}$$

2. $\boxed{C}$

3. $\alpha, \alpha, \alpha, \beta$

$$P(\alpha) = P'(\alpha) = P''(\alpha) = 0$$

$$P'(x) = 4x^3 + 3ax^2 - 6x + b$$

$$P''(x) = 12x^2 + 6ax - 6$$

$$2x^2 + ax - 1 = 0$$

$$-1, -1, -1, 2$$

$$\cancel{+2\alpha + 1 + 1 - 2 - 2 = -3}$$

$$1 + 1 - 2 + 1 - 2 - 2 = -3$$

$$-1 - \cancel{1} - \cancel{1} + \cancel{2} = -a$$

$$\boxed{a = 1}$$

$$P(-1) = 0, \quad 1 - 1 - 3 - b - 2 = 0$$

$$\boxed{b = -5} \quad \boxed{D}$$

4. $y = \tan^{-1} 4x$

$$y' = \frac{1}{1 + 16x^2} \times 4 = \boxed{D}$$

5. $\boxed{B}$

6. $\boxed{A}$

7. $\int_1^4 (4ax^2 - ax^3)\,dx = 1$

$\left[4a\frac{x^3}{3} - \frac{ax^4}{4}\right]_1^4 = 1$

$a \times \left[64\times\frac{4}{3} - 64 - \frac{4}{3} + \frac{1}{4}\right] = 1$

$a\left[\frac{60}{3} + \frac{1}{4}\right] = 1$

$a\left[\frac{243}{12}\right] = 1 \quad \Rightarrow \quad a = \frac{\cancel{12}\ 4}{\cancel{243}\ 81}$ $\boxed{B}$

8. $\cancel{4} - x^2 = x^4 - 4x^2 + \cancel{4}$

$x^4 - 3x^2 = 0.$

$x^2(x^2-3) = 0, \quad x = \pm\sqrt{3}, 0.$

$V = 2\pi \int_0^{\sqrt{3}} \left[(4-x^2) - (x^2-2)^2\right] dx$

$= 2\pi \int_0^{\sqrt{3}} (\cancel{4} - x^2 - x^4 + 4x^2 - \cancel{4})\,dx$

$= 2\pi \int_0^{\sqrt{3}} (3x^2 - x^4)\,dx.$ $\boxed{C}$

9. $\frac{dQ}{dt} = 4\text{ kg} \times 10\text{ L/min} - \frac{Q \times 14\text{ L/min}}{(2500 + (10\text{ L/min} - 14\text{ L/min})t)}$

$= 40 - \frac{14Q}{(2500 - 4t)}$

$= 40 - \frac{7Q}{2(625 - t)}$ $\boxed{A}$

10. $\boxed{D}$

Question 11

Question 11

a) $u = 1 + x$, $x = (u-1)$

$du = dx$

$$I = \int_0^1 2(u-1)u^{1/2}\,du$$

$$= \int_0^1 2u^{3/2} - 2u^{1/2}\,du$$

$$= \left[\frac{2u^{5/2}}{5} \times 2 - \frac{2u^{3/2}}{3} \times 2\right]_0^1$$

$$= -\frac{8}{15}$$

This question was done well overall.

- Only few forgot 2 at the front
- A very few forgot to integrate and they just sub in the value.

b) $\left(2x^2 - \frac{1}{x}\right)^{11}$

the general term is given by

$$^{11}C_k \, (2x^2)^{11-k} \left(-\frac{1}{x}\right)^k$$

$$= {}^{11}C_k \, 2^{11-k} \, x^{22-2k} \, (-1)^k \, x^{-k}$$

For x^4, $22 - 3k = 4 \Rightarrow \boxed{k = 6}$

the coefficient of x^4:

$$^{11}C_6 \, 2^5 (-1)^6$$

$$= 14784$$

- many students did find the correct value of k.
- some students expanded without using the general term
- some wrote the general term from backwards and got $\boxed{k = 5}$
- very few wrote $\boxed{-14784}$ which is marked incorrect.

(c) (i) $\dfrac{dy}{dx} = \dfrac{x^2}{y^2}$

$y^2 dy = x^2 dx$ (variable-separable)

integrate both sides,

$\dfrac{y^3}{3} = \dfrac{x^3}{3} + C$

$\cancel{y^3 = x^3}$

(ii) $y(0) = 2 \Rightarrow \boxed{C = \dfrac{8}{3}}$

- very well done on this part
- some students wrote $y = x+2$ instead

d) (i) $y = \cos^{-1}(\sin x)$

$y' = \dfrac{-\cos x}{\sqrt{1-\sin^2 x}}$

very well done

(ii) $y' = \dfrac{-\cos x}{|\cos x|} = \pm 1$

$y' = -1$ when $\cos x > 0$ 1^{st} & 4^{th} Quad.

$y' = 1$ when $\cos x < 0$ ~~2nd~~ / 3^{rd} Quad.

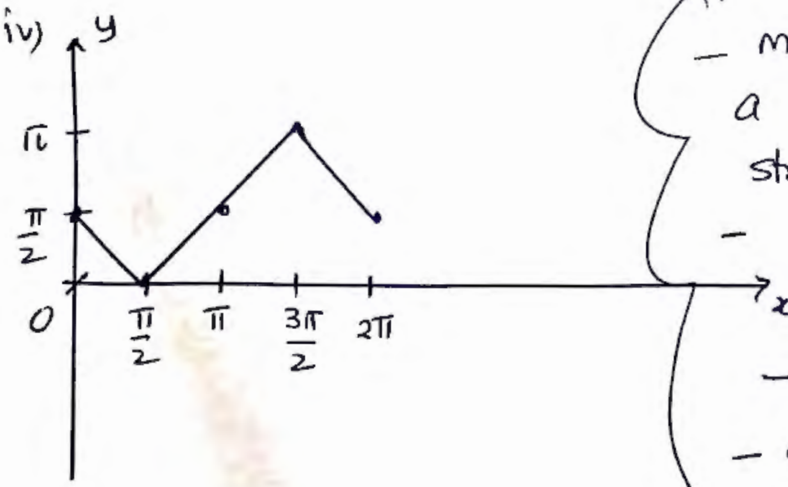
Poor explanation overall

(iii) $D_f : (-\infty, \infty)$

$R_f : [0, \pi]$

poor attempt overall

iv)



- many students drew a curve rather than straight line
- incorrect domain/range.
- incorrect shape
- overall, needs more practice on sketching inverse Trigonometric functions

Q12

a(i) $A\cos(x+\alpha)$

$= A[\cos x \cos\alpha - \sin x \sin\alpha]$

$= \sqrt{3}\cos x - \sin x$ ✓

equating coefficients of $\cos x$ and $\sin x$,

$A\cos\alpha = \sqrt{3}$
$A\sin\alpha = 1$
} square and add gives ✓

$A^2 = 3+1 = 4$

divide,

$A = 2 \quad (A>0)$

$\tan\alpha = \frac{1}{\sqrt{3}}$, $\alpha = \pi/6$; $0 < \alpha < \pi/2$

$\therefore \quad \sqrt{3}\cos x - \sin x = 2\cos(x + \pi/6)$

(ii) $2\cos(x+\pi/6) = 1 \quad 0 \le x \le 2\pi$

$\cos(x+\pi/6) = \frac{1}{2}$ $\quad \begin{array}{c|c} S & A \\ \hline T & C \end{array}$

$x + \pi/6 = \pi/3, \; 2\pi - \pi/3 = 5\pi/3$ ✓

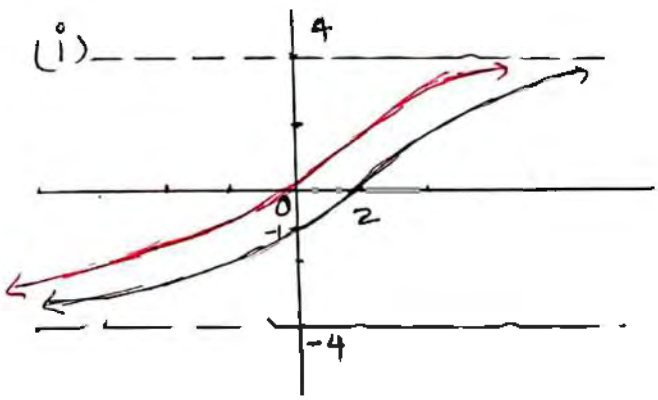
$x = \pi/3 - \pi/6, \; 5\pi/3 - \pi/6$

$x = \pi/6, \; \frac{9\pi}{6}$

$x = \pi/6, \; 3\pi/2$ ✓

b) $y = f(x+2)$

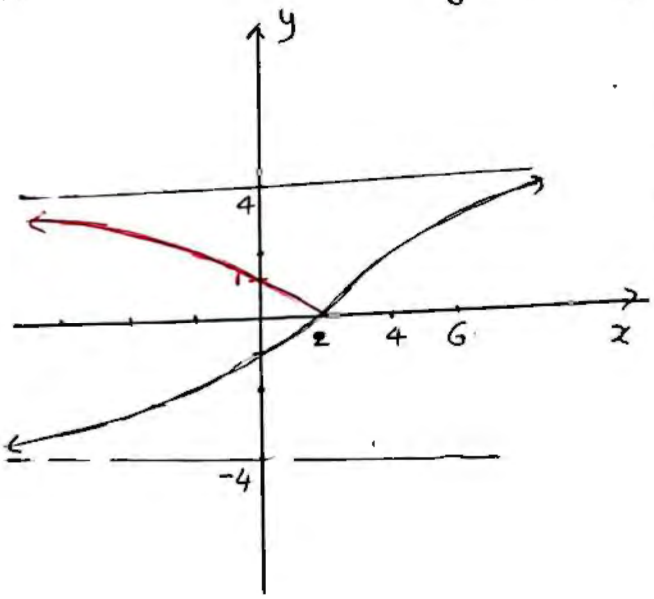
(i)



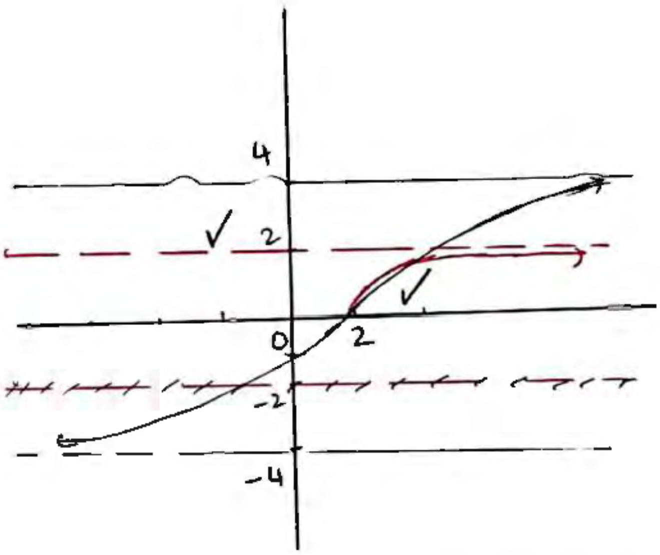
(ii) $y = |f(x)|$

y

(ii) $y = |f(x)|$



iii)



c) $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$ as $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$

$\theta = 90^\circ, \cos 90^\circ = 0$ ✓

$3k^2 + 2(-2-2k) = 0$

$3k^2 - 4k - 4 = 0$ ✓

$k = \dfrac{4 \pm \sqrt{16 + 4 \times 4 \times 3}}{6}$

$= \dfrac{4 \pm 8}{6} = 2, -\frac{1}{3}$ ✓

d) $(1+x)^n-1$ is divisible by x for $\forall n \geq 1$.

for $n=1$,
$(1+x)^1-1 = x$ which is divisible by x.

Let the result be true for $n=k$, $k \geq 1$, $k \in \mathbb{Z}$.

$(1+x)^k-1 = xP$, $P \in \mathbb{Z}$.

to prove that the result is true for $n=k+1$,

$$(1+x)^{k+1}-1 = (1+x)^k \cdot (1+x) - 1$$
$$= (xP+1)(1+x)-1$$
(using assumptions)
$$= xP + x^2P + \cancel{1} + x - \cancel{1}$$
$$= x(1+P+xP)$$
$$= xQ, \quad Q = 1+P+xP \in \mathbb{Z}^+.$$

$\therefore$ Using PMI, $(1+x)^n-1$ is divisible by all $n \in \mathbb{Z}$, $n \geq 1$, $x \in \mathbb{Z}^+$.

Maths Extension 1

Question 13

a) $4x^3 - 27x + k = 0$ has a double root

Let α, α, β be the roots

$P'(x) = 12x^2 - 27$

$P'(\alpha) = 0$

$12\alpha^2 - 27 = 0$ ✓

$\alpha^2 = \frac{27}{12} = \frac{9}{4}$

$\alpha = \pm \frac{3}{2}$

$2\alpha + \beta = 0$

$= -2\alpha$

$\therefore \alpha = \frac{3}{2}, \beta = -3$ ✓

$\quad " \quad -\frac{3}{2}, \beta = 3$

$\alpha^2 \beta = \frac{-k}{4}$

$\therefore k = -4\alpha^2\beta$

3 marks

For $\alpha = \frac{3}{2}, \beta = -3$

$k = -4\left(\frac{3}{2}\right)^2(-3)$

$\boxed{k = 27}$

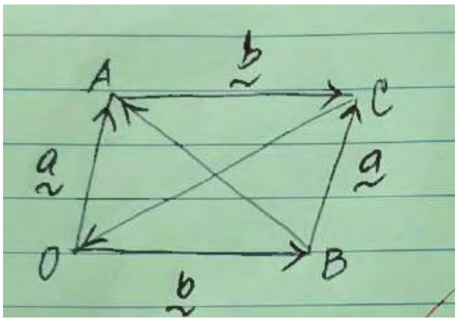
✓

For $\alpha = -\frac{3}{2}, \beta = 3$

$k = -4\left(-\frac{3}{2}\right)^2(3)$

$\boxed{k = -27}$

b)



OACB is a parallelogram

Let $\vec{OA} = \underset{\sim}{a}$, $\vec{OB} = \underset{\sim}{b}$

$\vec{OC} = \underset{\sim}{a} + \underset{\sim}{b}$

$\vec{BA} = \underset{\sim}{a} - \underset{\sim}{b}$

$|\vec{OC}|^2 + |\vec{BA}|^2$

$= |\underset{\sim}{a} + \underset{\sim}{b}|^2 + |\underset{\sim}{a} - \underset{\sim}{b}|^2$

$= (\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b}) + (\underset{\sim}{a} - \underset{\sim}{b}) \cdot (\underset{\sim}{a} - \underset{\sim}{b})$

3 marks

$= \underset{\sim}{a} \cdot \underset{\sim}{a} + \cancel{\underset{\sim}{a} \cdot \underset{\sim}{b}} + \cancel{\underset{\sim}{b} \cdot \underset{\sim}{a}} + \underset{\sim}{b} \cdot \underset{\sim}{b}$

$+ \underset{\sim}{a} \cdot \underset{\sim}{a} - \cancel{\underset{\sim}{a} \cdot \underset{\sim}{b}} - \cancel{\underset{\sim}{b} \cdot \underset{\sim}{a}} + \underset{\sim}{b} \cdot \underset{\sim}{b}$

$= 2|\underset{\sim}{a}|^2 + 2|\underset{\sim}{b}|^2$ ✓

$= |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2$

note that parallelogram has two pairs of parallel & equal sides.

Question 13

(c) $X \sim Bin(50, 0.08)$

(i) $\mu = E(X) = np$

$= 50 \times 0.08$

$= 4$ ✓

$Var(X) = npq$

$= 50 \times 0.08 \times 0.92$

$= 3.68$ ✓

$\sigma = \sqrt{3.68} = 1.92$ (2 d.p.) ✓

(ii) $P(\mu - \sigma \le X \le \mu + \sigma) = P(2.08 \le X \le 5.92)$

$= P(X = 3 \text{ or } X = 4 \text{ or } X = 5)$

$= {}^{50}C_3 (0.08)^3 (0.92)^{47} + {}^{50}C_4 (0.08)^4 (0.92)^{46} + {}^{50}C_5 (0.08)^5 (0.92)^{45}$

$= 0.5659$ (4 d.p)

(iii) $P(X > 5) = 1 - P(X=0 \text{ or } X=1 \text{ or } X=2 \text{ or } X=3 \text{ or } X=4 \text{ or } X=5)$

$= 1 - [0.92^{50} + {}^{50}C_1 (0.08)(0.92)^{49} + {}^{50}C_2 (0.08)^2 (0.92)^{48} + {}^{50}C_3 (0.08)^3 (0.92)^{47} + {}^{50}C_4 (0.08)^4 (0.92)^{46} + {}^{50}C_5 (0.08)^5 (0.92)^{45}]$

$= 0.2081$ (4 d.p.)

2 marks

(iv) $X \sim Bin(50, 0.04)$

$\mu = 50 \times 0.04 = 2$

$\sigma = \sqrt{50 \times 0.04 \times 0.96} = 1.39$

$\mu + \sigma = 3.39$

$P(X \le 3)$

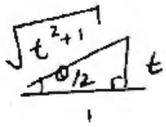
$= 0.96^{50} + {}^{50}C_1 (0.04)(0.96)^{49} + {}^{50}C_2 (0.04)^2 (0.96)^{48} + {}^{50}C_3 (0.04)^3 (0.96)^{47}$

$= 0.8609$ (4 d.p.)

2 marks

Question 14 :

a) i) $\frac{t}{1} = \tan\left(\frac{\theta}{2}\right)$



$\therefore \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$= \frac{2t}{\sqrt{1+t^2}} \times \frac{1}{\sqrt{1+t^2}} = \frac{2t}{1+t^2}$ 1

Done mostly well.

$\cos\theta = \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)$

$= \frac{1-t^2}{1+t^2}$ 1

ii) $LHS = \frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$

$= \frac{1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$

$= \frac{1+t^2+2t-1+t^2}{1+t^2+2t+1-t^2}$

$= \frac{2t^2+2t}{2t+2}$

$= \frac{2t(t+1)}{2(t+1)}$ 1

$= t$

$= \tan\theta$

$= RHS \quad \therefore$ proven 1

Done mostly well.

CFE for errors in part i) and some attempt was made.

b) i) $\boxed{\ddot{x} = 0}$

$\dot{x} = \int \ddot{x}\, dt$

$= C_1$

at $t=0$, $\theta = 0°$, $\dot{x} = V\cos\theta$

$\therefore \dot{x} = V\cos(0) = C_1$

$= V$

$\therefore \boxed{\dot{x} = V}$

$x = \int \dot{x}\, dt = Vt + C_2$

at $t=0$, $x=0$

$x = V(0) + C_2 = 0$

$\therefore \boxed{x = Vt}$

1

$\boxed{\ddot{y} = -10}$

$\dot{y} = \int \ddot{y}\, dt = -10t + C_3$

at $t=0$, $\dot{y} = V\sin\theta$, $\theta = 0°$

$\therefore \boxed{\dot{y} = -10t}$

$y = \int -10\, dt = -5t^2 + C_4$

at $t=0$, $y = 125$

$C_4 = 125$

$\therefore \boxed{y = -5t^2 + 125}$

1

NOTES:

- Most students showed some attempt.
- Students didn't show $\dot{x} = V\cos\theta$ and $\dot{y} = V\sin\theta$ initially, just disregarded the trig components without justification ($\theta = 0°$ initially)

ii) Showing some attempt at explaining displacement has horizontal and vertical component is sufficient for 1 m. 1

iii) $x = Vt$

$\therefore t = \frac{x}{V}$

$y = -5t^2 + 125$

$= -5\left(\frac{x}{V}\right)^2 + 125$ ← 1

at $x = 20\sqrt{5}$ $y = 100 + c$

$100 + c = -\frac{5(20\sqrt{5})^2}{V^2} + 125$ →

→ $-25 + c = \frac{-5(400 \times 5)}{V^2}$ ← 1

$V^2 = \frac{10000}{25 - c}$

$\therefore V = \frac{100}{\sqrt{25 - c}}$ (since $V > 0$)

NOTE: - Few students said $V > 0$.

- Other alternative methods were accepted.
- Not done as well.

iv) Just clears tower $\Rightarrow c = 0$

NOTE:
Done well

$$V = \frac{100}{\sqrt{25-0}}$$ 1

$$= \frac{100}{5}$$

$= 20$ m/s as required.

v) Horizontal range $\Rightarrow y = 0$, $V = 20$ m/s

$$y = \frac{-5x^2}{20^2} + 125 = 0$$

$x^2 = 10000$

$x = 100$ m (since $x > 0$) 1

$\therefore$ $100 - 20\sqrt{5}$ m past the tower 1

NOTE:
- Some students understood this while others got CFE for making conclusion on how far past tower.

vi) $y = 0$

$\Rightarrow$ $0 = 5t^2 - 125$

$125 = 5t^2$

$t = 5$ $(t > 0)$ 1

$$\therefore \tan\theta = \frac{\dot{y}}{\dot{x}} = \frac{-10(5)}{20} = \frac{-50}{20}$$

$$\theta = \tan^{-1}\left(\frac{-50}{20}\right)$$

$\approx 112°$ or $68°$ from the ground. 1

$$v = \sqrt{(-50)^2 + 20^2}$$

$= 10\sqrt{29}$ m/s. impact velocity. 1