

2010 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time 5 minutes
- Working time 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

- Attempt questions 1-7
- All questions are of equal value

Question	1	2	3	4	5	6	7	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/84	

Question 1 (12 marks) Start a new sheet of writing paper. Marks

a) Show that
$$\lim_{x \to 0} \frac{3x}{\tan 2x} = \frac{3}{2}.$$
 1

b)
$$\int_{0}^{\frac{\pi}{8}} \sin^2 4x \, dx$$
 3

c) Solve
$$\frac{4x-3}{x} \ge 5$$
 3

d) Find the acute angle between the lines y = -x-1 and 4x+5y=2. 3 Answer to the nearest minute.

e) Using
$$t = \tan \frac{\theta}{2}$$
, find the exact value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ showing all working.

Question 2 (12 marks) Start a new sheet of writing paper. Marks

a) Find the coordinates of the point, *P*, that divides the interval *AB* **3** $\underline{\text{externally}}$ in the ratio of 1: 4 if A (3, 1) and B (-1, -5).

b) Using the substitution,
$$u = x^4 + 1$$
, or otherwise, evaluate $\int_{0}^{1} x^3 e^{x^4 + 1} dx$. 3

c) Find the constant term in the expansion of
$$(x^2 - \frac{1}{2x^3})^{10}$$
 2

- d) Find the general solution to $2\cos\theta + \sqrt{3} = 0$. Express your answer in terms of π .
- e) Find x, giving reasons:



End of Question 2

2

Q	uesti	ion 3 (12 marks) Start a new sheet of writing paper.	Marks
a)	i)	Show that $x^3 + 2x - 17 = 0$ has a root between $x=2$ and $x=3$	1
	ii)	Using an approximation of $x = 2 \cdot 4$, use one application of Newton's method to find a better approximation for this root. Give your answer to two decimal places.	3
b)	i)	Express $\sin x - 2\cos x$ in the form $A\sin(x-\alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$.	2
	ii)	Hence, or otherwise, solve $\sin x - 2\cos x = \frac{\sqrt{5}}{2}$ for $0 \le x \le 2\pi$.	2

Give your answer(s) correct to 2 decimal places.

c) A particle moves on the x axis with velocity v m/s. The particle is initially at rest at x=1 m. Its acceleration is given by $\ddot{x} = 2v$ m/s². Find the velocity and acceleration of the particle at x = 10 metres.

Question 4 (12 marks)Start a new sheet of writing paper.Marksa)Given
$$y = 2 \sin^{-1} \frac{x}{3}$$
2i)State the domain and range of this function.2ii)Sketch the curve $y = 2 \sin^{-1} \frac{x}{3}$.2

b) Find the exact value of
$$\int_{0}^{2\sqrt{3}} \frac{1}{4+x^2} dx$$
 2

c) Prove that
$$\frac{2}{\tan A + \cot A} = \sin 2A$$
 2

d) i) For the binomial expansion of
$$(4+3x)^{15}$$
, show that: 2

$$\frac{T_{k+1}}{T_k} = \frac{16-k}{k} \times \frac{3}{4} \times x \; .$$

ii) Hence, find the greatest coefficient of $(4+3x)^{15}$, leaving your answer in 2 index form.

Qu	esti	ON 5 (12 marks) Start a new sheet of writing paper.	Marks
a)		The velocity <i>v m/s</i> of a particle moving along the x-axis is given by $v^2 = 16x - 4x^2 + 20$.	
	i)	Prove that the motion is simple harmonic.	2
	ii)	Find the centre of motion.	1
	iii)	Find the distance travelled in one complete oscillation.	1
b)	i)	Show that the equation of the normal at $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ is $x + py - ap^3 - 2ap = 0$	2
	ii)	The normals from $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ on the parabola $x^2 = 4ay$ meet at right angles. Prove that the locus of the points of intersection of these normals is the parabola $x^2 = ay - 3a^2$	3

c) Prove by mathematical induction that $\frac{1}{2!} + \frac{2}{3!} + ... + \frac{n}{(n+1)!} = \frac{(n+1)! - 1}{(n+1)!}$ 3

for all positive integers *n*.

Question 6 (12 marks) Start a new sheet of writing paper.

a) Newton's law of cooling states that a body cools according to the equation $\frac{dT}{dt} = -k(T-S)$,

where T is the temperature of the body at time t, S is the temperature of the surroundings and k is a constant.

- i) Show that $T = S + Ae^{-kt}$ satisfies the equation, where A is a constant. 1
- ii) A metal rod has an initial temperature of $350^{\circ}C$ and cools to $100^{\circ}C$ in 10 minutes. The surrounding temperature is $24^{\circ}C$.

(
$$\alpha$$
) Find the value of A and show that $k = \frac{-1}{10} \log_e \left(\frac{38}{163}\right)$ 2

 (β) Find how long it will take from the rod to cool to 25°C. 2

Question 6 continues on the next page

Question 6 Continued



In the diagram below, ABF and DCE are straight lines.



- i) Copy the diagram into your answer booklet.
- ii) Prove that AC is parallel to EF. 3

c) Given that
$$f(x) = \frac{(5-x)(1+x)}{5}$$
 and $h(x) = \log_e \{f(x)\}$

- i) Find the largest domain of y = h(x). 1
- ii) Find the equation of the inverse function $y = h^{-1}(x)$. 2
- iii) Find the domain of the inverse function $y = h^{-1}(x)$. 1

Question 7 (12 marks) Start a new sheet of writing paper.

a)

- The polynomial $P(x) = x^3 2x^2 + ax + b$ has (x+2) and (x-2) as factors, find the
- i) values of *a* and *b*.
- ii) remaining root of $P(x) = x^3 2x^2 + ax + b$
- A missile is launched from point A at an angle α and at a speed V towards a target at B, d metres away. Simultaneously a second missile is launched at speed W from B at an angle β , to intercept the first. The angles α and β are measured as in the diagram and are related by $\beta = 90^{\circ} - \alpha$.



The horizontal and vertical displacements of the projectiles from *A* and *B* are given by the following equations (DO NOT PROVE THESE RESULTS):

Missile from <i>A</i> :	Missile from <i>B</i>
$x = Vt \cos \alpha$	$x = d - Wt \cos \beta$
$y = -\frac{1}{2}gt^2 + Vt\sin\alpha$	$y = -\frac{1}{2}gt^2 + Wt\sin\beta$

- i) By equating y-components, show that if the missiles are to intersect, then the second missile must have speed $W = V \tan \alpha$.
- ii) Show that the time of intersection is $t = \frac{d \cos \alpha}{V}$ seconds after launch. 2

Question 7 continues on the next page

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Marks

2

1

b)

Question 7 Continued

c) Consider the function $f(x) = \frac{\log_e x}{x}$

- i) Find the coordinates of the stationary point on the curve y = f(x) 3 and determine its nature.
- ii) Hence show that $\pi^e < e^{\pi}$

2

End of Examination

Marks

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \ x > 0$$

c

$$\int e^{ax} dx \qquad \qquad = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx \qquad \qquad = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx \qquad = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx \qquad = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx \qquad = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Solutions for exams a	ind assessment tasks	-		Ver l
Academic Year	4112	Calendar Year	2010	
Course	Ext. 1.	Name of task/exam	Ext. I TRIAL EXA	1 m

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Question 1:	$\therefore \qquad \chi \left(4\chi - 3 \right) - 5\chi^2 > 0$
a) $\lim_{x \to 0} \frac{3x}{\tan 2x} = \lim_{x \to 0} \frac{3x}{\tan 2x}$	$\mathcal{X}\left[4\mathbf{x}-3-5\mathbf{x}\right] \geq 0$
$=\lim_{\lambda\to 0} \frac{3\times 2\lambda}{2} \tan 2\lambda$	$(-3-x) \geq 0$
$= \lim_{\lambda \to 0} \frac{3}{2} \left(\frac{2\lambda}{\tan 2\lambda} \right)$	$\begin{array}{c} \chi(\chi+3) \leqslant 0 \\ \vdots & -3 \leqslant \chi \leqslant 0 \\ \end{array}$
$= \frac{3}{2} \left(\cdot \right)$	d) $y = -x - 1$ $4x + 5y = 2$
= <u>3</u> 2.	$m = -1 \qquad m = -\frac{4}{5}$
b) $\int_{0}^{\frac{\pi}{8}} \sin^2 4\chi d\chi$	$\therefore \tan \varphi = \left(\frac{m_1 - m_2}{1 + m_1 m_2} \right)$
We know $\cos 8x = l - 2\sin^2 4x$	= -14
$\cdot \cdot S_{12}^{\prime 2} 4_{2} = \frac{1}{2} - \frac{1}{2} \cos 8_{2}$	$1 + (-i\chi - \frac{4}{5})$
$\int_{0}^{\frac{11}{5}} \left(\frac{1}{2} - \frac{1}{2}\cos 8x\right) dx$	$= \left \frac{-\frac{1}{5}}{\frac{9}{5}} \right $
$= \left[\frac{1}{2}x - \frac{1}{16}\sin 8x\right]^{\frac{1}{8}}$	$tano = \frac{1}{9}$
	· · · · · · · · · · · · · · · · · · ·
$\left(\begin{array}{c}2\left(\begin{array}{c}8\end{array}\right) \\ \left(\begin{array}{c}2\left(\begin{array}{c}8\end{array}\right) \\ \left(\begin{array}{c}6\end{array}\right) \\ \left(\begin{array}{c}8\end{array}\right) \\ \left(\begin{array}{c}8\end{array}\right) \\ \left(\begin{array}{c}0-0\\1\end{array}\right) \\ \left(\begin{array}{c}0-0\\1\end{array}\right) \\ \left(\begin{array}{c}1\\1\end{array}\right) \\ \left(\begin{array}{c}1\\1\end{array}\right$	e) $t = tan \frac{e}{2}$
$=\frac{\pi}{16}$	$\cos \Theta = \frac{1-t^2}{1+t^2}$
c) $\frac{4x-3}{x} \ge 5$	$\frac{1-\tan^2 15}{1+\tan^2 15} = \cos 30^\circ$
$\left \begin{array}{c} \chi^{2'} \left(\frac{4\chi - 3}{\chi} \right) > 5 \chi^2 \end{array} \right $	$\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \frac{\sqrt{3}}{2}$
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Ver 1

Solutions for exams	and assessment tas	sks	
Academic Year	4r 12	Calendar Year	2010
Course	Ext. 1.	Name of task/exam	TRIAL EXAM

4

a) $A(3, i) B(-1, -5) -1:4$ $x = \frac{m x_{2} + n x_{1}}{m + n}, y = \frac{m y_{2} + n y_{1}}{m + n},$ $x = \frac{m x_{2} + n x_{1}}{m + n}, y = \frac{m y_{2} + n y_{1}}{m + n},$ $x = \frac{m x_{2} + n x_{1}}{3}, y = \frac{(-i)(5) + 4}{3}(i)$ $x = \frac{1 + 12}{3}, y = \frac{5 + 4}{3}$ $x = \frac{13}{3}, y = \frac{5 + 4}{3},$ $x = \frac{13}{4}, y = \frac{5 + 4}{3},$ $x = \frac{14}{3}, y = \frac{14}{3}, x = \frac{14}{3},$ $x = \frac{14}{4}, x = \frac{14}{3},$ $x = \frac{14}{4}, x = \frac{14}{3},$ $x = \frac{14}{4}, x = \frac{14}{4},$ $x = 1, x = \frac{14}{4}, x = \frac{14}{4},$ $x = \frac{14}{4}, x = \frac{14}{4},$ $x = \frac{14}{4}, x = \frac{14}{4}, x$	Questin 2:	$T_{k+1} = {}^{0}C_{k} = {}^{20-2k} \left(-\frac{1}{2} \right)^{k} -\frac{3k}{2}$
$\begin{aligned} \chi &= \frac{m \chi_{2} + n \chi_{1}}{m + n}, \chi &= \frac{m \gamma_{2} + n \gamma_{1}}{m + n}, \\ \chi &= \left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right) + 4\left(3\right), \chi &= \left(\frac{-1}{2}\right)\left(\frac{-5}{3}\right) + 4\left(1\right), \\ \chi &= \frac{1 + 12}{3}, \chi &= \frac{5 + 4}{3}, \\ \chi &= \frac{1 + 12}{3}, \chi &= \frac{5 + 4}{3}, \\ \chi &= \frac{1 + 3}{3}, \chi &= \frac{5 + 4}{3}, \\ \chi &= \frac{1 + 3}{3}, \chi &= \frac{5 + 4}{3}, \\ \chi &= \frac{1 + 3}{3}, \chi &= \frac{5 + 4}{3}, \\ \chi &= \frac{1 + 3}{3}, \chi &= 3, \\ \frac{-1}{2}, \chi^{2} = \frac{3}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = 3, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = \frac{1 + 1}{3}, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = \frac{1 + 1}{3}, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = \frac{1 + 1}{3}, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = \frac{1 + 1}{3}, \\ \frac{-1}{2}, \chi^{2} = \frac{1 + 1}{3}, \chi^{2} = \frac{1}{3}, \chi^{2} = \frac{1}{3},$	a) $A(3,1) B(-1,-5) -1:4$	$= {}^{10}C_{k} \left(-\frac{1}{2} \right)^{k} \times \frac{20-5k}{x}$
$\begin{aligned} x_{\pm} = \frac{(-1)(-1)+4}{3} \begin{pmatrix} x \\ 2 \end{pmatrix}, y_{\pm} = \frac{(-1)(-5)+4}{3} \begin{pmatrix} y \\ 3 \end{pmatrix} \\ z = \frac{1}{3} \\ z = \frac{1+12}{3} \\ x = \frac{13}{3} \\ y_{\pm} = 3 \\ \vdots \\ p_{\pm} = \frac{13}{3} \\ y_{\pm} = 3 \\ \vdots \\ p_{\pm} = \frac{13}{3} \\ p_{\pm} = 3 \\ \vdots \\ p_{\pm} = \frac{13}{3} \\ p_{\pm} = 3 \\ \vdots \\ p_{\pm} = \frac{1}{3} \\ $	$X = \frac{m X_2 + n X_1}{m + n}, Y = \frac{m Y_2 + n Y_1}{m + n}$	for constant term $20-5k=0$ $\therefore k=4$ $\therefore \text{ constant term is } {}^{10}C \left(-\frac{1}{2}\right)^{4}$
$x = \frac{1+12}{3}, y = \frac{5+4}{3}$ $x = \frac{13}{3}, y = 3$ $x = (2k+i)\pi \pm \frac{\pi}{6}, k \text{ integer}$ $x = (2k+i)\pi \pm \frac{\pi}{6}, k = (2k+i)\pi \pm \frac$	$\mathcal{L} = \frac{(-1)(-1) + 4(3)}{3}, \mathcal{Y} = \frac{(-1)(-5) + 4(1)}{3}$	$= 210\left(\frac{1}{16}\right)$
$x = \frac{i3}{3}, y = 3$ $x = \frac{i3}{3}, \frac{y}{3}, \frac{y}{3}$ $x = \frac{i3}{3}, \frac{y}{3}, \frac{y}{3}, \frac{y}{3}$ $x = \frac{i3}{3}, \frac{y}{3}, \frac{x^{4}+i}{dx}$ $x = \frac{i4}{3}e^{x}\frac{x^{4}+i}{dx}$ $x = \frac{i4}{4}e^{x} = \frac{i4}{4}e^{x}$ $x = \frac{i4}{4}e^{x}(e^{x}-\frac{1}{4}e^{x})$ $x = \frac{1}{4}e^{x}(e^{x}-\frac{1}{4}e^{x})$ $x = \frac{1}{4}e^{x}(e^{x}-\frac{1}{4}e^{x})$ $x = \frac{1}{4}e^{x}(e^{x}-\frac{1}{2x^{3}})^{10}$	$\chi = \frac{1+12}{2}$, $Y = \frac{5+4}{2}$	= 105 8
$\frac{1}{2} = \frac{1}{4} e^{u} = \frac{1}{2}$ $\frac{1}{2} = \frac{1}{4} e^{(u-1)}$ $\frac{1}{2} = \frac{1}{2} e^{(u-1)}$ $\frac{1}{2} e^{(u-1)}$ $\frac{1}{2} e^{(u-1)}$ $\frac{1}{2} e^{(u-1)}$	3 3	d) $2\cos \theta + \overline{3} = 0$
$P\left(\frac{13}{3}, 3\right)$ $P = (2k+i)T \pm T_{6}, k \text{ integer}$ $P\left(\frac{13}{3}, 3\right)$ $P = (2k+i)T \pm T_{6}, k \text{ integer}$ $P = (2k+$	<u> </u>	$\frac{13}{2}$
b) $\int_{0}^{1} \chi^{3} e^{\chi^{4} + 1} d\chi$ $\int_{0}^{1} \frac{4}{4} \frac{x^{3}}{2} e^{\chi^{4} + 1} d\chi$ $= \int_{0}^{1} \frac{4}{4} \frac{x^{3}}{2} e^{\chi^{4} + 1} \frac{d\chi}{d\chi} = 4\chi^{3}$ $= \int_{1}^{1} \frac{4}{4} e^{\chi} d\chi$ $= \int_{1}^{1} \frac{4}{4} e^{\chi} d\chi$	$P(\frac{13}{3}, 3)$	o=(2k+1)∏ ± ₩, k integer
$\int_{0}^{1} \frac{4}{4} \frac{4x^{3}}{x^{3}} e^{x^{4}+1} \frac{4x}{4x} = 4x^{3}$ $= \int_{1}^{1} \frac{4}{4} e^{x} \frac{dx}{4x} = 4x^{3} \frac{dx}{4x} = 4x^{3}$ $= \int_{1}^{1} \frac{4}{4} e^{x} \frac{dx}{4x} = 4x^{3} \frac{dx}{4x}$ $= \int_{1}^{1} \frac{4}{4} \frac{e^{x}}{4x} = \frac{1}{4} \frac{e^{x}}{4x}$ $= \int_{1}^{1} \frac{4}{4} \frac{e^{x}}{4x} + \frac{1}{4} \frac{e^{x}}{4x}$ $= \int_{1}^{1} \frac{4}{4} \frac{e^{x}}{4x} + \frac{1}{4} \frac{e^{x}}{4x}$ $= \int_{1}^{1} \frac{1}{4} \frac{e^{x}}{4x} + \frac{1}{4x}$ $= \int_{1}^{1} \frac{1}{4} \frac{e^{x}}{4x} +$	b) $\int_{0}^{1} \chi^{3} e^{\chi^{4} + 1} d\chi$	e) $(9+x)x = 14x8$ (The product
$= \int_{-1}^{2} \frac{1}{4} e^{u} du = 4x^{3} dx$ $= \int_{-1}^{1} \frac{1}{4} e^{u} du = 4x^{3} dx$ u = 1 u low x = 0 $= \left[\frac{1}{4} e^{u}\right]^{2}$ u = 2 u low x = 1 $= \frac{1}{4} e^{2} - \frac{1}{4} e^{l}$ $= \frac{1}{4} e^{(e-1)}$ $c) \left(x^{2} - \frac{1}{2x^{3}}\right)^{10}$ $T_{k+1} = \int_{-1}^{10} C_{k} \left(x^{2}\right)^{10-k} \left(-\frac{1}{2x^{3}}\right)^{k}$ $T_{k+1} = \int_{-1}^{10} C_{k} \left(x^{2}\right)^{10-k} \left(-\frac{1}{2x^{3}}\right)^{k}$ Secants to a circle from an extornal point are equal). $qx + x^{2} = 112$ $x^{2} + 9x - 112 = 0$ (x - 7)(x + 16) = 0 x = 7, -16 $b_{u+} x > 0 as length$ $\therefore x = 7$ Page 2 of 10	$\int_{0}^{1} \frac{4}{4} \frac{4}{x} e^{\frac{1}{x}} e^{\frac{1}{x}} \frac{dx}{dx} = 4x^{3}$	of the intercepts of two intersecting
$\int_{1}^{4} 4^{2} + \frac{1}{2} + \frac{1}{2$	$= \left(\frac{1}{2} e^{\frac{1}{2}} du - 4x^{3} dx \right)$	Secarts to a circle from
$= \begin{bmatrix} \frac{1}{4} e^{x} \end{bmatrix}_{1}^{2} \qquad x = 2 \text{ when } x = 1 \qquad are equal),$ $= \frac{1}{4} e^{2} - \frac{1}{4} e^{1} \qquad qx + x^{2} = 112 \qquad x^{2} + 9x - 112 = 0 \qquad (x - 7)(x + 16) = 0 \qquad (x - 7)(x + 16) = 0 \qquad x = 7, -16 \qquad b_{0+} x > 0 as \text{ length},$ $T_{k+1} = {}^{10} C_{k} \left(\frac{x^{2}}{2x^{3}} \right)^{10-k} \left(-\frac{1}{2x^{3}} \right)^{k} \qquad \therefore x = 7 \qquad Page 2 \text{ of } 10$	L=1 when x=0	external point
$= \frac{1}{4} e^{2} - \frac{1}{4} e^{4}$ $= \frac{1}{4} e^{4} (e^{-1})$ $= \frac{1}{4} e^{4} (e^{-1})$ $(x - 7)(x + 16) = 0$ $(x - 7)(x + 16) = 0$ $x = 7, -16$ $b_{u+} x > 0 as length$ $T_{k+1} = {}^{10} C_{k} (x^{2})^{10-k} (-\frac{1}{2k^{3}})^{k}$ Page 2 of 10	$= \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} $	are equal).
$\frac{1}{4} = \frac{1}{4} = (e - 1)$ $\frac{1}{4} = \frac{1}{2} = 0$ $(x - 7)(x + 16) = 0$ $x = 7, -16$ $x = 7, -16$ $y = 7, -16$ $y = 7, -16$ $y = 7$ $x = 7$ Page 2 of 10	$=\frac{1}{4}e^{2}-\frac{1}{4}e^{1}$	$9x + x^{2} = 112$
c) $(\chi^{2} - \frac{1}{2\chi^{3}})^{10}$ $T_{k+1} = {}^{10}C_{k} (\chi^{2})^{10-k} (-\frac{1}{2\chi^{3}})^{k}$ $\chi = 7, -16$ $b_{1+} \chi > 0$ as length $\therefore \chi = 7$ Page 2 of 10	$= \frac{1}{4} e \left(e - 1 \right)$	x + 9x - 112 = 0 (x-7)(x+16)=0
$T_{k+1} = {}^{10}C_k \left(\chi^2\right)^{10-k} \left(-\frac{1}{2\chi^3}\right)^k$ $F_{k+1} = {}^{10}C_k \left(\chi^2\right)^{10-k} \left(-\frac{1}{2\chi^3}\right)^k$ $F_{k+1} = {}^{10}C_k \left(\chi^2\right)^{10-k} \left(-\frac{1}{2\chi^3}\right)^k$ $F_{k+1} = {}^{10}C_k \left(\chi^2\right)^{10-k} \left(-\frac{1}{2\chi^3}\right)^k$ $F_{k+1} = {}^{10}C_k \left(\chi^2\right)^{10-k} \left(-\frac{1}{2\chi^3}\right)^k$		x = 7, -16
$T_{k+1} = {}^{10}C_k \left(\chi^2\right)^{10-k} \left(-\frac{1}{2\chi^3}\right)^k \qquad \therefore x = 7$ Page 2 of 10	$(\chi^{-} - \frac{1}{2\chi^{3}})^{-}$	but x>0 as length
	$T_{k+1} = {}^{10}C_{k} \left(\chi^{2}\right)^{10-k} \left(-\frac{1}{2\chi^{3}}\right)^{k}$	$\therefore x = 7$ Page 2 of 10

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Solutions for exams	and assessment tas	ks			Ver I
Academic Year	Yr 12	Calendar Year	2010		
Course	Ext. 1.	Name of task/exam	TRIAL	EXAM	

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Question 3:	: 2 = tan & 2 = 0
\rightarrow	L = 1.107
P(x) = -5 P(x) = -5	$\therefore s_{10} x - 2 \cos x \equiv \sqrt{5} s_{10} (x - 1.107)$
P(3) = 16	ii) $\sin x - 2\cos x = \sqrt{\frac{5}{2}}$
since $P(2) < 0$ and $P(3) > 0$: 15 sin (x - 1.107) = 15
and P(x) is continuous,	$s_{1'n}(x-1.107) = \frac{1}{2}$
there exists a root between $X=2$ and $X=3$.	$x - 1.107 = \frac{11}{6}, \frac{5\pi}{6}$
 ") P(2.4) = 1.624	.x=1.63,3.73. (24p)
$P'(x) = 3x^{2} + 2$	
P'(2.4) = 19.28	$\frac{1}{2} = \frac{1}{2} = \frac{1}$
$\therefore Z_2 = Z_1 - \frac{P(Z_1)}{P'(Z_1)}$	x = xv $V dv = 2v$ dx
$= 2.4 - \frac{1.624}{19.28}$	$\frac{dv}{dx} = 2$
= 2.315	$\frac{dz}{dv} = \frac{1}{2}$
$\therefore A$ better root is $x = 2.32$ (2 dp)	$\chi = \int \frac{dv}{2} dv$
- 17	$X = \frac{1}{2}v + c$
b) $\sin x - 2\cos x \equiv A\sin(x-z)$	l = O + C
$s_1' = 2\omega s_1 = A s_1 \times 2\omega s_2 = A s_1$	$x = \frac{1}{2}v + i$
	at x = 10
$2 = A \sin \chi ()$	$ 0 = \frac{1}{2} v + i$
$1 + 4 = A^2 \left(\frac{2}{105} d + 5, \frac{2}{105} d \right) \left(\frac{1}{105} + 5 \right)^2$	$\frac{7}{2} v$
$5 = A^{2}$ $A = \sqrt{5}$ $A > 0$	$\ddot{x} = 36 \text{ m/s}^2$ Page 3 of 10

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a) $y = 2 \sin^{-1} \frac{x}{3}$ i) Domain: $-1 \le \frac{x}{3} \le 1$ $\therefore -3 \le x \le 3$ Range: $-\frac{\pi}{2} \le \sin^{-1} \frac{x}{3} \le \frac{\pi}{2}$ $-\pi \le 2 \sin^{-1} \frac{x}{3} \le \frac{\pi}{2}$ $-\pi \le 2 \sin^{-1} \frac{x}{3} \le \pi$ $-\pi \le 2 \sin^{-1} \frac{x}{3} \le \pi$ $= \frac{2}{\frac{1}{\cos A \sin A}}$ $= 2 \cos A \sin A$ $= \sin 2 A$ = R HS $\therefore Proved$ $d)'_1) (4 + 3\chi)'^5$ $T_{Z+1} = {}^{15}C_{K} (4)^{15-K} (3\chi)^{K-1}$ $T_{K} = {}^{15}C_{K-1} (4)^{15-(K-1)} (3\chi)^{K-1}$	Question 4:	e) Prove $\frac{2}{\tan A + \omega + A} = \sin 2A$
i) Domain: $-1 \leq \frac{x}{3} \leq 1$ $\therefore -3 \leq x \leq 3$ Range: $-\frac{\pi}{2} \leq \sin^{-1} \frac{x}{3} \leq \frac{\pi}{2}$ $-\pi \leq 2 \sin^{-1} \frac{x}{3} \leq \pi$ $= \frac{2}{\frac{1}{\cos A \sin A}}$ $= \frac{1}{\cos A \sin A}$ $= \frac{1}{\cos A \sin A}$	a) $y = 2 \sin^{-1} \frac{x}{3}$	$LHS = \frac{2}{S_{12}A_{12}}$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	i) Domain: $-1 \leq \frac{x}{3} \leq 1$	Cos A Sin A
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	3«x «3	$= \frac{2}{\frac{S_{1n}^{2}A + c_{0}c^{2}A}{c_{0}sA}}$
$-\pi \times 2 \sin^{-1} \frac{x}{3} \times \pi$ $-\pi \times 2 \sin^{-1} \frac{x}{3} \times \pi$ $= 2 \cos A \sin A$ $= \sin^{-1} 2 A$ $= R H S$ $\therefore Proved$ $d)'i) (4 + 3\chi)'^{5}$ $T_{\mu+i} = {}^{15}C_{\mu} (4)^{15-\mu} (3\chi)^{\mu}$ $T_{\mu} = {}^{15}C_{\mu-i} (4)^{15-(\mu-i)} (3\chi)^{\mu-i}$	Range : - TE J Sin 1 x J TZ	= _2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	-TT < 2 SIN 3 5 TT	Cos A s,. A
$ \begin{array}{c} = & s_{1}^{\prime} & 2A \\ = & RHS \\ & & \ddots & Proved \\ \end{array} $ $ \begin{array}{c} = & s_{1}^{\prime} & 2A \\ = & RHS \\ & & \ddots & Proved \\ \end{array} $ $ \begin{array}{c} d \end{pmatrix} 'i \end{pmatrix} (4 + 3\chi)^{15} \\ T_{\mu+1} = & {}^{15}C_{\mu} (4)^{15-\mu} (3\chi)^{\mu} \\ T_{\mu+1} = & {}^{15}C_{\mu} (4)^{15-\mu} (3\chi)^{\mu} \\ \end{array} $ $ \begin{array}{c} = & s_{1}^{\prime} & 2A \\ = & RHS \\ \hline d \end{pmatrix} 'i \end{pmatrix} (4 + 3\chi)^{15} \\ T_{\mu+1} = & {}^{15}C_{\mu} (4)^{15-\mu} (3\chi)^{\mu} \\ T_{\mu} = & {}^{15}C_{\mu-1} (4)^{15-(\mu-1)} (3\chi)^{\mu-1} \\ \end{array} $	- <i>TT & Y & TT</i>	= $2\cos A \sin A$
$ \begin{array}{c} T \\ T $	₩ ₩	$=$ si_2 A
$ \begin{array}{c} & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ \end{array} \right) \begin{array}{c} & & & \\ & & \\ \end{array} \right) \begin{array}{c} & & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \bigg) \begin{array}{c} & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \right) \begin{array}{c} & & \\ & & \\ \end{array} \bigg) \begin{array}{c} & & \\ & & \\ \end{array} \bigg) \begin{array}{c} & & \\ & & \\ \end{array} \bigg) \begin{array}{c} & & \\ \end{array} \bigg) \end{array} \bigg) \begin{array}{c} & & \\ \end{array} \bigg) \end{array}$ \bigg) \begin{array}{c} & & \\ \end{array} \bigg) \end{array} \bigg	··)	= RHS
$\frac{1}{1-\pi} = \frac{1}{2} \int_{0}^{2} \frac{1}{4+\chi^{2}} d\chi$ $\frac{1}{2} \int_{0}^{2} \frac{1}{4+\chi^{2}} d\chi$		
$T_{k+1} = {}^{15}C_{k} (4) {}^{15-k}(3x)^{k}$ $T_{k} = {}^{15}C_{k-1} (4) {}^{15-(k-1)}(3x)^{k-1}$ $T_{k} = {}^{15}C_{k-1} (4) {}^{15-(k-1)}(3x)^{k-1}$		$d)'i) (4+3\chi)'^{5}$
b) $\int_{0}^{2\sqrt{3}} \frac{1}{4 + \chi^{2}} d\chi$ $T_{k} = {}^{15}C_{k-1} \left(4\right)^{15-(k-1)} \left(3\chi\right)^{k-1}$	<i>μ</i> -π-	$T_{k+1} = {}^{15}C_{k} (4) {}^{15-k} (3x)^{k}$
	b) $\int_{0}^{2\sqrt{3}} \frac{1}{4+x^{2}} dx$	$T_{k} = {}^{l5}C_{k-l} \left(4\right)^{l5-(k-l)} \left(3_{k}\right)^{k-l}$
$= \frac{1}{2} \left[\frac{1}{4a_{n}} \frac{1}{2} \right]^{2\sqrt{3}} \qquad $	$=\frac{1}{2}\left[+a_{n}^{-1}\frac{x}{2}\right]^{2\sqrt{3}}$	$\frac{T_{k+1}}{T_{k}} = \frac{15!}{\frac{k!(15-k)!}{15!}} + \frac{4^{15-k}}{3^{k}} \times \frac{k}{x^{k}}$
$= \frac{1}{2} \left[\frac{1}{16-1} + \frac{1}{2} - \frac{1}{16} + \frac{1}{16} \right]^{-1} \left[\frac{1}{16} + \frac{1}{16}$	=	$\frac{1}{(k-1)!} \frac{4}{(k-1)!} \frac{4}{4} \frac{3}{3} \frac{1}{x}$
$= \frac{1}{2} \begin{bmatrix} 4a^{-1} \\ 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4a^{-1} \\ 3 \end{bmatrix} = \frac{15}{2} \begin{bmatrix} 4a^{-1} $	$\frac{1}{2} \left[\frac{1}{2} \left[\frac{1}{2} \left[\frac{1}{3} \right] \right]$	$=\frac{15!}{k!(15-k)!} \times (16-k)! \times (16-k)! \times 4 = 3 \times 15!$
$= \frac{1}{2} \frac{\pi}{3} = \frac{(16-k)}{k} \times 4 \times 3 \times 1$	$= \frac{1}{2} \frac{\pi}{3}$	$= \frac{(16-k)}{k} \times 4^{-1} \times 3^{1} \times 1^{1}$
$= \frac{\pi}{6}$ $\frac{T_{k+1}}{T_k} = \frac{16-k}{k} \times \frac{3}{4} \times \frac{3}{4}$ Page 4 of 10	$=\frac{\pi}{6}$	$\frac{T_{k+1}}{T_k} = \frac{16-k}{k} \times \frac{3}{4} \times \frac{3}{4}$ Page 4 of 10

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ii) for greatest coefficient	ii) Centre of motion is x=2
	(1) when Y=0
TL	$0 = 16x - 4x^2 + 20$
	$4\chi^2 - 16\chi - 20 = 0$
$\frac{16-k}{k} \times \frac{3}{4} > 1$	$\chi^2 - 4\chi - 5 = 0$
	(x+1)(x-5)=0
48-3k > 4k (as $k > 0$)	oscillates between -1 & 5
48>7k	i distance travelled is 12 m.
$k < \frac{48}{7}$	
. k = 6,5,	b) 1) x = 4ay
i. k = 6 is greatest	$Y = \frac{x^2}{4a}$
$T_7 = {}^{15}C_6 (4) {}^{15-6}(3)^6$	$\frac{dy}{dx} = \frac{2x}{4x}$
$= 5005 \times 4^{9} \times 3^{6}$	at x, 2,
= $5005 \times 2^{18} \times 3^{6}$	un L= Lap
Question 5:	$m_{targ} = \frac{4ap}{4a}$
\rightarrow μ^2 μ μ^2 μ^2	= P .
(a) $V = 16 \chi - + \chi + 20$	· · m rormal = - 1
i) $\frac{1}{2}v^2 = 8x - 2x^2 + 10$: equation normal :
$\frac{d}{dx}\left(\frac{1}{2}y^{2}\right) = \frac{d}{dx}\left(8x - 2x^{2} + 10\right)$	$y-y_{i}=m(x-x_{i})$
$\dot{x} = 8 - 4x$	$y - ap^2 = -\frac{1}{p} (x - 2ap)$
x = -4(x-2)	$Py - ap^3 = -x + 2ap$
which is of the form	$\therefore x + py - ap^3 - 2ap = 0$
$\dot{x} = -n^2 (x - h) \text{which is}$	ii) If normals meet at cirly
s+m with centre at x=h	angles then
i.e. $X = Q$.	$-\frac{1}{P} \times -\frac{1}{q} = -1$ Page 5 of 10
	$\therefore Pq = -1$

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egn of normals	$4 = \alpha \left(p^2 + pq_1 + q_2^2 \right) + 2\alpha$
$x + py - ap^{3} - 2ap = 0 ()$	
$x + qy - aq^3 - 2aq = 0$ (2)	$\frac{y-2a}{a} = p^2 + (-1) + q^2$
y (p-q) - ap = + aq = 2ap + 2aq = 0	$\frac{y-2a}{a} + 1 = p^2 + q^2$
$Y(p-q)-a(p^{3}-q^{3})-2a(p-q)=0$	$\frac{y-2a+a}{a} = (p+q)^2 - 2pq$
$Y(p-q) - a(p-q)(p^2+pq+q^2) - 2a(p-q) = o$	$\frac{y_{-\alpha}}{\alpha} = \left(\frac{x}{\alpha}\right)^2 - 2(-1)$
Since p ≠ q can divide by (p-q)	$y - a = x^2$
$y - a(p^2 + pq + q^2) - 2a = 0$	$\int \frac{1}{a} = \frac{1}{a^2} + 2$
$y = a(p^{2} + pq + q^{2}) + 2a$	$a(y-a) = x^2 + 2a^2$
	$\therefore x^2 = ay - a^2 - 2a^2$
$x = -py + ap^{-} + 2ap$	$\chi^2 = \alpha \gamma - 3\alpha^2$
$= -\rho \left[\alpha \left(p^2 + pq + q^2 \right) + 2q \right] + ap + 2af$	
$= -\alpha p^{3} - p^{2}q_{\alpha} - \alpha pq_{\alpha} - 2 \alpha p + 3$	c) Prove
$\lambda = - \cos(\alpha + \alpha)$	$\frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(n+i)!} = \frac{(n+i)!}{(n+i)!}$
$-\alpha p q (p + q)$	for all positive intro o
pt intersection of Normals :	Clark
$\left(-\alpha \rho q \left(\rho + q\right), \alpha \left(\rho^2 + \rho q + q^2 + q\right)\right)$	Step 1: prove true for n=1
	$LHS = \frac{1}{21}$ $RHS = (1+1)! - 1$
Locus :	$= \frac{1}{2} \qquad (1+i)!$
we know pg = -1 and	= 2! -1
X = -apq(p+q) Y = ap + pq + q + 2	2!
$ x = -\alpha(-i)(p+q)$	$=\frac{2-1}{2}$
$x = \alpha(p+q)$	$\therefore L = R = L$
$\frac{x}{q} = p + q$	intrue for n=1:
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Step 2 : Assume true for n=k	Question 6 :
$\frac{1}{2!} + \frac{2}{3!} + \dots + \frac{k}{(k+1)!} = \frac{(k+1)! - 1}{(k+1)!}$	a) $\frac{dT}{dt} = -k(T-s)$
Step 3 : prove true for n=k+1	i) $T = S + Ae^{-kt}$
i.e. prove,	1-5=Ae ()
$\frac{1}{2!} + \frac{2}{3!} + \dots + \frac{k+1}{(k+2)!} = \frac{(k+2)! - 1}{(k+2)!}$	$\frac{dT}{dt} = A e^{-kt} (-k)$
LHS = $\frac{1}{2!} + \frac{2}{3!} + \dots + \frac{k}{(k+1)!} + \frac{k+1}{(k+2)!}$	= (T-S) (-k) from ()
$= (\underline{k}+i)! - 1 + \underline{k}+i$	$\frac{dT}{dt} = -k(T-s)$
(L+i)! (k+2)!	ii) t=0 T=350°C
= (k+2)[(k+1)! - 1] + k+1	t=10 mis T= 100°C
(k+2)!	S = 24°C
= (k+2)! - (k+2) + k+1	$(-1) T = 24 + Ae^{-kt}$
(k+ 2)!	$350 = 24 + Ae^{\circ}$
= (k+2)! - k - 2 + k + 1	A = 326
(k+2)!	$T = 24 + 326 e^{-k \times 10}$
= (h+2) = 1	$76 = 326e^{-10k}$
$\frac{(k+2)!}{(k+2)!}$	$\frac{76}{326} = e^{-10k}$
= RHS	$ln\left(\frac{76}{30}\right) = -10k$
iby the principle of	(526)
mathematical induction, proved	$k = -\frac{1}{10} ln(\overline{326})$
tor all positive integers, n.	$k = -\frac{1}{10} L_n \left(\frac{38}{163} \right)$ Page 7 of 10

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$(\beta) T = 24 + 326e^{-kt}$	c) $f(x) = (5-x)(1+x)$
-kt 25 = 24 + 326 c	$h(x) = k_n \{ f(x) \}$
$\frac{1}{326} = e^{-kt}$	
$lo(\frac{1}{2k}) = -kt$	(5-x)(1+x)
(326)	5 50
$t = -\frac{1}{k} \ln \left(\frac{1}{326} \right)$	(5-x)(1+x) > 0
t = 39.7 mins	
ы);)	
F	·1 < x < 5
E	") $f: y = l_{n} \left[\frac{(5-x)(1+x)}{5} \right]$
B G ·	f^{-1} : $x = ln\left[\frac{(5-y)(1+y)}{5}\right]$
A . W	$e^{x} = (\frac{5-y}{1+y})$
	$5e^{x} = (5-y)(1+y)$
	$5e^{x} = 5 + 4y - y^{2}$
ii) Let < BAC = x	$-5e^{x} = y^{2} - 4y - 5$
·· < BDC = x (angles standing	$-5e^{x} = y^{2} - 4y + 4 - 9$
are equal)	$9-5e^{x} = (y-2)^{2}$
······································	$y-2 = \pm \sqrt{9-5e^{x}}$
	$y = 2 \pm \sqrt{9 - 5e^{x}}$
··· < BAC = < E FG and they are	for function, either of the w
ACIEF	both not both. Page 8 of 10

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iii) Domain: 9-5ex >0	$(b)')y = -\frac{1}{2}gt^{2} + Vtsin \alpha$ (1)
5e ^x < 9	$y = -\frac{1}{2}gt^2 + wt \sin \beta$
$x lne \leq ln \frac{9}{5}$	$-\frac{1}{2}gt^{2}+vtsind = -\frac{1}{2}gt^{2}+wtsin\beta$
x ≤ ln 5	Vtsind = wtsing
Question 7:	V sind = W sin B
a) i) $P(x) = x^{3} - 2x^{2} + ax + b$	$\beta = 90 - \alpha$
P(-2) = 0 and $P(2) = 0$	$V S_{1n} \alpha = W S_{1n} (40 - \alpha)$ $V S_{1n} \alpha = W \cos \alpha$
-8 - 8 - 2a + b = 0 (1) 8 - 8 + 2a + b = 0	$W = V \frac{S_{1} d}{40 s d}$
	W = V tan à
2a+b=16 () 2a+b=0 (2)	ii) Vtcos ~ = d-Wtcos ß
2b = 16 () + (2) $b = 8$	$V t \cos \alpha + w t \cos \beta = d$
a=-4	$t(V\cos\alpha + W\cos\beta) = d$
ii) $P(x) = x^3 - 2x^2 - 4x + 8$	$t = \frac{d}{(v \cos d + v \tan d \cos B)}$
$= (x+2)(x-2)(x-\alpha)$	=d
$(\chi^{2}-4)\chi^{3}-2\chi^{2}-4\chi+8$ $\chi^{3}-4\chi$	$(V\cos \alpha + V \tan \alpha \cos(90-\alpha))$
$\frac{-2\chi^2}{+8}$	$= \underline{d}$ $(V\cos \alpha + V \sin \alpha \cdot \sin \alpha)$
	t = d
\therefore other root is $x=2$. \therefore roots are 2, 2, -2	$\frac{Y\cos^2 + V \sin^2 x}{\cos x}$ Page 9 of 10

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$t = \frac{d \cos \alpha}{v \left(\cos^2 \alpha + s_{x_{x_{x_{x_{x_{x_{x_{x_{x_{x_{x_{x_{x_$
$t = \frac{d \cos \alpha}{V}$
c) $f(x) = \frac{ln x}{x}$
i) $f'(x) = \frac{x(\frac{1}{x}) - \ln x(i)}{x^2}$
$= \frac{1 - \ln x}{\sqrt{2}}$
for stat. $pts f'(x) = 0$
$\frac{1-\ln x}{x^2} = 0$
$l - \ln x = 0$ $L_{0.14} = 1$
$\cdot \cdot e = x$
$= \frac{1}{e}$
\cdot Stat. pt $(e, \frac{1}{e})$
$f''(x) = \frac{x^2(-\frac{1}{x}) - (1 - \ln x)^2 x}{x^4}$
$= -\frac{X-2X+2X\ln X}{X^4}$

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$f''(x) = -\frac{3x + 2x \ln x}{x^4}$
when $x = e$
$f''(x) = -\frac{3e+2elne}{e^4}$
$= -\frac{3e+2e}{e^4}$
$= -\frac{1}{e^3}$
< 0
i max at (e, t) .
ii) Since max value is $\frac{1}{e}$
then $\frac{\ln x}{x} < \frac{1}{e}$
at $x = T$
$\frac{L_n T}{T_n} < \frac{1}{e}$
e hom < m
La The < Th
take e of both sides
e ^{hn} < e ^{TT}
. Tr < e"
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