NAME:______________________________

TEACHER'S NAME:______________________________

Mrs Hickey
Mrs Choong
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Mrs Quarles
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Pymble Ladies' College

1999 TRIAL H.S.C. EXAMINATION

MATHEMATICS

3/4 UNIT

Time Allowed: 2 Hours
plus 5 minutes reading time

INSTRUCTIONS TO CANDIDATES:

All questions must be attempted.
All necessary working must be shown
Start each question on a new page.
Put your name and your teacher's name on every sheet of paper.
Marks may be deducted for careless or untidy work.
Only approved calculators may be used.
DO NOT staple different questions together.
Hand this question paper in with your answers.
All rough working paper must be attached to the back of the last question.
All questions are of equal value.

There are seven (7) question in this paper.

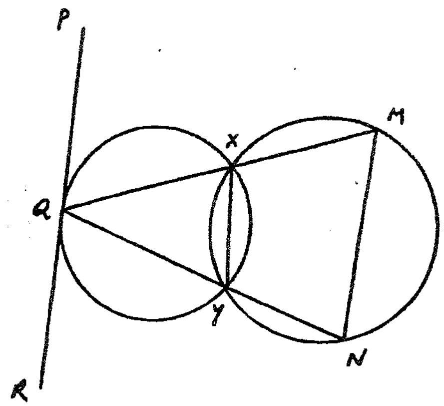
QUESTION 1

		Marks
(a)	Differentiate $\tan^{-1} 3x$	1
(b)	Solve $\dfrac{x^2 - 4}{x} < 3$	3
(c)	Find the exact value of $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4 - x^2}}$	2
(d)	Find $\displaystyle\int \frac{x}{x+1}\,dx$ (You may use the substitution $u = x + 1$ if you wish)	3
(e)	Use the substitution $u = \sqrt{x}$ to find $\displaystyle\int \frac{dx}{\sqrt{x}\,(1+x)}$	3

QUESTION 2 (Start a new page) — Marks

(a) Prove that $\dfrac{\cos 2\theta}{\sin\theta} + \dfrac{\sin 2\theta}{\cos\theta} = \operatorname{cosec}\theta$ — 3

(b) — 4



Not to scale

Two circles intersect at X and Y. PR is a tangent to the smaller circle, touching it at Q. QX is produced to M and QY is produced to N, as shown.

(i) Copy (or trace) the diagram onto your working paper.

(ii) Prove that PR is parallel to MN.

(c) $P(6t, 3t^2)$ is a variable point on the parabola $x^2 = 12y$, and S is the focus. The line joining P and S is produced to Q so that $PS = SQ$. — 5

(i) Find the coordinates of Q

(ii) Find the equation of the locus of Q as P moves on the parabola.

QUESTION 3 (Start a new page) — Marks

(a) Two of the zeros of the cubic polynomial $P(x) = 3x^3 - bx^2 - 27x + 9$ are reciprocals of each other, and two of the zeros of $P(x)$ are opposites of each other. — 4

(i) Find the value of b.

(ii) Factorise $P(x)$ completely.

(b) A particle, when x metres from the origin on a straight line, has acceleration $a\ ms^{-2}$ given by $a = -2x(x^2 + 1)$. — 8

Initially the particle is at rest at $x = 1$.

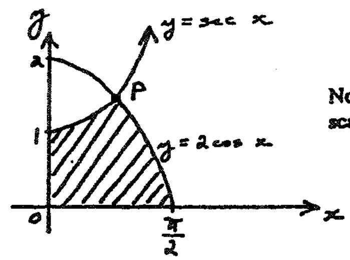
(i) In which direction will the particle first move? Why?

(ii) Show that its velocity $v\ ms^{-1}$ is given by $v^2 = 3 - 2x^2 - x^4$.

(iii) ~~When~~ Where does the particle next come to rest? Briefly describe its subsequent motion.

QUESTION 4 **(Start a new page)** **Marks**

(a) 7



Not to scale

P is the point of intersection of the graphs of $y = \sec x$ and $y = 2\cos x$ in the domain $0 \le x < \frac{\pi}{2}$.

(i) Verify that P is the point $\left(\frac{\pi}{4}, \sqrt{2}\right)$.

(ii) The shaded region is rotated about the x axis. Show that the volume of the solid of revolution so formed is $\frac{\pi^2}{2}$ cubic units.

(b) Prove, by mathematical induction, that $3^n + 7^{n+1}$ is divisible by 4 for all positive integers n. 5

QUESTION 5 **(Start a new page)** **Marks**

(a) (i) Express $\cos 3t + \sin 3t$ in the form $A\cos(3t - \alpha)$ where $A > 0,\quad 0 < \alpha < \frac{\pi}{2}$. 5

(ii) Hence, or otherwise, solve $\cos 3t + \sin 3t = 1$ for $0 \le t \le \pi$.

(b) A particle moves in a straight line such that its displacement x metres from the origin after t seconds is given by $x = \cos 3t + \sin 3t$. 7

(i) Show that the particle is moving in simple harmonic motion and write down the period of the motion.

(ii) Find the initial position of the particle.

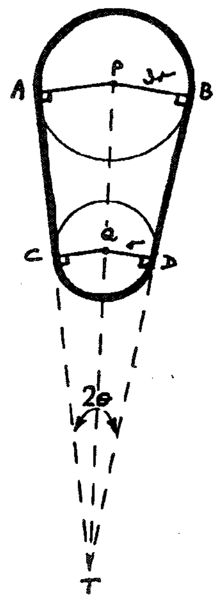
(iii) Find the speed of the particle when it first returns to its initial position.

(iv) What is the maximum speed of the particle?

QUESTION 6 (Start a new page)

Marks

(a) 8



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A taut belt passes around two discs of radii r *cm* and $3r$ *cm* as shown in the diagram. The straight sections of the belt are inclined to each other at an angle of 2θ radians.

(i) Explain, using similar triangles *TPB* and *TQD*, why $BD = 2DT$.

(ii) If the total length of the belt is L *cm*, show that $BD = \frac{1}{2}L - 2r(\theta + \pi)$.

(iii) Given that $L = 176$ and $r = 4$, show that $\cot\theta = 11 - \pi - \theta$.

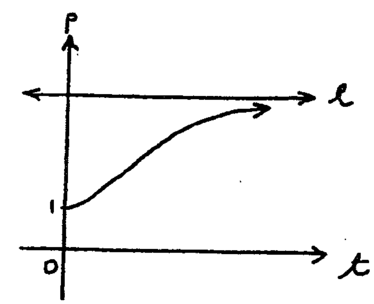
(b) Using $\theta = 0{\cdot}1$ as a first approximation to the solution of the equation $\cot\theta = 11 - \pi - \theta$, use Newton's method once to obtain a closer approximation. 4

QUESTION 7 (Start a new page)

Marks

(a) Use the substitution $u = e^x$ to show that $\int_0^{\ln 10} \frac{3}{1+2e^{-x}}\,dx = 6\ln 2$ 4

(b) 8



Not to scale

The sketch above shows the graph of an increasing function

$P = \frac{3}{1+2e^{-t}}, \quad t \geq 0$

(i) Copy this diagram onto your working paper and state the equation of the asymptote *l*.

(ii) Sketch the inverse function $f^{-1}(t)$ on the same axes as $P = f(t)$.

(iii) Find $f^{-1}(t)$ and verify that $f^{-1}(1) = 0$ and $f^{-1}(2{\cdot}5) = \ln 10$.

(iv) Evaluate $\int_1^{2\cdot 5} f^{-1}(t)\,dt$.

(You may use your result from part (a) above).

END OF PAPER