**Mrs Gibson**
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# PYMBLE LADIES' COLLEGE

## YEAR 12

## MATHEMATICS EXTENSION 1

## HSC TRIAL EXAMINATION 2001

**Time Allowed: 2 hours + 5 mins reading time**

**Test [illegible]: 16 August 2001**

**Instructions:**

- All questions should be attempted.
- Write your name and your teacher's name on each page
- Start each question on a new page.
- **DO NOT** staple the questions together.
- Only approved calculators may be used.
- A standard integral sheet is attached.
- Marks might be deducted for careless or untidy work.
- Hand this question paper in with your answers.
- ALL rough working paper must be attached to the back of the last question.
- Staple a coloured sheet of paper to the back of each question.
- There are seven (7) questions in this paper.
- All questions are of equal value.

## MARKING GUIDELINES

- **Provide answers which are complete, accurate and comprehensive.**
- **Leave your answers in exact form unless otherwise stated.**
- **Include all necessary working. Correct answers will not necessarily gain full marks unless necessary working is shown. Relevant working might gain marks even if your answer is wrong.**
- **Take care with mathematical notation.**
- **Show relevant information clearly and unambiguously on sketches if required.**
- **Present well set out solutions using a logical set of steps in which justification is included where necessary.**

**QUESTION 1** **Marks**

(a) Differentiate $\dfrac{1}{1+x^2}$ **1**

(b) The polynomial $P(x) = 2x^3 - x + a$ is divisible by $x + 2$. **1**

Find the value of $a$.

(c) A, B and P are the points (-1,8), (6,-6) and (4,-2) respectively. **2**

The point P divides the interval AB internally in the ratio $k:1$.

Find the value of $k$.

(d) Solve $x - 1 = \sqrt{x+1}$ **3**

(e) Evaluate $\displaystyle\int_1^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}}\,dx$ **3**

(f) Solve $|\,3 - 3x\,| > x + 3$ **2**

**QUESTION 2** **Start a new page** **Marks**

(a) Find the exact value of $cos\left(sin^{-1}(-\frac{1}{3})\right)$ **2**

(b) Given that $\log_b a = 2$ and $\log_c b = 3$, find the value of $\log_a$ c. **2**

(c) Find the value of $\int_0^3 \frac{t}{\sqrt{1+t}}\, dt$ **4**

using the substitution $t = u^2 - 1$ where $u > 0$

(d) $A$ and $B$ are acute angles such that $cos\, A = \frac{3}{5}$ and $sin\, B = \frac{1}{\sqrt{5}}$. **4**

Without finding the size of either angle, show that $A = 2B$, and

use this result to find the exact value of $sin\, 3B$.

**QUESTION 3** **Start a new page** **Marks**

(a) Write down the value of the **prime** number $b$ such that

$$\sum_{n=1}^{3} \log_2 2n = a + \log_2 b$$ **1**

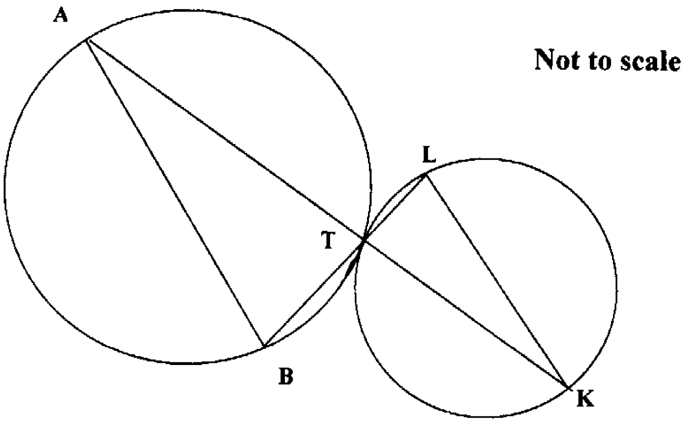
(b) The diagram shows two circles touching externally at T.

AB is any diameter of the first circle, and AT and BT are produced to meet the second circle again at K and L respectively.

Copy the diagram onto your answer paper, then prove that

(i) KL is a diameter of the second circle **1**

(ii) LK is parallel to AB **2**



(c) Evaluate $\int_0^{\frac{\pi}{4}} (\cos x + \sec x)^2 \, dx$ **4**

(d) The perimeter of an equilateral triangle of side $a$ *cm* is increasing at a constant rate of 6 *cm/sec* as the triangle is being enlarged. **4**

Find the rate at which the area of the triangle is increasing at the instant the perimeter is 24 *cm*. (The triangle remains equilateral.)

**QUESTION 4** **Start a new page** **Marks**

(a) A certain population N is changing at a rate given by the equation

$$\frac{dN}{dt} = 0 \cdot 5\,(N - 100).$$

(i) Show that $N = 100 + Ae^{0 \cdot 5t}$ is a solution of this equation, and find the value of $A$ given that the initial value of $N$ is 500. **2**

(ii) Find the value of $N$ when $t = 10$. **1**

(b) A function $f(x)$ has an inverse whose equation is $f^{-1}(x) = \frac{2x-2}{x-2}$. **3**

What is the equation of $f(x)$?

Explain the geometrical significance of your answer.

(c) (i) Sketch $f(x) = \sin x$ and its inverse $g(x) = \sin^{-1} x$ on the same axes for $0 \le x \le \frac{\pi}{2}$. **1**

(ii) Show that the tangent at $x = 1$ on $f(x)$ and the tangent at $y = 1$ on $g(x)$ are equally inclined to $y = x$. **4**

(iii) What is the angle between these two tangents? **1**

**QUESTION 5** **Start a new page** **Marks**

(a) A particle travels in a straight line executing simple harmonic motion about O according to the equation $x = a\cos nt$.

(i) Show that the velocity $v$ and displacement $x$ of the particle at any time $t$ are related by the equation $v^2 = n^2(a^2 - x^2)$. **2**

(ii) **Hence** show that the acceleration of the particle can be given as $\ddot{x} = -n^2 x$. **1**

(b) A particle executes simple harmonic motion about O according to the above equations. Initially it is at $x = 2$. As it passes through O its speed is 2 m/sec. How long does it take to get to O for the first time? **3**

(c) Draw a large and accurate sketch of the curve $y = \dfrac{x+4}{x(x+8)}$, showing all essential features such as intercepts on axes and asymptotes. **4**

Show that there are no stationary points. (You do not need to find the coordinates of any inflection points.)

(d) Find the area bound by the curve $y = \dfrac{x+4}{x(x+8)}$ and the $x$ - axis between $x=1$ and $x=2$. **2**

You may use the substitution $u = x(x+8)$ to evaluate this area if you wish.

**QUESTION 6** **Start a new page** **Marks**

(a) A curve has equation $f(x) = 3x - 4x^3$.

(i) Show that the equation of a tangent at the point on the curve where $x = a$ is $y = (3 - 12a^2)x + 8a^3$. **2**

(ii) How many tangents can be drawn to this curve from the point (1,0)? **3**

(You must show full working to substantiate your answer.)

(b) P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$.

The tangent at P and a line through Q parallel to the $y$ axis meet at point R.

The tangent at Q and a line through P parallel to the $y$ axis meet at point S.

(i) Draw a neat diagram showing all information given above. **1**

(ii) Show that the equation of the tangent at P is $y = px - ap^2$. **2**

(iii) Show that PQRS is a parallelogram **2**

(iv) Show that the area of PQRS is $2a^2|p - q|^3$ square units. **2**

**QUESTION 7** **Start a new page** **Marks**

(a) A particle moves in a straight line towards the centre O experiencing an acceleration that is inversely proportional to the cube of the distance from O, namely $a = -\dfrac{4}{x^3}$.

(i) If the particle starts from rest at $x = 2$, find an expression for the velocity of the particle in terms of $x$. **3**

Make sure you justify the sign of your expression.

(ii) Hence find an expression that relates elapsed time $t$ and displacement $x$, and find the time the particle takes to reach $x = 1$ ( for the first time, if it does so more than once). **3**

(b) (i) Prove by induction that for all integers $n \geq 1$ **3**

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(ii) Use this result to evaluate $2^2 + 4^2 + 6^2 + \ldots + 100^2$ **2**

(iii) Hence evaluate $1^2 + 3^2 + 5^2 + \ldots\ldots + 99^2$ **1**

**END OF PAPER**

# STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

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Q1 (a) $\frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \frac{-2x}{(1+x^2)^2}$ ①

$\boxed{1}$

(b) $P(x) = 2x^3 - x + a$

$P(-2) = 0 \Rightarrow 2(-2)^3 - (-2) + a = 0$

$-16 + 2 + a = 0$

$\therefore a = 14$ ①

$\boxed{1}$

c) A P B

$$x: \underbrace{-1 \quad 4}_{5}\underbrace{\quad 6}_{2}$$

$\frac{AP}{PB} = \frac{5}{2} = \frac{2\frac{1}{2}}{1} \quad \therefore k = 2\frac{1}{2}$ ②

$\boxed{2}$

(d) $x - 1 = \sqrt{x+1}$ $\quad x - 1 \geq 0$

$x \geq 1$

$(x-1)^2 = x + 1$

$x^2 - 2x + 1 = x + 1$ ①

$x^2 - 3x = 0$

$x(x-3) = 0$ ①

$x = 0$ or $3$

but since $x \geq 1$, only $x = 3$ ①

$\boxed{3}$

(e) $\int_1^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}}\,dx = \left[\sin^{-1}\frac{x}{2}\right]_1^{\sqrt{2}}$ ①

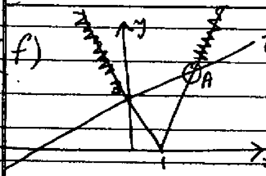
$= \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}\frac{1}{2}$

$= \frac{\pi}{4} - \frac{\pi}{3}$ ①

$= -\frac{\pi}{12}$ ①

$\boxed{3}$

(f)



at A, $x + 3 = 3x - 3$

$2x = 6$

$x = 3$

so $x < 0$ or $x > 3$

① ①

$\boxed{2}$

12

Q2 (a) $\cos \sin^{-1}\left(-\frac{1}{3}\right) = \cos\alpha$

$= \frac{2\sqrt{2}}{3}$

½ quadrant
½ sign
½ $\sqrt{8} = 2\sqrt{2}$
½ ratio

②

$\boxed{2}$

(b) $\log_b a = 2 \Rightarrow b^2 = a$ (½)

$\log_c b = 3 \Rightarrow c^3 = b$ (½)

$\therefore c^6 = b^2 = a \Rightarrow c = a^{\frac{1}{6}}$ (½)

$\therefore \log_a c = \frac{1}{6}$ (½)

②

$\boxed{2}$

(c) $\int_0^3 \frac{t}{\sqrt{1+t}}\,dt$

$t = u^2 - 1$
$dt = 2u\,du$
$t = 0,\ u = 1$
$t = 3,\ u = 2$ ①

$= \int_1^2 \frac{(u^2-1)2u\,du}{\sqrt{u^2}}$ ①

$= 2\int_1^2 (u^2-1)\,du$

$= 2\left[\frac{u^3}{3} - u\right]_1^2$ ①

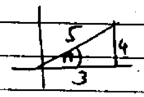
$= 2\left[\frac{8}{3} - 2 - \frac{1}{3} + 1\right]$

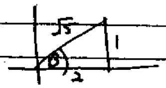
$= 2\left(\frac{4}{3}\right)$ ①

$= \frac{8}{3}$

$\boxed{4}$

(d) $\cos A = \frac{3}{5}$ $\quad \sin B = \frac{1}{\sqrt{5}}$





$\cos 2B = 1 - 2\sin^2 B$

$= 1 - 2\left(\frac{1}{5}\right)$

$= \frac{3}{5} = \cos A \quad \therefore A = 2B$ ①

$\sin 3B = \sin(B + 2B) = \sin(B + A)$

$= \sin B \cos A + \cos B \sin A$ ③

$= \frac{1}{\sqrt{5}} \cdot \frac{3}{5} + \frac{2}{\sqrt{5}} \cdot \frac{4}{5} = \frac{11}{5\sqrt{5}}$

$\boxed{4}$

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Q3 (a) $\sum_{1}^{3} \log_2 2n = \log_2 2 + \log_2 4 + \log_2 6$

$= 1 + 2 + \log_2 6$

$\boxed{1}$ $= 1 + 2 + (\log_2 2 + \log_2 3)$

$= 4 + \log_2 3$

$\therefore\ b = 3$ ①

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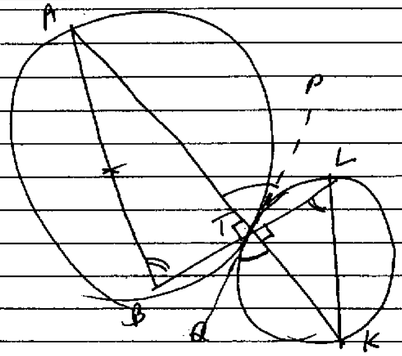
(b)



(i) since AB is diameter

$\angle ATB = 90°$ ($\angle$ in semicircle)

$\therefore \angle LTK = 90°$ (vert opp $\angle$'s)

$\boxed{3}$ hence $\angle LTK$ is also $\angle$ in a semicircle at circum.

so LK is diameter ①

(ii) Construct tangent PTQ

$\angle PTA = \angle TBA$ ($\angle$ between tangent + chord = $\angle$ in alt. sg)

similarly $\angle QTK = \angle FLK$ ( - - - - - - ) ①

but $\angle PTA = \angle QTK$ (vert. opp)

$\therefore \angle TBA = \angle TLK$ ①

these are alternate equal $\angle$'s

$\therefore$ AB // LK

(c) $\int_0^{\frac{\pi}{4}} (\cos x + \sec x)^2 dx$ $\cos 2x = 2\cos^2 x - 1$

$= \int_0^{\frac{\pi}{4}} (\cos^2 x + 2 + \sec^2 x) dx$ (½)

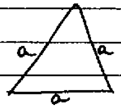
$= \int_0^{\frac{\pi}{4}} \frac{1}{2}(1 + \cos 2x) + 2 + \sec^2 x \ dx$ (½)

$\boxed{4}$ $= \left[\frac{1}{4}\sin 2x + \frac{5x}{2} + \tan x\right]_0^{\frac{\pi}{4}}$ ②

$= \left(\frac{1}{4} + \frac{5\pi}{8} + 1\right) - 0$ ①

$= \frac{5}{4} + \frac{5\pi}{8}$

(d)



$P = 3a$

$\frac{dP}{dt} = 3\frac{da}{dt} = 6$ $\therefore \frac{da}{dt} = 2$

$A = \frac{1}{2} a^2 \sin 60$

$= \frac{\sqrt{3}}{4} a^2$ ①

$\boxed{4}$ $\frac{dA}{dt} = \frac{dA}{da} \cdot \frac{da}{dt}$ ①

$= \frac{\sqrt{3}}{2} a \cdot 2$ ①

$= \sqrt{3}\, a$

$= 8\sqrt{3}$ when $P = 24$ ①

// 12

24 (a) $\frac{dN}{dt} = 0.5(N-100)$

(i) $N = 100 + Ae^{0.5t} \Longrightarrow Ae^{0.5t} = N-100$

$\frac{dN}{dt} = 0.5Ae^{0.5t}$ ①

$= 0.5(N-100)$ as required

when $t=0$, $N=500$

$\boxed{3}$ $\therefore 500 = 100 + Ae^{0}$.

so $A = 400$ ①

(ii) $N = 100 + 400e^{0.5t}$

when $t = 10$,

$N = 100 + 400e^{5}$

$= 59465.26\ldots$ ①

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(b) $f^{-1}(x) = \frac{2x-2}{x-2} = y$

so $f(x)$: $x = \frac{2y-2}{y-2}$ ①

$xy - 2x = 2y - 2$

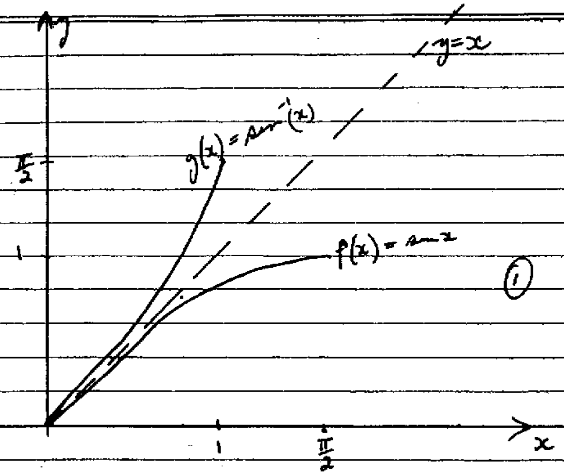
$y(x-2) = 2x-2$

$\boxed{3}$ $y = \frac{2x-2}{x-2} = f(x)$ ①

Since $f(x) = f^{-1}(x)$ then the function is symmetric about $y=x$ ①

or the fn is the inverse of itself

(c) (i)

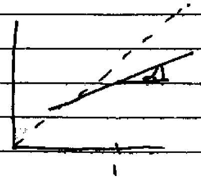


$\boxed{1}$ ①

(ii) $f(x) = \sin x$

$f'(x) = \cos x$.

$= \cos 1$ at $x = 1$



(β)

$\therefore \tan\alpha = \cos 1$

$\alpha = 0.49536\ldots^{c}$ ①

$\boxed{4}$ so angle between tangent & $y=x$ is $\frac{\pi}{4} - 0.49\ldots$

$= 0.29^{c}$ (½)

gradient of tangent at $y=1$ on $g(x)$ is $\frac{1}{\cos 1}$ ①

so $\tan\beta = \frac{1}{\cos 1}$

$\beta = 1.0754\ldots^{c}$ (½)

$\therefore$ angle between tangent & $y=x$ is

$1.0754 - \frac{\pi}{4} = 0.29^{c}$ (½)

$\boxed{1}$ (iii) Required angle $= 0.29 \times 2 = 0.58^{c}$ ①

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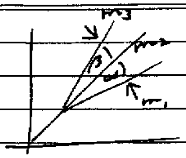
(c) Alternative: for (ii)



$f(x) = \sin x$

$f'(x) = \cos x$ (½)

$= \cos 1$ when $x = 1$

$= m_1$

So $\tan\alpha = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$

$= \left|\dfrac{1 - \cos 1}{1 + \cos 1}\right|$ ①

for $g(x) = \sin^{-1} x$      $g(x) = y = 1$

$g'(x) = \dfrac{1}{\sqrt{1-x^2}}$      $x = \sin 1$

$= \dfrac{1}{\sqrt{1-\sin^2 1}}$ at $y=1$

$= \dfrac{1}{\cos 1}$ since $1-\sin^2 1 > 0$ ①

$= m_3$

So $\tan\beta = \left|\dfrac{\frac{1}{\cos 1} - 1}{1 + \frac{1}{\cos 1}}\right|$ ①

$= \left|\dfrac{1 - \cos 1}{\cos 1 + 1}\right| = \tan\alpha$

$\therefore \alpha = \beta$ as required. (½)

Since $\alpha$ + $\beta$ are both acute

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Ⓠ5 (a) (i) $x = a\cos nt$

$v = \dfrac{dx}{dt} = -an\sin nt$

$v^2 = a^2n^2\sin^2 nt$

$= a^2n^2\left[1 - \cos^2 nt\right]$ ①

$= a^2n^2\left[1 - \dfrac{x^2}{a^2}\right]$

$= a^2n^2 - n^2x^2$ ①

$\boxed{3}$ so $v^2 = n^2(a^2 - x^2)$

(ii) $\ddot{x} = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \dfrac{d}{dx}\left[\dfrac{n^2}{2}(a^2 - x^2)\right]$

$= \dfrac{n^2}{2} \cdot -2x$ ①

$\ddot{x} = -n^2x$

(b) $x = a\cos nt$

when $t=0$, $x=2$ $\therefore$ $2 = a\cos 0$

$\therefore a = 2$ ①

so $x = 2\cos nt$

when $x=0$, $v^2=4$

$\therefore$ $4 = n^2(4-0)$

$\boxed{3}$ $\therefore n^2 = 1$ ①

So period $= \dfrac{2\pi}{n} = 2\pi$

so takes $\frac{1}{4}(2\pi)$ secs to get to 0 ①

ie $\dfrac{\pi}{2}$ secs.

OR $x = 2\cos t$

when $x=0$, $\cos t = 0$

$\therefore t = \dfrac{\pi}{2}$

(c) $y = \frac{x+4}{x(x+8)}$

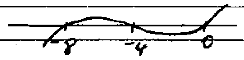
when $y=0$, $x = -4$

$x \neq 0$

Vert. asymp. $x = 0$ & $x = -8$ ①

horiz. asymp. $y = 0$

Sign:



$\frac{dy}{dx} = \frac{(x^2+8x)1 - (x+4)(2x+8)}{x^2(x+8)^2}$ ②

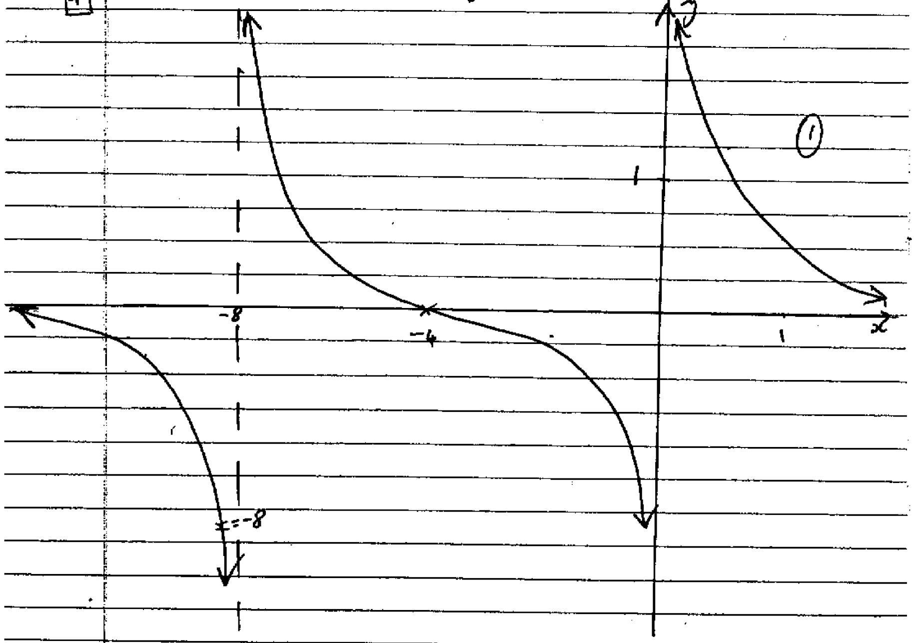
$= 0$ when $x^2+8x-2x^2-16x-32=0$

$x^2+8x+32 = 0$

but $(x+4)^2 + 16 \neq 0$

$\therefore \frac{dy}{dx} \neq 0$ so no S.P's.

$\boxed{4}$



①

O

(d) $A = \int_1^2 \frac{x+4}{x^2+8x}\,dx$

$\boxed{2}$

$= \frac{1}{2}\ln(x^2+8x)\Big]_1^2$ ①

$= \frac{1}{2}(\ln 20 - \ln 9)$

$= \frac{1}{2}\ln\frac{20}{9}$ ①

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**Q6** (a) (i) $f(x) = 3x - 4x^3$

$f'(x) = 3 - 12x^2$

$f'(a) = 3 - 12a^2$ and $f(a) = 3a - 4a^3$ ①

$\boxed{2}$ ∴ eqn is: $y - (3a - 4a^3) = (3 - 12a^2)(x - a)$

$y = (3 - 12a^2)x + 3a - 4a^3 - 3a + 12a^3$ ①

$y = (3 - 12a^2)x + 8a^3$

(ii) From $(1, 0)$:

$0 = (3 - 12a^2)1 + 8a^3$

How many 'a' values satisfy $8a^3 - 12a^2 + 3 = 0$ (½)

Let $P(a) = 8a^3 - 12a^2 + 3$

$P'(a) = 24a^2 - 24a$

$= 24a(a - 1)$

$= 0$ when $a = 0$ or $1$

$\boxed{3}$ $P''(a) = 48a - 24$

$P''(0) = -24$

$< 0$ ∴ max at $(0, 3)$ ①

$P''(1) = 24$

$> 0$ ∴ min at $(1, -1)$

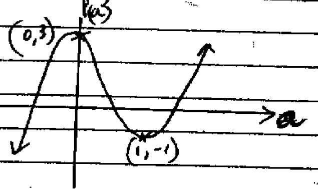
since $P(0) > 0$

and $P(1) < 0$ ①

there must be a root between 0 & 1

∴ $P(a) = 0$ has 3 real roots

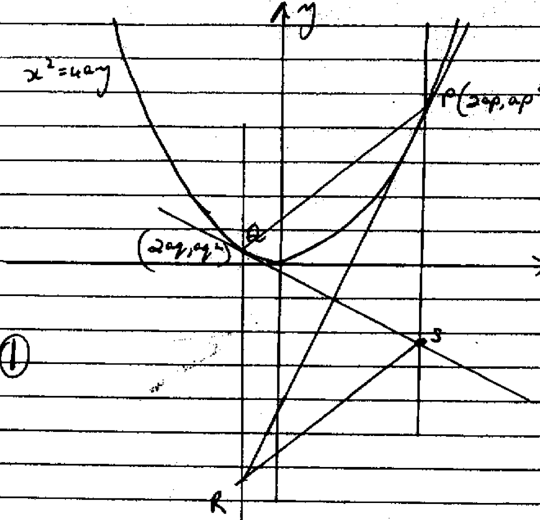
so $\underline{\underline{3}}$ tangents can be drawn.



(½)

(b)

(i)



No coord deduct ½

Award ½ if PS not proven

$\boxed{1}$ ①

(ii) at P: $\frac{dy}{dp} = 2ap$ ∴ $\frac{dy}{dx} = 2ap \cdot \frac{1}{2a}$

$\frac{dx}{dp} = 2a$ $= p$ ①½

$\boxed{2}$ ∴ Eqn PR is

$y - ap^2 = p(x - 2ap)$ (½)

$y = px - 2ap^2 + ap^2$

$y = px - ap^2$

(iii) $\underline{PS \parallel QR}$ given equation $\underline{\text{of } QS}$ is $y = qx - aq^2$

at S, $x = 2ap$ ∴ $y = q(2ap) - aq^2$ (½)

∴ $PS = ap^2 - (2apq - aq^2)$

$= a(p^2 - 2pq + q^2) = a(p - q)^2$ (½)

$\boxed{2}$ at R, $x = 2aq$ ∴ $y = p(2aq) - ap^2$ (½)

∴ $QR = aq^2 - (2apq - ap^2)$

$= a(q^2 - 2pq + p^2) = a(q - p)^2$

since $a(p-q)^2 = a(q-p)^2$ (½)

then $PS = QR$ so PQRS is a parallelogram

(one pr of opp. sides equal and parallel).

Aw $\frac{1}{2}$ for (2ap(

(iv) $A_{PQRS} = PS \times$ perp dist from P to Q

$= a(p-q)^2 \times |2ap - 2aq|$ ①

$= a(p-q)^2 \times 2a\,|p-q|$

$= 2a^2(p-q)^2|p-q|$

$\boxed{2}$ $= 2a^2\,|(p-q)^3|$ ①

12 $= 2a^2|p-q|^3$

(iii) alternative method.

$M_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$ $M_{SR} = \frac{2apq - ap^2 - 2apq + aq^2}{2aq - 2ap}$

$= \frac{p+q}{2}$ $= \frac{p+q}{2}$

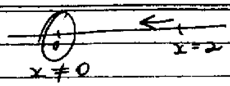
$\therefore$ SR ∥ PQ

QR ∥ PS

PQRS is a parallelogram.

* if all that's given is QR ∥ PS ∥ y axis - Aw $\frac{1}{2}$

Ⓠ7 (a) (i) $\ddot{x} = -\frac{4}{x^3}$



$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{-4}{x^3}$

$\frac{1}{2}v^2 = 2x^{-2} + C$ ($\frac{1}{2}$)

when $t=0$, $v=0$ & $x=2$

$\therefore\ 0 = \frac{1}{2} + C \Rightarrow C = -\frac{1}{2}$ ($\frac{1}{2}$)

$\boxed{3}$

$\frac{1}{2}v^2 = \frac{2}{x^2} - \frac{1}{2}$

$v^2 = \frac{4}{x^2} - 1$ ①

Now at $t=0$, $v=0$ & $\ddot{x} < 0$ $\therefore$ moves left

But since motion is not defined for $x=0$,
particle can not ever stop

so $v = -\sqrt{\frac{4}{x^2} - 1}$ ①

$= \frac{-\sqrt{4-x^2}}{x}$ since $x > 0$

(ii) $\frac{dx}{dt} = \frac{-\sqrt{4-x^2}}{x}$ ($\frac{1}{2}$)

$\frac{dt}{dx} = \frac{-x}{\sqrt{4-x^2}}$ ($\frac{1}{2}$)

$\boxed{3}$

$t = \sqrt{4-x^2} + C$ ①

when $t=0$, $x=2$ so $c=0$

$t = \sqrt{4-x^2}$ ($\frac{1}{2}$)

when $x=1$, $t = \sqrt{3}$ ($\frac{1}{2}$)

(b) (i) Let $P(n)$ be the proposition that for integral $n \geq 1$

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Test $P(1)$: $LHS = 1^2 = 1$

$RHS = \frac{1(2)(3)}{6} = 1 = RHS$ ($\frac{1}{2}$)

$\therefore$ $P(1)$ is true

Let $k$ be a value of $n$ for which $P(k)$ is true

ie $1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

Then for $P(k+1)$

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ ($\frac{1}{2}$)

$\boxed{3}$

$= \frac{k+1}{6}\left[k(2k+1) + 6(k+1)\right]$

$= \frac{k+1}{6}\left[2k^2 + 7k + 6\right]$ ①

$= \frac{k+1}{6}(2k+3)(k+2)$ ($\frac{1}{2}$)

$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$

as required

So provided $P(k)$ is true, it is established that $P(k+1)$ is true.

$\therefore$ Since $P(1)$ is true, then $P(2)$ is true, hence $P(3)$ is true etc ($\frac{1}{2}$)

$\therefore$ $P(n)$ is true for all integers $n \geq 1$.

(ii) $2^2 + 4^2 + 6^2 + \cdots + 100^2 = 2^2\left[1^2 + 2^2 + \cdots + 50^2\right]$ ① so using $n=50$,

$\boxed{2}$

$= \frac{4 \times 50 \times 51 \times 101}{6} = 171\ 700$ ①

$\boxed{1}$

(iii) $1^2 + 3^2 + \cdots + 99^2 = (1^2 + 2^2 + \cdots + 100^2) - (2^2 + 4^2 + 6^2 + \cdots + 100^2)$

$= \frac{100 \times 101 \times 201}{6} - 171\ 700 = 166\ 650$ ①

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