Multiple Choice.

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1. Solve the inequation $\frac{4}{5-x} \le 1$.

(A) $1 < x < 5$

(B) $1 \le x \le 5$

(C) $x \le 1$ or $x \ge 5$

(D) $x \le 1$ or $x > 5$

2. Find the constant term in $\left(x - \frac{2}{x}\right)^6$.

(A) -8

(B) -160

(C) 8

(D) 160

3. For what value of *p* is the expression $4x^3 - x + p$ divisible by $x + 3$?

(A) 105

(B) -105

(C) 108

(D) -108

4. Find $\int \sin x \cos^3 x \, dx$.

(A) $4\cos^4 x + C$

(B) $-4\cos^4 x + C$

(C) $\frac{1}{4}\cos^4 x + C$

(D) $\frac{-1}{4}\cos^4 x + C$

5. $\frac{d}{dx}\left(\sin^{-1} 2x\right) =$

(A) $\frac{2}{\sqrt{1-x^2}}$

(B) $\frac{4}{\sqrt{1-4x^2}}$

(C) $\frac{2}{\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-2x^2}}$

6. Find $\int \sin^2 x\, dx$.

(A) $x - \frac{1}{2}\sin 2x + c$

(B) $\frac{1}{4}\left(2x - \sin 2x\right) + c$

(C) $\frac{1}{2}\left(x - \sin 2x\right) + c$

(D) $\frac{1}{2}\left(x - 2\sin 2x\right) + c$

7. Evaluate $\int_{-1}^{0} x\sqrt{1+x}\, dx,$ using the substitution $u = 1 + x.$

(A) $\frac{-4}{15}$

(B) $\frac{4}{15}$

(C) $\frac{-15}{4}$

(D) $\frac{15}{4}$

8. The equation $\sin x = 1 - 2x$ has a root near $x = 0.3$
Using one application of Newton's method to obtain a better approximation, correct to two decimal places, gives:

(A) 0.33

(B) 0.34

(C) 0.43

(D) 0.71

9. If α, β and γ are the roots of the equation $x^3 - x^2 + 4x - 1 = 0$, the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$ is:

(A) -5

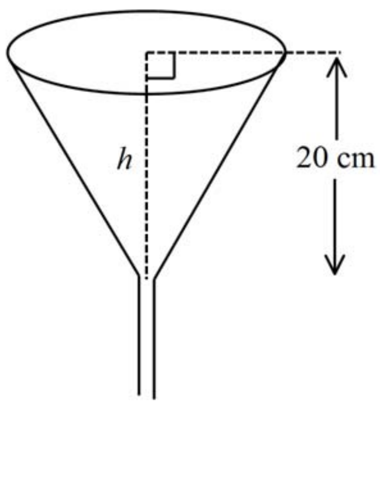
(B) -1

(C) 3

(D) 7

10. Water is running out of a conical funnel at the rate of 5 cm^3/s. At any time t seconds, the depth of water is h cm and the volume of water in the funnel is V cm^3.

Given that $V = \dfrac{\pi h^3}{12}$, find the rate at which the water level is changing when it is 5 cm from the top.



(A) $\dfrac{4}{45\pi}$ cm/s

(B) $\dfrac{-4}{45\pi}$ cm/s

(C) $\dfrac{4}{5\pi}$ cm/s

(D) $\dfrac{-4}{5\pi}$ cm/s

Question 11. (15 marks).
Use a **separate** writing booklet.

Marks

(a) The interval AB, where A is $(-3,3)$ and B is $(5,-1)$ is divided internally in the ratio $m:n$ by the point $P(-1,2)$. Find the values of m and n. **2**

(b) Newton's law of cooling states that a body cools according to the equation $\frac{dT}{dt} = -k(T - S)$, where T is the temperature of the body in degrees Celsius at time t minutes, S is the temperature, in degrees Celsius, of the surroundings and k is a constant.

(i) Show that $T = S + Ae^{-kt}$ satisfies the equation, where A is a constant. **1**

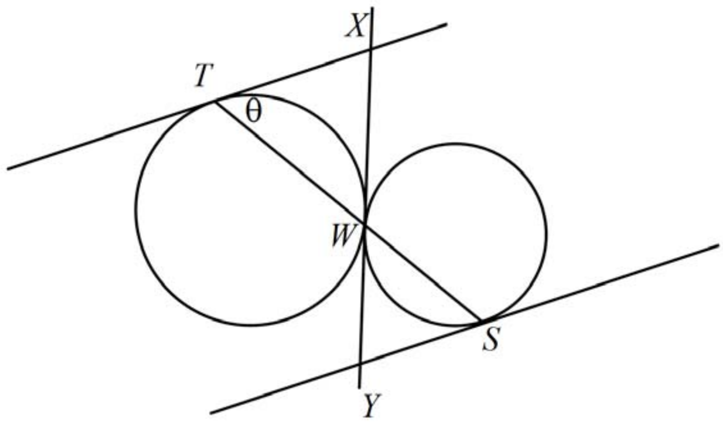
(ii) A metal rod has an initial temperature of 470°C and cools to 250°C in 10 minutes. The surrounding temperature is 30°C.

(α) Find the value of A and show that $k = \frac{1}{10}\log_e 2$. **2**

(β) How long will it take the rod to cool to 70°C, giving your answer correct to the nearest minute? **2**

Question 11 continued on page 5.

(c)



In the diagram above, two circles touch one another externally at point *W*.
A straight line through *W* meets one of the circles at *T* and the other at *S*. The tangents at *T* and *S* meet the common tangent at the point *W*, at *X* and *Y* respectively.

Let $\theta = \angle XTW$.

(i) Explain why $\angle XWT$ is θ. **1**

(ii) Prove that $TX \parallel YS$. **2**

(d) (i) Rewrite $\cos x + \sqrt{3}\sin x$ in the form of $r\cos(x-\alpha)$. **1**

(ii) Hence, or otherwise, solve $\cos x + \sqrt{3}\sin x = 1$ for $0 \le x \le 2\pi$. **2**

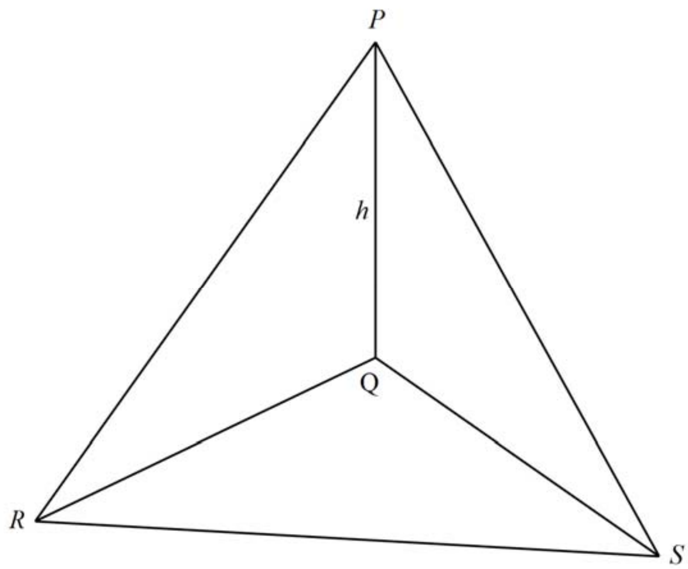
(iii) Sketch $y = \cos x + \sqrt{3}\sin x$ for $0 \le x \le 2\pi$. **1**

(iv) Find the possible ***positive*** values of *m* such that $\cos x + \sqrt{3}\sin x = mx$ has 2 solutions for $0 \le x \le 2\pi$. **1**

Question 12. (15 marks). **Marks**
Use a **separate** writing booklet.

(a) Find the exact value of $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{3}{5}\right)$. **3**

(b) In the diagram below, *PQ* is a vertical tower standing on horizontal ground.
A point *R* is due west of the tower *PQ*. From the base of the tower, *Q*, another point *S* is on a bearing of 150°.
The angles of elevation to the top of the tower *P* from *R* and *S* are 30° and 45° respectively.
R and *S* are 65 m apart. Let *h* be the height of the tower *PQ*.



(i) Copy and complete the diagram, with all the given information, in your answer booklet. Show that $\angle RQS$ is 120°. **2**

(ii) Show that $RQ=\sqrt{3}\,h$ m. **1**

(iii) Hence show that the height of the tower $h=\frac{65}{\sqrt{4+\sqrt{3}}}$ m. **3**

Question 12 continued on page 7.

(c) Using the principal of Mathematical Induction, show that $3^{3n} + 2^{n+2}$ is divisible by 5 for all positive integers n. **3**

(d) A particle moves in a straight line and its displacement, x cm, from a fixed origin after t seconds, is given by $x = \sin t - \sin t \cos t - 2t$.

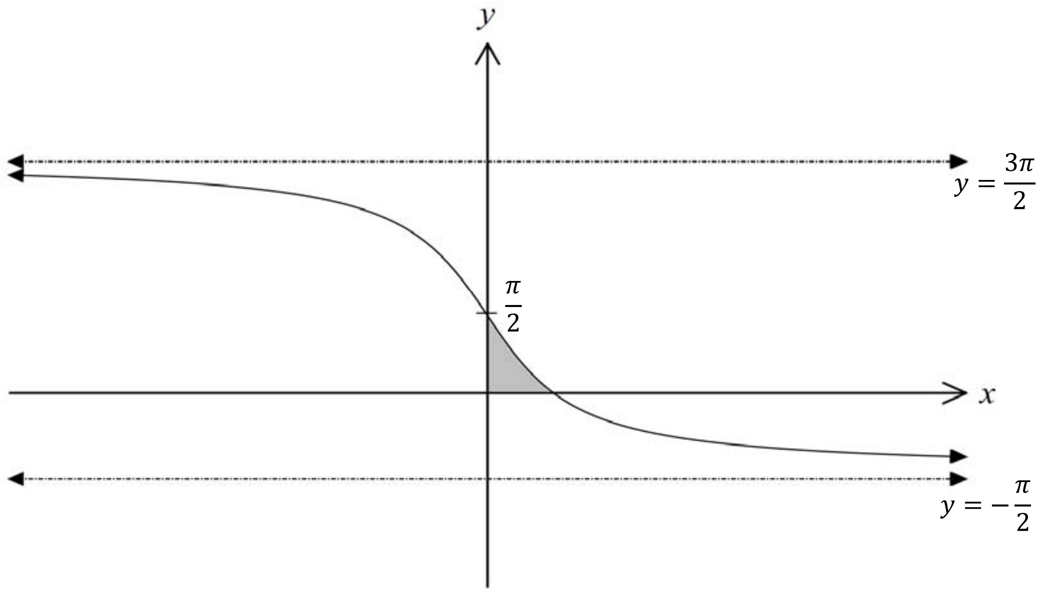
(i) Let v cm s^{-1} be the velocity of this particle. Show that $v = -2\cos^2 t + \cos t - 1$. **1**

(ii) Hence show that the particle never comes to rest and always moves in one particular direction. **2**

Question 13. (15 marks).
Use a **separate** writing booklet.

Marks

(a)



The graph of the curve $f(x) = \frac{\pi}{2} - 2\tan^{-1} x$ is drawn above. It cuts the y-axis at $\left(0, \frac{\pi}{2}\right)$.

(i) Write down the domain of the inverse function of $f(x) = \frac{\pi}{2} - 2\tan^{-1} x$. **1**

(ii) Find the equation of the inverse function of $f(x) = \frac{\pi}{2} - 2\tan^{-1} x$. **1**

(iii) Find the volume generated when the shaded region is rotated about the y-axis. **2**

Question 13 continued on page 9.

(b) Two curves $y = e^x - 1$ and $y = 2e^{-x}$ intersect at the point P.

(i) Sketch these two curves on the same number plane. **2**

(ii) Show that the coordinates of P are $(\ln 2, 1)$. **2**

(iii) Find the acute angle between the two curves at the point P. Give your answer correct to the nearest degree. **2**

(iv) Find the area bound by these two curves and the y-axis. **2**

(c) The normal at any point $P\ (2at, at^2)$ on the parabola $x^2 = 4ay$ cuts the y-axis at Q. This normal is produced to a point R such that $PQ=QR$. **3**

By first finding the equation of the normal at P, show that R has the coordinates $\left(-2at,\ at^2 + 4a\right)$.

Question 14. (15 marks).
Use a **separate** writing booklet.

Marks

(a) A particle undergoing simple harmonic motion has equation $\frac{d^2x}{dt^2} = -4(x-2)$ where x metres is the displacement of the particle from the origin O at time t seconds.
The particle is at rest at the origin.

(i) Show that $v^2 = -4x^2 + 16x$, where $v\ ms^{-1}$ is the velocity of the particle. **2**

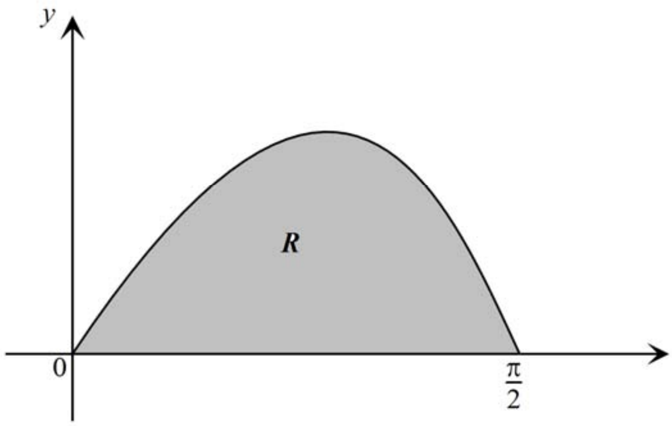
(ii) Find the period and amplitude of the motion. **2**

(iii) Find the distance travelled by the particle in the first π minutes of its motion. **1**

(b) If $2^x = 5^y = 10^z$, show that $\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$. **2**

Question 14 continued on page 11.

(c)



The diagram above shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{1+\cos x}$, $0 \le x \le \dfrac{\pi}{2}$.

The finite region R, shown shaded in the diagram, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{1+\cos x}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(i) Copy and complete the table in your answer booklet, giving the missing value of y correct to 5 decimal places. **1**

(ii) Use the trapezoidal rule to obtain an estimate for the area R, giving your answer correct to 4 decimal places. **2**

(iii) By using the substitution $u = 1+\cos x$, or otherwise, show that **3**

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$ where k is a constant.

(iv) Hence, calculate the error of the estimate in part (ii), giving your answer correct to 2 significant figures. **2**

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