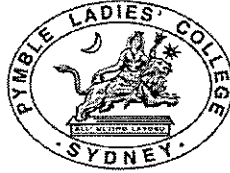


Mr Antonio  
Mrs Collett  
Mrs Israel  
Mrs Kerr  
Mrs Williams

Name: .....

Teacher:.....



# *Pymble Ladies' College*

## HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION 2014

# Mathematics Extension 1

### General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using pencil for Questions 1-10.
- Write using black or blue pen for Questions 11-14. Black pen is preferred.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- In Questions 11-14, show relevant mathematical reasoning and/or calculations.

Total Marks – 70

**Section I** Pages 1-4

10 marks

- Attempt all Questions 1-10
- Allow about 15 mins for this section

**Section II** Pages 5-11

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

<b>Mark</b>	<b>/70</b>
<b>Highest Mark</b>	<b>/70</b>
<b>Rank</b>	

**Section I**

**10 marks**

**Attempt Questions 1-10**

**Allow about 15 minutes for this section.**

Use the multiple choice answer sheet for Questions 1-10.

---

**1** The roots of the equation  $x^3 - 5x^2 + 4 = 0$  are  $\alpha, \beta$  and  $\gamma$ .

The value of  $\alpha + \beta + \gamma$  and the value of  $\alpha\beta\gamma$  are respectively.

- (A) 5 and 4
- (B) 5 and -4
- (C) -5 and 4
- (D) -5 and -4

**2** Evaluate  $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$ .

- (A)  $\frac{4\pi}{3}$
- (B)  $\frac{\pi}{3}$
- (C)  $\frac{-2\pi}{3}$
- (D)  $\frac{-\pi}{3}$

**3** When the polynomial  $P(x) = x^4 + ax + 2$  is divided by  $x^2 + 1$  the remainder is  $2x + 3$ .

The value of  $a$  is

- (A) 1
- (B) 2
- (C) 0
- (D) 3

- 4 Given the points  $A(7, 14)$  and  $B(1, 2)$ ,  $C$  is a point on  $AB$  produced such that  $AB : BC = 2 : 1$ .

Find the coordinates of  $C$ .

- (A)  $(-5, -10)$
- (B)  $(-2, -4)$
- (C)  $(3, 6)$
- (D)  $(5, 10)$

- 5 Find  $\int \frac{1}{\sqrt{1-3x^2}} dx$ .

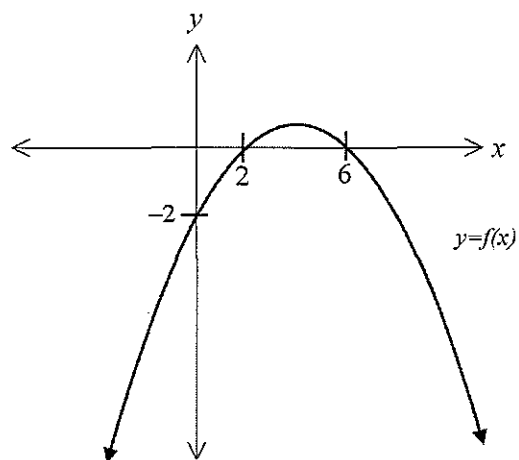
- (A)  $3\sin^{-1}(3x) + C$
- (B)  $\frac{1}{3}\sin^{-1}(3x) + C$
- (C)  $\sqrt{3}\sin^{-1}(\sqrt{3}x) + C$
- (D)  $\frac{1}{\sqrt{3}}\sin^{-1}(\sqrt{3}x) + C$

- 6 Evaluate  $\int_0^{\frac{\pi}{6}} \sin^2\theta d\theta$ .

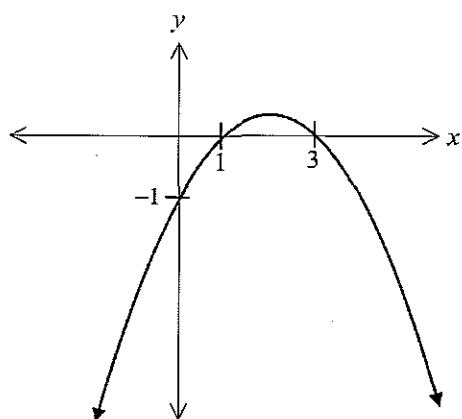
- (A)  $\frac{\pi}{12} - \frac{\sqrt{3}}{8}$
- (B)  $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$
- (C)  $\frac{1}{24}$
- (D) 1

7 The figure on the right shows the graph of  $y = f(x)$ .

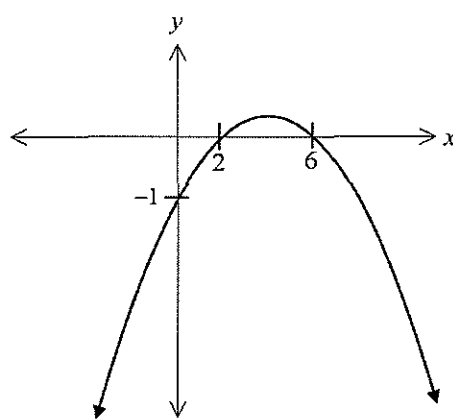
If  $2f(x) = g(x)$ , which of the following may represent the graph of  $y = g(x)$ ?



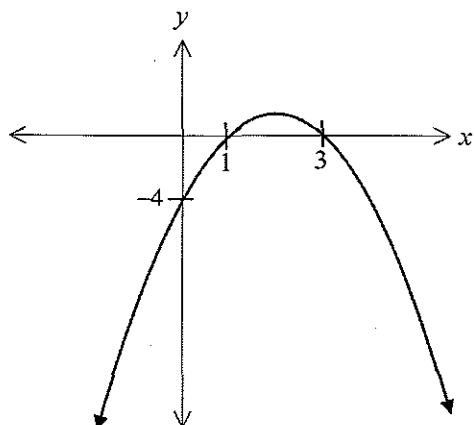
(A)



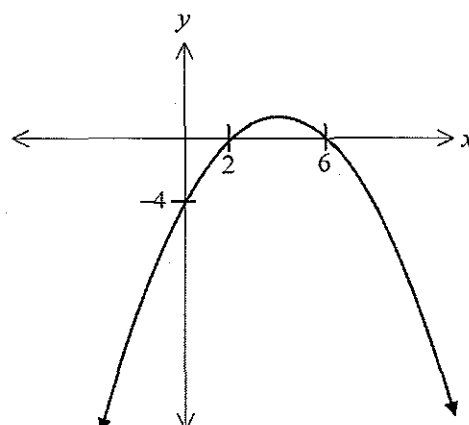
(B)



(C)



(D)



8 If  $\int_{-a}^a f(x) dx = 0$ , then which one of the following statements is false?

(A)  $f(x)$  is an odd function

(B)  $\int_0^a f(x) dx = \int_{-a}^0 f(-x) dx$

(C)  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

(D) The area bounded by the curve  $y = f(x)$ , the  $x$  axis and the lines  $x = a$  and  $x = -a$  is twice the area bounded by the curve  $y = f(x)$ , the  $x$  axis and the lines  $x = 0$  and  $x = a$ .

9 For  $0^\circ \leq \theta \leq 90^\circ$ , the least value of  $\frac{30}{3\sin^2 \theta + 2\sin^2(90^\circ - \theta)}$  is

(A) 5

(B) 6

(C) 10

(D) 15

10 Given  $n$  is an integer, the general solution of  $\tan\left(2x + \frac{\pi}{4}\right) = \sqrt{3}$  is

(A)  $x = \frac{(12n+1)\pi}{24}$

(B)  $x = \frac{(3n+1)\pi}{6}$

(C)  $x = \frac{(12n-1)\pi}{24}$

(D)  $x = \frac{(6n+1)\pi}{6}$

## Section II

60 marks

Attempt Questions 11-14

Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11.** (15 marks). Use a **Separate Booklet**.

**Marks**

(a) Given  $f(x) = x^4 + x^2 - 80$ .

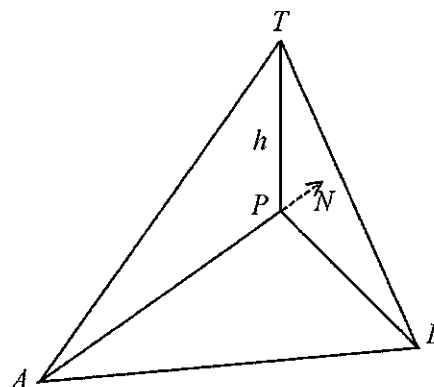
2

Assume there is a zero near  $x = 3$ . Use Newton's method once to find a better approximation to the root correct to 2 significant figures.

- (b) From a point  $A$  due south of a tower,  $TP$ , the angle of elevation of the top of the tower,  $T$  is  $25^\circ$  and from a point  $B$  due east of the tower is  $32^\circ$ .

The distance from  $A$  to  $B$  is 50 metres.

Let the height of tower  $TP$  be  $h$  metres.



- (i) Copy the diagram in your answer booklet and complete with all given information.

1

- (ii) Find an expression for  $PA$  in terms of  $h$ .

1

- (iii) Find the height of the tower,  $h$ , correct to 1 decimal place.

3

Question 11 continues on page 6.

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(c) The function  $f(x)$  is defined as  $f(x) = \frac{3x-4}{x+2}$ , where  $x \neq -2$ .

(i) Find an expression for  $f^{-1}(x)$ . 2

(ii) Write down the domain of  $f^{-1}(x)$ . 1

(d) Solve  $\frac{4}{(x-1)^2} > 1$ . 3

(e) Find  $\int \frac{\ln x}{2x} dx$  using the substitution  $u = \ln x$ . 2

**End of Question 11**

(a) Find the term independent of  $x$  in the expansion of  $\left(2x + \frac{1}{x^2}\right)^6$ . **2**

(b) (i) Show that  $\tan x = \frac{\sin 2x}{1 + \cos 2x}$ . **2**

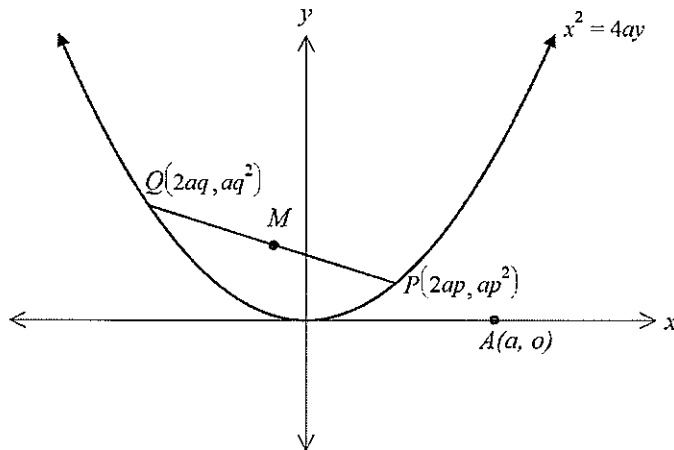
(ii) Hence evaluate  $\tan \frac{\pi}{12}$  in simplest form. **2**

(c) Prove by mathematical induction that  $8^n - 3^n$  is divisible by 5, where  $n$  is a positive integer. **3**

**Question 12 continues on page 8.**



(d)



In the diagram above, the points  $P(2ap, ap^2)$  and  $Q(2aq, aq^2)$  lie on the parabola with equation  $x^2 = 4ay$ .

- (i) Write down the coordinates of the midpoint  $M$  of the chord  $PQ$ . 1
  
- (ii) Show that the equation of the chord  $PQ$  is  $y = \frac{(p+q)x}{2} - apq$ . 2
  
- (iii) Show that the condition for the chord  $PQ$  produced to pass through the point  $A(a, 0)$  is  $p+q = 2pq$ . 1
  
- (iv) Find the cartesian equation of the locus of  $M$ , as the points  $P$  and  $Q$  move on the parabola subject to the constraint that  $PQ$  pass through  $A(a, 0)$ . 2

End of Question 12

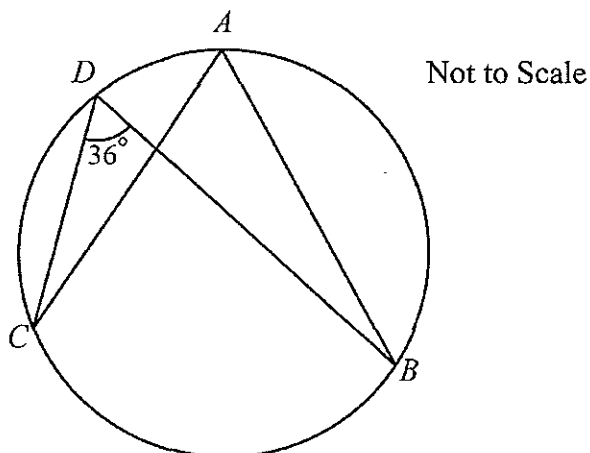
- (a) Find the acute angle between the tangents on the curve  $y = \tan^{-1} x$  at the points where  $x = 0$  and  $x = 1$ . Answer correct to the nearest degree. 2

- (b) During a chemical reaction, the amount,  $R$  kg, of chemical formed at time  $t$  hours is modelled by the differential equation

$$\frac{dR}{dt} = 4 - \frac{R}{15}.$$

- (i) Show that  $R = 60 - 50e^{-\frac{t}{15}}$  is a solution to  $\frac{dR}{dt} = 4 - \frac{R}{15}$ . 2
- (ii) How long will it take for 20 kg of the chemical to form? Give your answer correct to 2 significant figures. 2

- (c) In the figure below,  $BD$  is a diameter of the circle  $ABCD$ . If  $AB=AC$  and  $\angle BDC = 36^\circ$ , find  $\angle ABD$ . 3



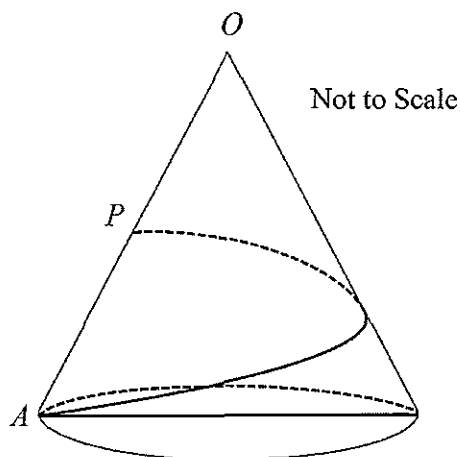
Question 13 continues on page 10.

(d) A thin sheet of smooth metal is in the shape of a sector of a circle with  $OA$ ,  $OB$  as bounding radii each of length 10 cm, and the angle  $AOB$  is  $60^\circ$ .

(i) Find the length of the arc  $AB$ .

1

(ii) The sheet is now bent to form a right circular cone by welding the radii  $OA$  and  $OB$  together (and inserting a circular disc to close in the cone at the base).



(α) Find the volume of the cone in terms of  $\pi$ .

3

(Note: The volume of a right circular cone is,  $\frac{1}{3}\pi r^2 h$ .)

(β) On the surface of this cone a thin string is pulled tight starting with one end fixed at the point  $A$  and passing once round the cone to the other end  $P$  which is at the midpoint of  $OA$  (as shown in diagram).

2

Find the exact length of this string.

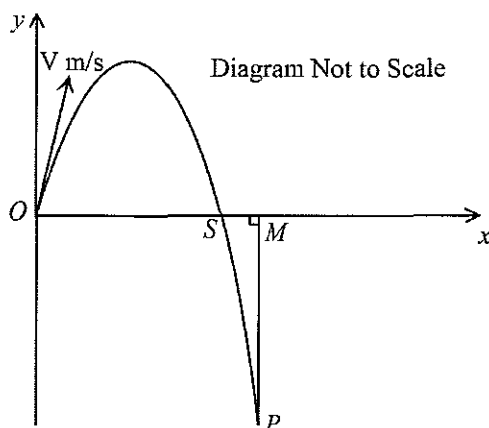
End of Question 13

(a) Solve  $\sin x - 3\cos x = 3$  for  $0^\circ \leq x \leq 360^\circ$ . 3

(b) A projectile is fired from a point  $O$  with initial speed of  $V$  m/s at an angle of elevation  $\theta$ . If  $x$  and  $y$  are the horizontal and vertical displacements of the projectile in metres from  $O$  at time  $t$  seconds later then

$$x = Vt \cos \theta \text{ and } y = Vt \sin \theta - \frac{1}{2}gt^2 \text{ where } g \text{ m/s}^2 \text{ is the acceleration due to gravity.}$$

The projectile falls to a point  $P$  below the level of  $O$  such that  $PM = OM$ .



- (i) Prove that the time taken to reach  $P$  is  $2V \frac{(\sin \theta + \cos \theta)}{g}$  seconds. 1
- (ii) Show that the distance  $OM$  is  $\frac{V^2}{g}(\sin 2\theta + \cos 2\theta + 1)$  metres. 2
- (iii) If  $OS = r$ ,  $OM = \frac{4r}{3}$  and  $r > 0$ , prove that  $\sin 2\theta - 3\cos 2\theta = 3$ . 3
- (iv) Hence, by using Question 14 part (a), find the value of  $\theta$ . 2
- (v) Find an expression for the horizontal and vertical components of the velocity. 1
- (vi) If the magnitude of the velocity of the projectile at  $P$  is  $kV$  m/s, find the exact value of  $k$ . 3

End of Paper

1. B  
6. A

2. D  
7. D

3. B  
8. C

4. B  
9. C

5. D  
10. A

Q 11

$$a) f(x) = x^4 + x^2 - 80$$

$$f'(x) = 4x^3 + 2x$$

$$x_2 = 3 - \frac{f(3)}{f'(3)}$$

$$= 3 - \frac{3^4 + 3^2 - 80}{4 \times 3^3 + 2 \times 3}$$

$$= 2.912 \dots$$

$$= 2.9$$

(2)

b) ii) In  $\triangle APT$ ;

$$\tan 25^\circ = \frac{h}{PA}$$

$$PA = h \cot 25^\circ \quad (1)$$

iii) In  $\triangle BPT$ ;

$$\tan 32^\circ = \frac{h}{PB}$$

$$PB = h \cot 32^\circ$$

In  $\triangle APB$ ;

$$50^2 = (h \cot 25^\circ)^2 + (h \cot 32^\circ)^2$$

$$h^2 = \frac{50^2}{\cot^2 25^\circ + \cot^2 32^\circ}$$

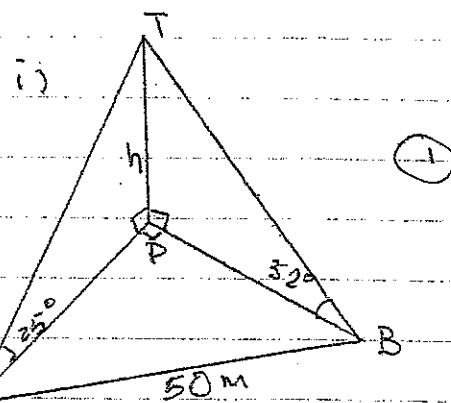
$$= 349.16 \dots$$

$$h = 18.68 \dots$$

$$= 18.7 \text{ m} \quad (h \text{ is a height of tower so } h > 0)$$

$$= 18.7 \text{ m}$$

(3)



$$c) f(x) = \frac{3x-4}{x+2}$$

$$ii) x = \frac{3y-4}{y+2}$$

$$xy + 2x = 3y - 4$$

$$xy - 3y = -4 - 2x$$

$$3y - xy = 2x + 4$$

$$y = \frac{2x+4}{3-x}$$

$$f^{-1}(x) = \frac{2x+4}{3-x} \quad (2)$$

iii) Domain: all real  $x$ ;  $x \neq 3$ . (1)

$$d) \frac{4}{(x-1)^2} > 1 \quad ; \quad x \neq 1$$

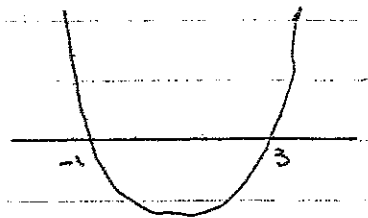
$$4 > x^2 - 2x + 1$$

$$x^2 - 2x - 3 < 0$$

$$(x-3)(x+1) < 0$$

$$-1 < x < 3$$

$\therefore -1 < x < 3$  except  $x = 1$ . (3)



$$e) \int \frac{\ln x}{2x} dx \quad ; \quad u = \ln x$$

$$= \frac{1}{2} \int \ln x \cdot \frac{1}{x} dx$$

$$du = \frac{1}{x} dx$$

$$= \frac{1}{2} \int u du$$

$$= \frac{1}{2} \left( \frac{1}{2} u^2 \right) + C$$

$$= \frac{1}{4} u^2 + C$$

$$= \frac{1}{4} (\ln x)^2 + C \quad (2)$$

Q 12

$$a) \left( 2x + \frac{1}{x^2} \right)^6 \\ = \sum_{r=0}^6 {}^6C_r (2x)^r (x^{-2})^{6-r}$$

Term independent of  $x$

$$= {}^6C_4 (2x)^4 (x^{-2})^2 \\ = {}^6C_4 \cdot 2^4 \\ = 240$$

(2)

$$b) i) \text{ RHS} = \frac{\sin 2x}{1 + \cos 2x} \\ = \frac{2 \sin x \cos x}{1 + 2 \cos^2 x - 1} \\ = \frac{2 \sin x \cos x}{2 \cos^2 x} \\ = \frac{\sin x}{\cos x} \\ = \tan x \\ = \text{LHS}$$

(2)

$$ii) \tan \frac{\pi}{12} \\ = \frac{\sin \frac{\pi}{6}}{1 + \cos \frac{\pi}{6}} \\ = \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} \\ = \frac{\frac{1}{2}}{\frac{\sqrt{3} + 2}{2}} \\ = \frac{1}{\sqrt{3} + 2} \\ = 2 - \sqrt{3}$$

(2)

c) Let the statement be  $8^n - 3^n = 5P$  where  $P$  is an integer.

When  $n = 1$ ;  
 $8^1 - 3^1 = 8 - 3$

$= 5$  which is divisible by 5.

Assume the statement is true for  $n = k$ ;

i.e.  $8^k - 3^k = 5P \Rightarrow 8^k = 5P + 3^k$

Prove that the statement is true for  $n = k + 1$ ;

i.e. prove that  $8^{k+1} - 3^{k+1} = 5Q$  where  $Q$  is an integer.

LHS =  $8^{k+1} - 3^{k+1}$

$= 8(8^k) - 3(3^k)$

$= 8(5P + 3^k) - 3(3^k)$ ; from \*

$= 5 \cdot 8P + 8 \cdot 3^k - 3 \cdot 3^k$

$= 5 \cdot 8P + 5 \cdot 3^k$

$= 5(8P + 3^k)$ ; since  $P$  is an integer,  $8P$  is an integer, since  $k$  is a positive integer,  $3^k$  is an integer.

$= 5Q$ ;  $Q = 8P + 3^k$  is also an integer.

Since the statement is true for  $n = 1$ ,

assume true for  $n = k$  and proved true for  $n = k$ .

So the statement is true for  $n = 1 + 1 = 2$ ,

$n = 2 + 1 = 3, \dots$ ,  $\therefore$  the statement is

true for all positive integers of  $n$ .

1. for intro, \* and conclusion

1. for case  $n = 1$

1. for correct steps showing

$8^{k+1} - 3^{k+1} = 5Q$ ,  $Q$  an integer

3



ds is Midpoint M of PQ

$$= \left( \frac{2ap + 2aq}{2}, \frac{ap^2 + aq^2}{2} \right)$$

$$= \left( ap + aq, \frac{ap^2 + aq^2}{2} \right) \quad (1)$$

$$\text{ii) } m_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$$

$$= \frac{a(p+q)(p-q)}{2a(p-q)}$$

$$= \frac{1}{2}(p+q)$$

Equation of chord PQ is

$$\frac{p+q}{2} = \frac{y - ap^2}{x - 2ap}$$

$$\left( \frac{p+q}{2} \right) x - 2ap \left( \frac{p+q}{2} \right) = y - ap^2$$

$$\frac{(p+q)x}{2} - (ap^2 + apq) + ap^2 = y$$

$$y = \frac{(p+q)x}{2} - ap^2 - apq + ap^2$$

$$y = \frac{(p+q)x}{2} - apq \quad (2)$$

iii) If chord PQ passes through A(a, 0),

$$\text{then } 0 = \frac{1}{2}a(p+q) - apq \quad (1)$$

$$\frac{1}{2}a(p+q) = apq$$

$$\frac{1}{2}(p+q) = pq$$

$$p+q = 2pq$$

$$\text{iv) } x_M = ap + aq$$

$$x_M = a(p+q)$$

$$x_M = 2apq \quad \therefore p+q = 2pq$$

$$y_M = \frac{1}{2}a(p^2 + q^2)$$

$$= \frac{1}{2}a[(p+q)^2 - 2pq]$$

$$= \frac{1}{2}a(p+q)^2 - apq$$

$$= \frac{1}{2a}[a^2(p+q)^2] - apq$$

$$= \frac{1}{2a}(x_M)^2 - \frac{1}{2}x_M$$

$$2ay = x^2 - ax$$

$$x^2 - ax - 2ay = 0 \quad (2)$$

Q 13

$$a) y = \tan^{-1} x$$

$$y' = \frac{1}{1+x^2}$$

$$m_1 = \frac{1}{1+0^2} = 1 \quad ; \quad m \text{ of tangent at } x=0$$

$$m_2 = \frac{1}{1+1^2} = \frac{1}{2} \quad ; \quad m \text{ of tangent at } x=1$$

$$\tan \theta = \frac{1 - \frac{1}{2}}{1 + (1)(\frac{1}{2})}$$

$$\tan \theta = \frac{1}{3}$$

The acute angle,  $\theta = 18.43 \dots$

$$= 18^\circ$$

$$b) i) R = 60 - 50e^{-\frac{t}{15}}$$

$$\Rightarrow 50e^{-\frac{t}{15}} = 60 - R$$

$$\frac{dR}{dt} = 0 - 50 \left( \frac{-1}{15} e^{-\frac{t}{15}} \right)$$

$$= \frac{1}{15} (50e^{-\frac{t}{15}})$$

$$= \frac{1}{15} (60 - R)$$

$$= 4 - \frac{R}{15} \quad (2)$$

$$ii) 20 = 60 - 50e^{-\frac{t}{15}}$$

$$-40 = -50e^{-\frac{t}{15}}$$

$$\frac{-t}{15} = \ln \frac{4}{5}$$

$$t = -15 \ln \frac{4}{5}$$

$$= 3.347 \dots$$

$$\approx 3.3 \text{ hours} \quad (2)$$

c)  $\angle BAC = \angle BDC$  (angles at the circumference standing on the same chord BC)

$= 36^\circ$

$\angle ACB = \angle ABC$  (angles opposite equal sides AB and AC in isosceles  $\triangle ABC$ )

$\angle ACB + \angle ABC + \angle BAC = 180^\circ$  (angles sum of  $\triangle ABC$ )

$2\angle ACB + 36^\circ = 180^\circ$

$\angle ACB = 72^\circ$

$\angle DCA = \angle DBA$  (angles at the circumference standing on the same arc AD)

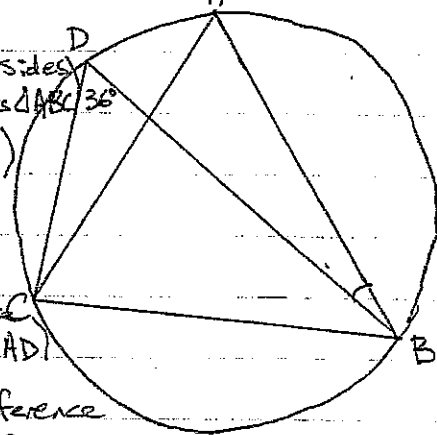
$\angle DCB = 90^\circ$  (angles at the circumference in a semicircle, BD is the diameter)

$\angle DCA + \angle ACB = \angle DCB$  (adjacent angles)

$\angle DBA + 72^\circ = 90^\circ$

$\angle DBA = 90^\circ - 72^\circ$

$= 18^\circ$



(3)

$$d) \text{ i) } \angle AOB = 60^\circ = \frac{\pi}{3}$$

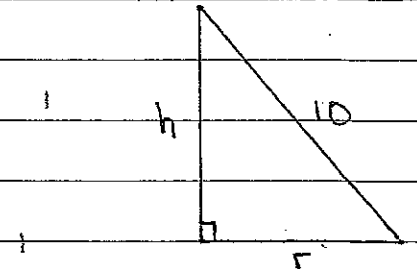
$$\begin{aligned} \text{Length of arc AB} &= 10 \times \frac{\pi}{3} \\ &= \frac{10\pi}{3} \text{ cm} \end{aligned} \quad (1)$$

ii) a) Let  $r$  be the radius of the cone and  $h$  be the height of the cone.

$$2\pi r = \frac{10\pi}{3}$$

$$r = \frac{5}{3} \text{ cm}$$

$$\begin{aligned} h &= \sqrt{10^2 - \left(\frac{5}{3}\right)^2} \\ &= \frac{5\sqrt{35}}{3} \text{ cm} \end{aligned}$$

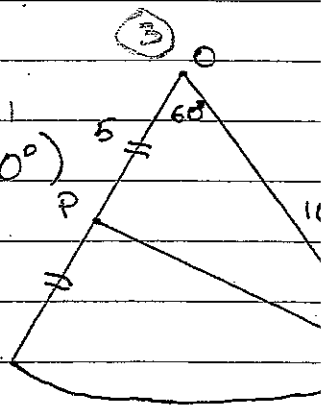


$$\begin{aligned} \text{Volume of cone} &= \frac{1}{3} \times \pi \times \left(\frac{5}{3}\right)^2 \times \frac{5\sqrt{35}}{3} \\ &= \frac{125\sqrt{35}\pi}{81} \text{ cm}^3 \end{aligned}$$

b) In  $\triangle BPO$ ;

$$\begin{aligned} PB^2 &= 5^2 + 10^2 - 2(5)(10)(\cos 60^\circ) \\ &= 125 - 100 \times \frac{1}{2} \\ &= 75 \end{aligned}$$

$$\begin{aligned} PB &= \sqrt{75} ; \text{ PB is a length so } PB > 0, PB \neq -\sqrt{75} \\ &= 5\sqrt{3} \end{aligned}$$



$\therefore$  Length of string,  $PB = 5\sqrt{3} \text{ cm}$ .

1 - correct set up

2 - correct set up and acute sol<sup>n</sup>

3 - All correct

Q 14

a)  $\sin X - 3 \cos X = 3$

$\sqrt{10} \sin(X - \theta) = 3$  ;  $\tan \theta = 3 \Rightarrow \theta \approx 71.56$

$\sin(X - \theta) = \frac{3}{\sqrt{10}}$  &  $-\theta \leq X - \theta \leq 360^\circ - \theta$

$X - \theta = 71.56 \dots$  OR  $X - \theta = 180^\circ - 71.56 \dots$

$X = 143.13 \dots$  OR  $X = 180^\circ$

$\therefore X = 143^\circ$  OR  $180^\circ$  (nearest degree) } ③

b)  $\begin{cases} x = vt \cos \theta \\ y = vt \sin \theta - \frac{1}{2}gt^2 \end{cases}$

i) Since  $PM = OM$ , at P  $x = -y$  :

$vt \cos \theta = \frac{1}{2}gt^2 - vt \sin \theta$

$\frac{1}{2}gt^2 = vt(\cos \theta + \sin \theta)$

$t = 0$  OR  $\frac{1}{2}gt = v(\sin \theta + \cos \theta)$

However at P,  $t \neq 0$ ;

$\frac{1}{2}gt = v(\sin \theta + \cos \theta)$  ①

r/w

$t = \frac{2v(\sin \theta + \cos \theta)}{g}$

ii) When  $t = \frac{2v(\sin \theta + \cos \theta)}{g}$ ,

$x = \frac{v \cos \theta - 2v(\sin \theta + \cos \theta)}{g}$  ①

$= \frac{v^2}{g} (2 \sin \theta \cos \theta + 2 \cos^2 \theta)$

$= \frac{v^2}{g} (\sin 2\theta + \cos 2\theta + 1)$  ;  $\begin{matrix} \sin 2\theta = 2 \sin \theta \cos \theta \\ \text{and } \cos 2\theta = 2 \cos^2 \theta - 1 \end{matrix}$  ①

$\Rightarrow 2 \cos^2 \theta = \cos 2\theta + 1$

②

iii)  $OS = r$

$\Rightarrow$  When  $y = 0$ ,  $x = r$

$vt \sin \theta - \frac{1}{2}gt^2 = 0$

$t(v \sin \theta - \frac{1}{2}gt) = 0$

$\frac{1}{2}gt = v \sin \theta$  ;  $t \neq 0$

$t = \frac{2v \sin \theta}{g}$  ①

When  $t = \frac{2v \sin \theta}{g}$ ,

$$x = v \cos \theta \cdot \frac{2v \sin \theta}{g} = r$$

$$\frac{2v^2 \sin \theta \cos \theta}{g} = r \quad \text{--- (1)}$$

$$\frac{v^2 \sin 2\theta}{g} = r \quad \text{--- *}$$

$$OM = \frac{4r}{3} = \frac{v^2}{g} (\sin 2\theta + \cos 2\theta + 1); \text{ from (i)}$$

$$\frac{4v^2 \sin 2\theta}{3g} = \frac{v^2 \sin 2\theta}{g} + \frac{v^2}{g} (\cos 2\theta + 1); \text{ from (i)}$$

$$\frac{v^2 \sin 2\theta}{3g} = \frac{v^2}{g} (\cos 2\theta + 1) \quad \text{--- (1)}$$

$$\frac{\sin 2\theta}{3} = \cos 2\theta + 1$$

Tying together.

$$\sin 2\theta = 3\cos 2\theta + 3$$

$$\sin 2\theta - 3\cos 2\theta = 3 \quad \text{--- (3)}$$

iv)  $2\theta = 143.13^\circ \text{ OR } 180^\circ$

$$\theta = 71.56^\circ \text{ OR } 90^\circ \quad \text{--- (1)}$$

However  $\theta$  is the angle of elevation

and the projectile is not fired vertically,  $0^\circ < \theta < 90^\circ$

$$\therefore \theta = 71.56^\circ$$

(1) Explanation

and exclusion

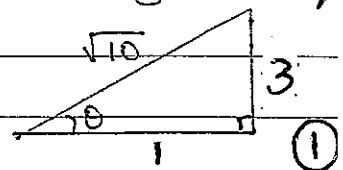
v)  $x = vt \cos \theta \Rightarrow \dot{x} = v \cos \theta$

vi)  $y = vt \sin \theta - \frac{1}{2}gt^2 \Rightarrow \dot{y} = v \sin \theta - gt$

vii) From (a),  $\tan \theta = 3$

$$\Rightarrow \sin \theta = \frac{3}{\sqrt{10}}$$

$$\& \cos \theta = \frac{1}{\sqrt{10}}$$



$$\dot{x} = \frac{v}{\sqrt{10}} \quad \text{and} \quad \dot{y} = \frac{3v}{\sqrt{10}} - g \left( \frac{2v}{g} (\sin \theta + \cos \theta) \right)$$

$$\dot{y} = \left( \frac{3}{\sqrt{10}} - \frac{6}{\sqrt{10}} - \frac{2}{\sqrt{10}} \right) v = \frac{-5v}{\sqrt{10}} \quad \text{--- (1)}$$

$$(\text{Velocity})^2 = \dot{x}^2 + \dot{y}^2 = (kV)^2$$

$$\left( \frac{v}{\sqrt{10}} \right)^2 + \left( \frac{-5v}{\sqrt{10}} \right)^2 = k^2 v^2$$

$$\frac{26v^2}{10} = k^2 v^2$$

$$k = \sqrt{\frac{13}{5}} \quad \text{--- (1)}$$

since the magnitude of  $\dot{y}$  is  $kV$ ,  $k > 0$