RAVENSWOOD SCHOOL FOR GIRLS

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

37 Copies	Miss Walker Mrs Jackson

2000

MATHEMATICS

3 UNIT (ADDITIONAL)
AND
3/4 UNIT (COMMON)

Time allowed—Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt **ALL** questions.
- **ALL** questions are of equal value.
- Write your Student Number on every page of the question paper and your answer sheets.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied.
- Board approved calculators may be used.
- The answers to the seven questions are to be handed in separately, clearly marked Question 1, Question 2, ... etc
- ***The question paper must be handed to the supervisor at the end of the examination.***

QUESTION 1. Use a SEPARATE writing booklet. **Marks**

(a) Let $A\,(-3, 6)$ and $B\,(1, 10)$ be points on the number plane. Find the coordinates of the point C, which divides the interval AB externally in the ratio $5 : 3$. **2**

(b) Find the obtuse angle between the lines $3y = 2x + 1$ and $y = -3x + 5$, correct to the nearest degree. **3**

(c) Use the substitution $u = 2x - 1$ to evaluate $\int_0^1 x(2x-1)^4\,dx$. **4**

(d) Solve the inequality $\dfrac{x}{x-3} < 4$. **3**

QUESTION 2. Use a SEPARATE writing booklet.

(a) Evaluate $\displaystyle\int_{-3}^{3} \frac{1}{9 + x^2}\,dx$. **3**

(b) Consider the function $y = \cos^{-1}(2x) - \frac{\pi}{2}$. **3**

(i) State the domain of this function.

(ii) State the range of this function.

(iii) Sketch the graph of this function.

(c) Find $\displaystyle\lim_{\theta \to 0} \frac{\theta + \sin 2\theta}{3\theta}$. **2**

(d) Use the table of standard integrals to find $\displaystyle\int \frac{dx}{\sqrt{x^2 - 4}}$. **1**

(e) Consider the polynomial $P(x) = x^3 - 5x + c$. **3**

(i) Find the value of c if $x + 2$ is a factor of $P(x)$.

(ii) For this value of c, find $Q(x)$ such that $P(x) = (x + 2)Q(x)$.

QUESTION 3. Use a SEPARATE writing booklet.

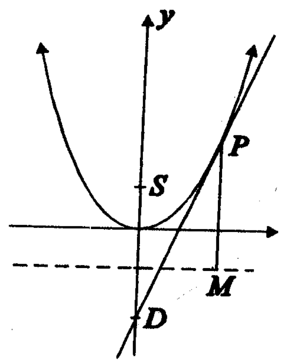
Marks

(a) If α, β, γ are the roots of the equation $x^3 + 2x^2 - x - 5 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. 2

(b) Find the coefficient of x^2 in the expansion of $(3 - 2x)(2 + x)^4$. 3

(c) The point $P(2ap, ap^2)$ lies on the parabola defined by $x^2 = 4ay$. 4

The line PM is drawn parallel to the axis of the parabola to meet the directrix in M. S is the focus of the parabola.

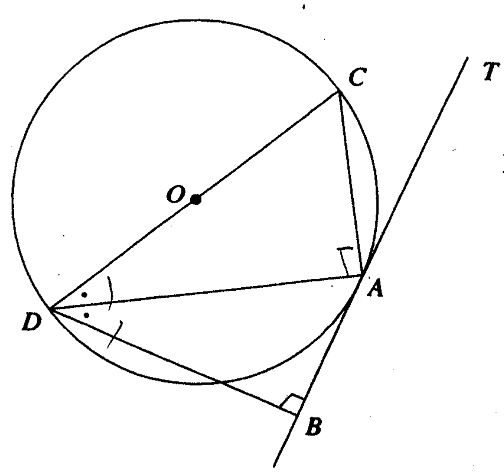


(i) State why SP is equal to PM.

(ii) The tangent at P meets the y-axis at D. Find the coordinates of D.

(iii) Show that SPMD is a rhombus.

(d) 3



O is the centre of a circle. *TAB* is a tangent to the circle at *A*. *AD* bisects the angle *CDB*.
Copy or trace the diagram into your Writing Booklet.
Prove that the angle *ABD* is a right angle.

QUESTION 4. Use a SEPARATE writing booklet.

Marks

(a) Due to the general ageing of the community, the numbers in the local high school were declining at a rate proportional to the amount by which the numbers in the school exceeded 600. This is expressed by the equation **4**

$$\frac{dN}{dt} = k(N - 600),$$

where N is the number of students enrolled t years after 1990.

There were 1100 students enrolled at the beginning of 1990 and 900 students enrolled at the beginning of year 2000.

(i) Prove that $N = 600 + Ae^{kt}$ satisfies this equation.

(ii) Find the value of A.

(iii) Find the value of k correct to 4 significant figures.

(iv) How many students would you expect to be enrolled at the beginning of the year 2010 if the decline continued under the same conditions?

(b) Prove, using mathematical induction, that $7^n - 4^n$ is divisible by 3, where n is a positive integer. **4**

(c) (i) Using the identities for the expansions of $\sin(A + B)$, $\sin 2A$ and $\cos 2A$, prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **4**

(ii) Hence solve the equation $3\sin\theta - 4\sin^3\theta = -1$ for $0 \le \theta \le 2\pi$.

QUESTION 5. Use a SEPARATE writing booklet.

Marks

(a) A particle P moves in a straight line in simple harmonic motion. The acceleration in metres per second per second is given by 6

$$\ddot{x} = 2 - 3x$$

where x metres is the displacement of the particle from the origin.

Initially the particle is at $x = 1$ moving with a velocity of $\sqrt{5}$ m s^{-1}.

(i) Using integration show that the velocity v m s^{-1} of the particle is given by

$$v^2 = 4 + 4x - 3x^2.$$

(ii) Find the amplitude of motion.

(iii) Find the centre of motion.

(iv) Find the maximum speed of the particle.

(v) Find the period of the motion.

(b) (i) Prove that $e^{2x} - e^x = 56$ has a root between 2 and 3. 6

(ii) Taking $x = 2$ as an approximation, use one application of Newton's method to find a better approximation correct to three significant figures.

(iii) By considering $e^{2x} - e^x = 56$ as a quadratic equation in e^x, solve the equation, giving your answer correct to three significant figures.

QUESTION 6. Use a SEPARATE writing booklet. **Marks**

(a) A factory machining car parts finds that 98% are machined correctly. From a sample of 40 car parts, calculate to 3 decimal places the probability that **3**

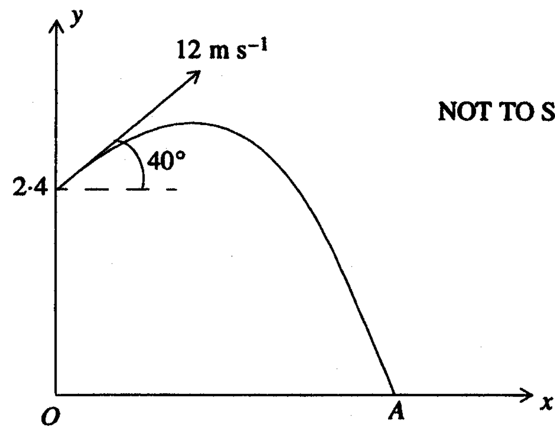
(i) exactly 38 of the parts are correctly machined.

(ii) less than three parts are incorrectly machined.

(b) If r is a positive integer and $1 \le r \le 10$, find the largest value of r which satisfies **3**

$$\binom{10}{r} 3^{10-r} \times 2^r > \binom{10}{r-1} 3^{11-r} \times 2^{r-1}.$$

(c) **6**



In an Olympic trial, a shot putter releases the shot from a height of 2·4 metres above ground level at an angle of 40° to the horizontal, and with a speed of 12 metres per second.

Take the origin O at a point on the ground directly under the point of release of the shot.

The equations of motion of the shot are

$$\ddot{x} = 0, \quad \ddot{y} = -g.$$

(i) Using calculus, show that the position of the shot at time t is given by

$$x = 12\cos 40^\circ t, \quad y = 2{\cdot}4 + 12\sin 40^\circ t - \frac{1}{2}gt^2.$$

(ii) The shot lands at a point A on the ground. Find the length of OA to the nearest centimetre. (Take $g = 9{\cdot}8$).

QUESTION 7. Use a SEPARATE writing booklet.

Marks

(a) A new car, value \$35 000, is bought on a lease arrangement. The interest is 13% per annum reducible, calculated fortnightly (assume 26 fortnights in a year). Repayments are made every fortnight. At the end of three years, there is still 40% of the original value of the car to be repaid. **6**

(i) If the fortnightly repayments are \$$M$, show that the amount owing after the first repayment is \$$(35\,000 \times 1{\cdot}005 - M)$.

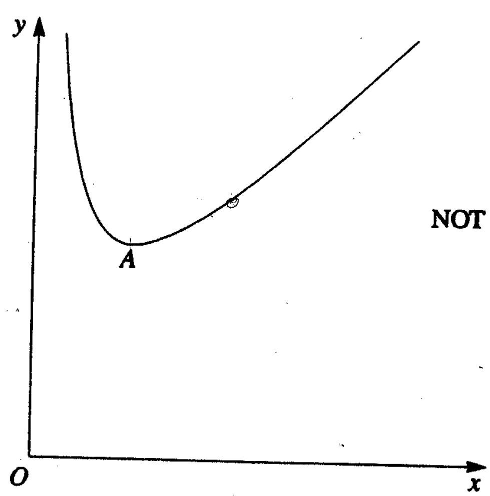
(ii) Show that the amount owing at the end of three years is

$$35\,000 \times 1{\cdot}005^{78} - 200M(1{\cdot}005^{78} - 1) \text{ dollars.}$$

(iii) Hence find the fortnightly repayments correct to the nearest cent.

(b) Consider the function $f(x) = 4x + \frac{1}{x}$ for $x > 0$. **6**

The diagram shows the graph of the function and $A\left(\frac{1}{2}, 4\right)$ is the stationary point.



NOT TO SCALE

(i) What is the largest domain for which the function $f(x)$ has an inverse function $f^{-1}(x)$?

(ii) Copy or trace the graph of $y = f(x)$ into your Writing Booklet.

On the same set of axes, draw the graph of $y = f^{-1}(x)$.

(iii) Find the inverse function $f^{-1}(x)$.

End of paper