Student Number: ____________

SCECGS REDLANDS

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
JULY 1999

3 UNIT MATHEMATICS

(3 Unit Additional and 4 Unit Common)

Time allowed - Two hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- All questions may be attempted.
- ***Begin a new answer booklet*** for each question.
- All questions are of equal value. A mark scheme is provided for your use.
- **All necessary working**, and any formulae used should be shown in every question.
- Marks may be deducted for **careless or badly arranged work**.
- **Only NSW Board of Studies approved calculators** may be used.
- **Standard integrals** are printed on the last page.
- **Clearly write** your **student number** and the **question number**, and the label of each part attempted on the front of every booklet that you hand in. A second booklet used for a question should be slid inside the first. Write "Book 1 of 2", "Book 2 of 2", etc.
- **A blank clearly numbered** booklet **must** be handed in for **any questions not attempted.**
- You may ask for extra answer booklets if you need them.

Students are advised that this is a *Trial Examination* only and cannot in any way guarantee the content of the Higher School Certificate Examination. We hope that this paper will assist you in your preparation for the final Higher School Certificate Examination.

ERRATA

TRIAL HSC 3 UNIT MATHEMATICS EXAMINATION
JULY 1999

Please make the following corrections to your printed Examination Paper :

QUESTION 6. (c) **The y-displacement-time should be written as** $y = (V \sin \alpha)t - \frac{g}{2}t^2$.

Part (iii) should read "Hence show that the uphill range up the road is given by the equation

$$OR = \frac{2V^2 \cos \alpha}{g \cos^2 \theta} . \sin(\alpha - \theta)$$ **."**

In Part (iv) show that the shell will reach nearly ***2 km*** **up the road (instead of** ***2.4 km*** **up the road).**

QUESTION 7. (a) **Delete the words "the ordinates" .**

QUESTION 1. *(Start a new answer booklet)* **Marks**

(a) If $y = e^{\sin x}$ find 3

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$.

(b) Make t the subject of the formula $N = P + Ae^{kt}$. 2

(c) On a number plane diagram sketch the region which represents the simultaneous solution of $y \geq x^2$ and $y \leq |x|$. 3

(d) Using the table of standard integrals (printed at the end of this paper), evaluate $\displaystyle\int_2^3 \frac{1}{\sqrt{x^2 - 4}}\,dx$. Give your answer in *exact form.* 2

(e) Evaluate as an *exact value* $\displaystyle\int_1^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\,dx$. 2

QUESTION 2. *(Start a new answer booklet)* **Marks**

(a) Evaluate $\lim_{x \to 0} \frac{\sin(ax)}{x}$. 1

(b) Use the substitution $u = 8 - 4t$ to evaluate 4

$\int_1^2 t\sqrt{8 - 4t}\,dt$. Express as an *exact value.*

(c) A particle is moving in a straight line subject to the 3
displacement-time equation $x = 4\cos(3t + \frac{\pi}{4})$.

(i) Show that the motion is simple harmonic.

(ii) Write down the period of the motion.

(iii) Write down the velocity when the particle is at $x = 4$.

(d) (i) *Find the general solution* of the equation 4
$\sqrt{3}t^2 - 4t + \sqrt{3} = 0$ where $t = \tan\theta$.
Express your solution in radians and in *exact form* .

(ii) Sketch on a number line those solutions which lie in the interval $-2\pi < \theta < 2\pi$.

QUESTION 3. ***(Start a new answer booklet)*** **Marks**

(a) **Using the substitution** $s = \sin\theta$ **evaluate as an** ***exact value*** 3

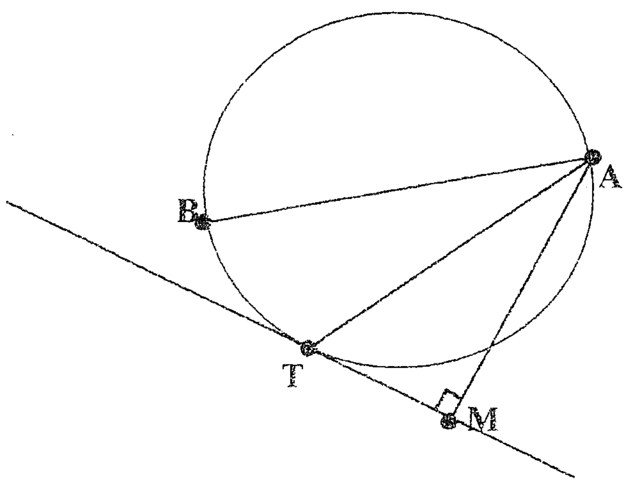
$$\int_0^{\frac{3\pi}{4}} \cos\theta \sin^5\theta \, d\theta \quad .$$

(b) **Show that** $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$. **Hence find** 5

(i) $\int \cos^2(3x)\,dx$

(ii) $\int \cos^4(3x)\,dx$.

(c) **AB is a diameter of a circle and T any third point on the circumference. M is the foot of the perpendicular from A to the tangent at T.** 4



NOT TO SCALE

(i) **Copy the diagram into your answer booklet .**

(ii) **Giving full reasons, prove that TA is the bisector of** $\angle MAB$.

[*HINT : Let* $\angle TAB = \theta$ *and join TB*]

QUESTION 4. *(Start a new answer booklet)* **Marks**

(a) *Briefly* sketch the graph of the function 4
$f(x) = (x-1)^2 + 1$.

(i) Explain why the function does not have an inverse function for all real values of x.

(iii) Suggest a suitable restriction to enable an inverse function to exist with a maximum domain, and *briefly* sketch the inverse function $y = f^{-1}(x)$ on the same diagram as $y = f(x)$.
Label your sketch clearly.

(iv) Find the equation of the inverse function $y = f^{-1}(x)$.

(b) *Consider* the function $f(x) = 3\cos^{-1}\frac{x}{2}$. 2

(i) Sketch the graph of $y = f(x)$.

(ii) State the domain and range of $y = f(x)$.

(c) (i) *Show* that $\sin 2t - \cos 2t$ may be expressed in the form $R\sin(2t - \alpha)$ where $\alpha = \frac{\pi}{4}$ and $R = \sqrt{2}$. 6

(ii) *Hence or otherwise* express *in exact form* the solutions of the equation $2\sin 2t - 2\cos 2t = \sqrt{6}$ in the interval $0 \le t \le \pi$.

QUESTION 5. *(Start a new answer booklet)* **Marks**

(a) A particle moves in a straight line with velocity v *m/s* satisfying the equation $v^2 = 6 + 4x - 2x^2$ where x is the displacement in metres from a fixed point O at time t seconds. **5**

(i) Show that the motion is simple harmonic about $x = 1$ as centre of motion .

(ii) Find the period of the motion.

(iii) By the method of completion of squares, or otherwise, find the amplitude of the motion.

(b) International Rugby star and celebrity Richie "Dazzajax" Jordan wishes to save for a spectacular retirement from international competition at age 35. **7**

Richie plans to invest $100 000 of his earnings, on each birthday from his 21st birthday to his 35th birthday, into an investment fund which guarantees a compound interest growth of 5% per annum.

As a 35th birthday present to himself, Richie plans to withdraw immediately his entire savings from the fund, buy a yacht, and sail around the world. This includes withdrawing the final $100 000 which did not have time to earn any interest.

(i) Write down an expression for the value of his first $100 000 investment on his 35th birthday.

(ii) Show that on his 35th birthday the total value of his savings is $\$2\,000\,000 \times \left[(1.05)^{15} - 1\right]$.

(iii) Find the percentage of interest earned on his total investment in the fund.

QUESTION 6. *(Start a new answer booklet)*

Marks

(a) **For which value of x does $\sin^{-1} x = \cos^{-1} x$?** **1**

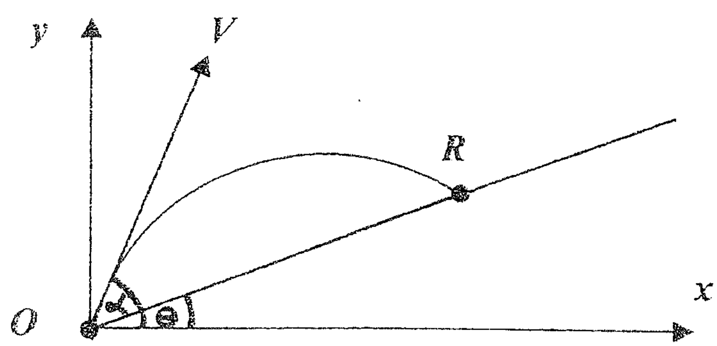
(b) **N is the number of animals in an endangered species. The animal population is found to be diminishing approximately in accordance with the rule $\frac{dN}{dt} = -k(N - 1000)$. In two years the population has dropped from *2500* to *2000* animals.** **5**

(i) **By integration show that** $N = 1000 + 1500\, e^{-kt}$ **with** $k = -\frac{1}{2}\ln\frac{2}{3}$.

(ii) **Find to the nearest month how long we might expect for the animal population to drop from *2500* to *1300* .**

(c) **An artillery shell is fired with an initial velocity of V *m/s* with a projection angle α from the base of a straight uphill road inclined at an angle θ above horizontal ($0^o < \theta < \alpha < 90^o$). The shell strikes the road at point R. You may assume that air resistance may be neglected and that the particle moves according to the following displacement-time equations :** **6**

$$x = (V\cos\alpha)t \quad \text{and} \quad y = \frac{g}{2}t^2 - (V\sin\alpha)t \ .$$



NOT TO SCALE

(i) **Copy the diagram into your answer booklet.**

(ii) **Show that the equation of the flight path of the shell is given by the equation** $y = (\tan\alpha)x - \frac{g\sec^2\alpha}{2V^2}x^2$.

(iii) **Hence show that the uphill range up the road is given by** $OR = \frac{2V^2\sin(\alpha - \theta)}{g\cos\theta}$.

(Hint: The equation of the road is $y = (\tan\theta)x$.**)**

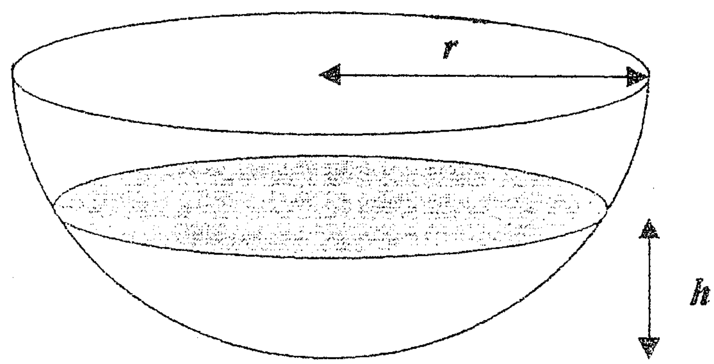
(iv) **If** $V = 200$ *m/s* , $\alpha = 45^o$, $\theta = 30^o$, **using** $g \approx 10 m/s^2$ **show that the shell will reach nearly [illegible] km on the road.**

QUESTION 7. *(Start a new answer booklet)* Marks

(a) Show that the volum... of the solid generated by rotating the circle $x^2 + y^2 = r^2$ about the x-axis between the ordinates $x = r - h$ and $x = r$ (where $0 < h < r$) is given by the formula $V = \pi\left[rh^2 - \frac{h^3}{3}\right]$ *units*^{3} . 4

Include a *large clear sketch* in your answer.

(b) Water is trickling into an upright hemispherical bowl of radius r centimetres at the constant rate of C *cm*^{3}*/minute.* After t minutes the depth is h centimetres. 8



(i) *Using part (a)* above show that the rate at which the depth is increasing after t minutes is given by

$$\frac{dh}{dt} = \frac{C}{\pi h\left[2r - h\right]} \ cm/min .$$

(ii) Water trickles into a hemispherical bowl of radius *20 cm* at the rate of *1 litre/minute.* Show that at the instant when the surface area of the water in the bowl reaches $100\pi\, cm^2$ the depth of water in the bowl is $20 - 10\sqrt{3}\, cm$.

(Hint: A good start might be to let the radius at the surface of the water be y *cm* and look carefully at your diagram for *part (a)* .)

(iii) Hence show that the depth of water is increasing at a rate of $\frac{10}{\pi}\, cm/\min$ when the surface area of water in the bowl reaches $100\pi\, cm^2$.