SAINT IGNATIUS' COLLEGE RIVERVIEW

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1997

3/4 UNIT MATHEMATICS

Time Allowed: 2 Hours
(plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

1. Attempt ALL questions.
2. All questions are of equal value.
3. All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
4. Standard integrals are provided.
5. Board-approved calculators may be used.
6. Each question attempted is to be returned in a SEPARATE Writing Booklet, clearly marked Question 1, Question 2, etc. on the cover.
7. Each Booklet must show your Student number and the name of your Class Teacher.
8. You may ask for extra Writing Booklets if you need them.

QUESTION 1 USE A SEPARATE ANSWER BOOK

a) Solve $\dfrac{x-1}{x} \geq 0$ (3)

b) Find the acute angle between the two lines $x-2y-1=0$ and $3x+y+2=0$. (Answer to the nearest minute) (3)

c) Use the " t " results to solve the equation $\sin\theta - 2\cos\theta = -1$ for $0 \leq \theta \leq 2\pi$. (Answer to 3 significant figures). (3)

d) Divide the interval AB externally in the ratio 5:2, given A is (-3,2) and B is (1,5) (3)

QUESTION 2 **USE A SEPARATE ANSWER BOOK**

a) Find the value of the constants p and q if $x^2 - 4x + 3$ is a factor of $x^3 + px^2 - x + q$ (3)

b) On old 727 jet planes, there were 56 rows of seats. Each row had three seats on each side of a central aisle. Three friends took a flight on a 727 jet plane with random seat allocation. Find the number of seating arrangements possible if:-

i) all three friends must sit together. (1)

ii) all three must sit in separate rows. (1)

iii) no more than two can sit together. (2)

c) Use the process of Mathematical Induction to prove for $n \geq 1$:- (5)

$$\frac{1}{1 \times 5} + \frac{1}{5 \times 9} + \frac{1}{9 \times 13} + \dots\dots\dots\dots\dots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1}$$

QUESTION 3 USE A SEPARATE ANSWER BOOK

a) i) Find $\frac{d}{dx}\left(x\cos^{-1}x - \sqrt{1-x^2}\right)$ (2)

ii) Hence, find $\int \cos^{-1}x \, dx$ (1)

b) The hole in the ozone layer can be approximated to a circle of radius r kilometres. If the radius of the hole is increasing at the rate of 0.6 km/year, find the rate at which the area of the hole is increasing when the hole is 1500 km in diameter. (3)

c) Use the substitution $v = \ln x$ to evaluate $\int_1^{e^2} \frac{dx}{x\left(1+(\ln x)^2\right)}$ (3)

d) i) Sketch $y = 3\cos x$ and $y = x$ for $0 \le x \le 2\pi$, on the same set of axes. (1)

ii) An approximate solution to the equation $3\cos x - x = 0$ is $x = 1.15$. (2)

Use one application of Newton's Method to find a better approximation to the solution.

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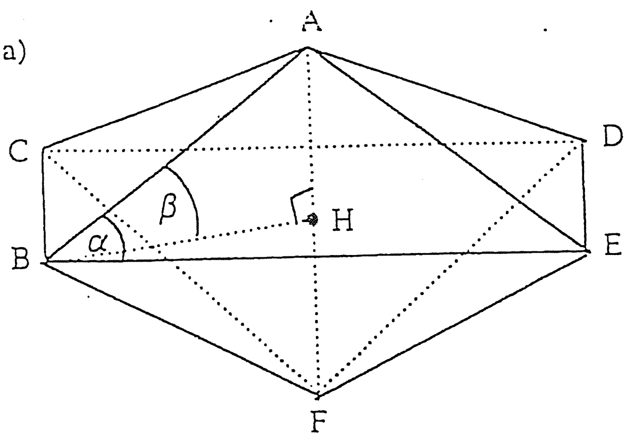
QUESTION 4 USE A SEPARATE ANSWER BOOK

a)



The figure represents an octahedron which has eight congruent faces in the shape of isosceles triangles.

BE = 8cm
AHF is a diagonal
$\angle ABH = \beta$
$\angle ABE = \alpha$
AB = AE

DIAGRAM NOT TO SCALE

i) Show that $AB = 4\sec\alpha$ (2)

ii) Hence, show that the length of diagonal AF is $8\sin\beta\sec\alpha$ (2)

b) State the domain and range of $y = \sin^{-1}\left(\frac{x}{2}\right) + \frac{3\pi}{2}$ (2)

c) The velocity, v metres/second, of a particle moving along the x-axis at time t seconds, is given by $v = \frac{1}{\sqrt{4-t^2}}$.

The particle is initially $\frac{3\pi}{2}$ metres to the right of O.

i) For which values of t is the velocity defined? (1)

ii) Find an equation for the displacement, x metres from O, in terms of t. (2)

iii) Sketch the graph of displacement versus time. (2)

iv) Describe the motion of the particle as $t \to 2$ (1)

QUESTION 5 — USE A SEPARATE ANSWER BOOK

a) Find the term independent of x in the expansion $\left(x+\frac{3}{x^2}\right)^{12}$ (3)

b) The parabola $x^2=4ay$ has S as the focus.

P is a variable point $\left(2ap,ap^2\right)$ on the parabola and the line joining P and S is produced to Q so that PS = SQ. (3)

i) Write down the co-ordinates of Q in terms of p.

ii) Find the cartesian equation of the locus of Q.

c) The rate at which a drug is being expelled from the body at time t hours is given by the equation $\frac{dM}{dt}=-k\left(M-0.04\right)$ where k is a constant and M is measured in grams.

i) Show that $M=0.04+M_0e^{-kt}$ is a solution to the equation $\frac{dM}{dt}=-k\left(M-0.04\right)$ given M_0 is a constant. (2)

ii) Initially 4 grams was ingested. Find the value of M_0 (1)

iii) After 10 hours, 1.6 grams was still in the body. Find the value of k. (2)

iv) Show that the drug will never be entirely eliminated from the body. (1)

QUESTION 6 USE A SEPARATE ANSWER BOOK

a) Evaluate $\int_0^{\frac{\pi}{4}} \sin^2 3\theta . d\theta$ (2)

b) In an HSC examination of 3 hours duration, there are 10 questions. Henry decides that the time it will take to complete each question is 10% longer than the time it took to complete the previous question. If the time taken to complete the first question is x minutes:

i) Show that $x\left(1.1^{10}-1\right)=18$ (2)

ii) Hence, find the number of minutes he should allow for the first question. (1)

c)

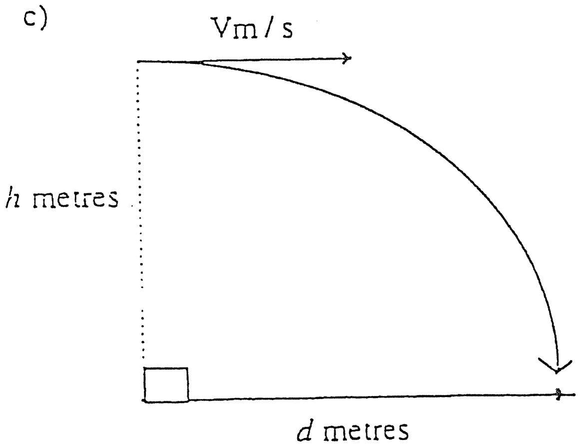


DIAGRAM NOT TO SCALE

A plane wants to drop water bombs on spot fires to extinguish them before a bushfire ensues.

The plane is travelling at V metres/second horizontally at a height of h metres.

A spot fire is ahead, d metres as the crow flies. The water bomb is dropped at this instant.

Take $g=-10 m/s^2$

i) Derive equations for $\dot{x}$, $\dot{y}$, x and y, the horizontal and vertical components of velocity and displacement (4)

ii) Show that $5d^2 = V^2 h$ if the water bomb hits the spot fire. (2)

iii) If a plane travelling at 50 m/s horizontally sights a spot fire 600m away, as the crow flies, at what height should it be travelling in order for a water bomb to hit the fire? (1)

QUESTION 7 USE A SEPARATE ANSWER BOOK

a) i) Expand $(1+x)^n$ (1)

ii) Consider the identity $(1+x)^n(1+x)^n=(1+x)^{2n}$. By comparing co-efficients of x^n on both sides, show that:

$$\binom{n}{o}^2+\binom{n}{1}^2+\binom{n}{2}^2+\binom{n}{3}^2+\dots\dots\dots\dots+\binom{n}{n}^2=\binom{2n}{n}$$ (2)

b) At high tide, the water level reaches the 13 metre mark on a wharf pylon whereas at low tide the water level drops to the 5 metre mark. Assuming the water level flows in simple harmonic motion, that high tide was at 6.50am and then low tide was at 12.30pm:

i) Find the amplitude and period of the motion. (1)

ii) Find at what time the water level dropped below 7 metres. (3)

c) Given the differential equation $\ddot{x}=-e^{-\frac{x}{2}}$, (where $\ddot{x}=\frac{d^2x}{dt^2}$):-

i) Find an expression for $\dot{x}$ in terms of x, given that $x=0$ and $\dot{x}=2$ when $t=0$. (2)

ii) Show that $x=4\,ln\left(\frac{t+2}{2}\right)$ (3)

END OF EXAMINATION