Total marks (84)
Attempt Questions 1 - 7
All questions are of equal value

Answer all questions in a SEPARARTE writing booklet.

QUESTION 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the exact value of $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$. **3**

(b) Find the coordinates of the point dividing the interval joining (-3, 5) to (2, -8) internally in the ratio 3 : 2. **2**

(c) Solve $\frac{x-2}{x+5} < 2$. **3**

(d) Find the gradients of the two lines that make an angle of 45º with the line whose equation is $y = 2x - 1$. **3**

(e) Find $\lim_{x \to 0} \frac{\sin \pi x}{x}$. **1**

QUESTION 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) For what values of x is $|x| + x = 0$? **1**

(b) Use the substitution $u = x + 1$ to evaluate $\int_{-1}^{3} x\sqrt{x+1}\,dx$. **4**

(c) Prove the identity $\dfrac{\cos 2A}{\sin A} + \dfrac{\sin 2A}{\cos A} = \operatorname{cosec} A$. **3**

(d) Use the substitution $t = \tan\dfrac{\theta}{2}$, or otherwise, to solve for $0° \le \theta \le 360°$: **4**

$2\sin\theta + 3\cos\theta = 2$

QUESTION 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The polynomial $P(x) = 2x^3 - x^2 - 5x + k$ has a factor $(x-2)$.

(i) Find the value of *k*. **1**

(ii) For this value of *k*, solve the equation $P(x) = 0$. **3**

(b) (i) Show that the equation $5\log_e x + x = 9$ has a root between 3 and 4. **1**

(ii) Taking $x = 3$ as a first approximation, use Newton's method to find a second approximation to the root. **3**

(c) Prove, using the Principle of Mathematical Induction, that, for all positive integers *n*, **4**

$$1\times 2 + 2\times 5 + 3\times 8 + ... + n(3n-1) = n^2(n+1)$$

QUESTION 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the general solution, in terms of π, of the equation **4**

$$\sin 2x - \cos x = 0$$

(b) Consider the function $f(x) = 2\cos^{-1}(x-1)$.

(i) State the domain and range of $y = f(x)$. **2**

(ii) Sketch the graph of $y = f(x)$. **1**

(c) (i) Sketch the graph of the function $f(x) = |2x - 1|$. **1**

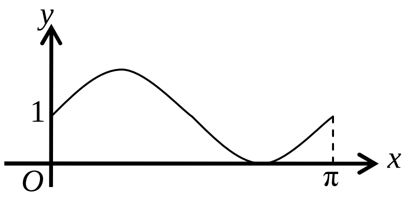
(ii) What is the largest domain containing the value $x = 1$ for which $f(x)$ has an inverse function $f^{-1}(x)$? **1**

(iii) Find $f^{-1}(x)$ in terms of x and state its domain and range. **2**

(iv) Sketch the graph of $y = f^{-1}(x)$. **1**

QUESTION 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

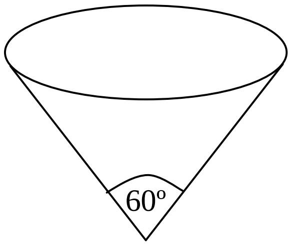
(a)



The diagram shows the graph of $y = 1 + \sin 2x$ for $0 \le x \le \pi$. **4**

This graph is rotated about the *x*-axis. Find the volume of the solid formed.

(b)



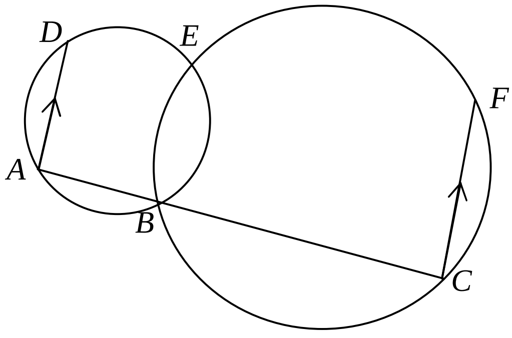
An inverted conical vessel has a vertical angle of 60°. Water is poured into the vessel at a constant rate of 8 cm³/minute.

(i) Show that, at a depth of *h* cm, the volume *V* cm³ of water is given by **2**

$$V = \frac{1}{9}\pi h^3.$$

(ii) At what rate is the water level rising when the depth is 4 cm? **2**

(c)



ABC is a straight line. *AD* is parallel to *CF*. **4**

Copy the diagram neatly into your answer booklet.

Prove that *D*, *E* and *F* are collinear.

QUESTION 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

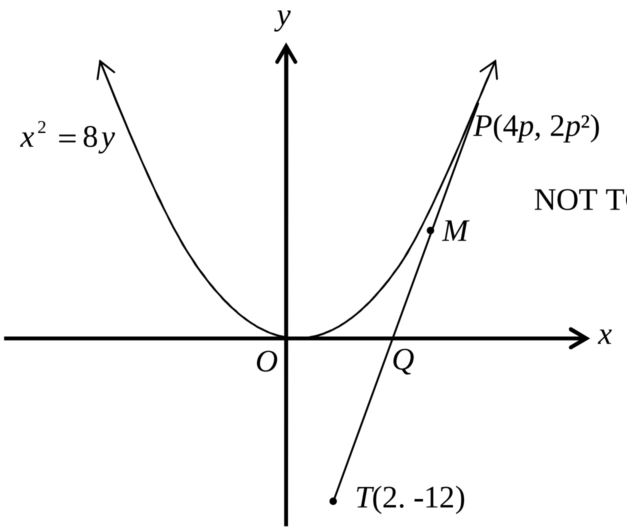
(a) A particle is moving in a straight line. Its velocity v m/s at a position x metres from an origin O is given by:

$$v^2 = 4(27 - 3x^4)$$

(i) Determine the position of the particle when it is instantaneously at rest. **1**

(ii) Find its acceleration when $x = 1$. **2**

(b)



The point $P(4p, 2p^2)$ lies on the parabola $x^2 = 8y$.
PT is a tangent to the parabola at P.

(i) Find the equation of the tangent at P. **2**

(ii) The tangent at P meets the x-axis at Q. Find the equation of the locus of M, the midpoint of PQ, as P moves around the parabola. **3**

(iii) If the tangent at P passes through $T(2, -12)$, show that $p^2 - p - 6 = 0$. **1**

(iv) Hence find the equations of the two tangents to the parabola that pass through $T(2, -12)$. **3**

QUESTION 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the exact value of $\cos\left(2\tan^{-1}\frac{3}{5}\right)$ **2**

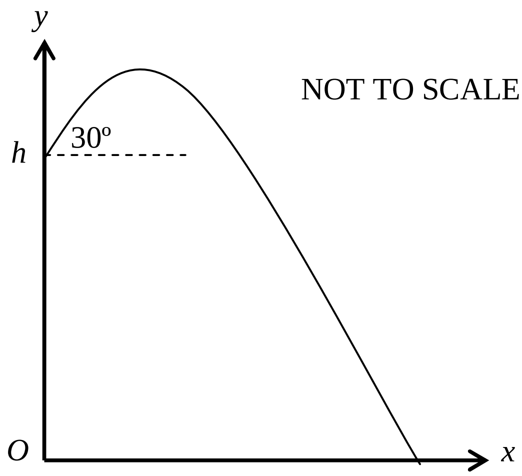
(b) Find the coefficient of x^3 in the expansion of $(3+2x)^5$. **2**

(c) Using the Binomial Theorem expansion **3**

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots\ldots + \binom{n}{r}x^r + \ldots\ldots + \binom{n}{n}x^n,$$

prove that $\displaystyle\sum_{r=0}^{n} \frac{1}{r+1}\binom{n}{r} = \frac{1}{n+1}\left(2^{n+1}-1\right)$.

(d)



A stone is thrown from the top of a vertical cliff, h metres high, at 20 m/s and at an angle of 30° above the horizontal.
Take acceleration due to gravity as 10 m/s².

(i) Starting from the acceleration equations $\ddot{x} = 0$ and $\ddot{y} = -10$, show that **2**

$$x = 10\sqrt{3}\,t \quad \text{and} \quad y = 10t - 5t^2 + h$$

where x and y are measured in metres from the base of the cliff.

(ii) The stone hits the ground after 6 seconds. Find the height of the cliff. **1**

(iii) Find the acute angle at which the stone strikes the ground. **2**

End of paper

STANDARD INTEGRALS

$\int x^n \, dx$	$= \frac{1}{n+1} x^{n+1}, \; n \neq -1; \; x \neq 0, \text{if } n < 0$
$\int \frac{1}{x} \, dx$	$= \ln x, \quad x > 0$
$\int e^{ax} \, dx$	$= \frac{1}{a} e^{ax}, \; a \neq 0$
$\int \cos ax \, dx$	$= \frac{1}{a} \sin ax, \; a \neq 0$
$\int \sin ax \, dx$	$= -\frac{1}{a} \cos ax, \; a \neq 0$
$\int \sec^2 ax \, dx$	$= \frac{1}{a} \tan ax, \; a \neq 0$
$\int \sec ax \tan ax \, dx$	$= \frac{1}{a} \sec ax, \; a \neq 0$
$\int \frac{1}{a^2 + x^2} \, dx$	$= \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx$	$= \sin^{-1} \frac{x}{a}, \; a > 0, \; -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx$	$= \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx$	$= \ln\left(x + \sqrt{x^2 + a^2}\right)$

NOTE: $\ln x = \log_e x, \quad x > 0$