Total marks (84) Attempt Questions 1 - 7 All questions are of equal value

Answer all questions in a SEPARARTE writing booklet.

| QUESI | TION 1 (12 marks) Use a SEPARATE writing booklet. | Marks |
|-------|---|-------|
| (a) | Factorise $x^3 + 27$. | 1 |
| (b) | Draw the graph of the relation $ x+2y = 4$. | 2 |
| (c) | Find the coordinates of the point which divides the interval joining $(2, -1)$ to $(5, 3)$ <i>externally</i> in the ratio $3 : 1$. | 2 |
| (d) | Solve $\frac{3x-1}{x+2} > 4$. | 3 |

(e) The line y = 8 - 2x cuts the parabola $y = x^2$ at the point (2, 4). Find the acute **4** angle between the line y = 8 - 2x and the tangent to the parabola at (2, 4).

(a) Find
$$\int \frac{x}{\sqrt{16-x^4}} dx$$
 using the substitution $u = x^2$. 3

(b) Find
$$\int_{0}^{\frac{\pi}{4}} \sin^{2} 2x \, dx$$
. 3

(c) Find the exact value of
$$\sin\left(2\tan^{-1}\frac{2}{3}\right)$$
. 3

(d) Consider the function
$$f(x) = \frac{\pi}{2} + \tan^{-1}(x-1)$$
.
(i) What is the range of $y = f(x)$?

(ii) Sketch the graph of
$$y = f(x)$$
. 2

(a) Write down the general solution of the equation
$$\cos(\pi x) = \frac{\sqrt{3}}{2}$$
. 2

(b) Using the expansion the expansion of sin(A+B), find the exact value of $sin105^{\circ}$.

(c) Sketch the graph of
$$y = \sec x$$
 for $0 \le x \le 2\pi$.

| (d) | (i) | Express $6\sin\theta - 8\cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$ and θ and α are in degrees. | 2 |
|-----|-------|--|---|
| | (ii) | Hence solve the equation $6\sin\theta - 8\cos\theta = 4$ for $0^\circ \le \theta \le 360^\circ$. | 2 |
| | (iii) | What is the minimum value of $6\sin\theta - 8\cos\theta$, and what is the least positive value of θ for which it occurs? | 2 |

QUESTION 4 (12 marks) Use a SEPARATE writing booklet. Marks

(a) A parabola has equation
$$8ay = x^2 - 4ax - 20a^2$$
.

- (i) By expressing the equation in the form $(x-h)^2 = 4A(y-k)$, find 2 the coordinates of the vertex.
- (ii) Write down the equation of the directrix.



The points $P(4p, 2p^2)$ and $Q(4q, 2q^2)$ lie on the parabola $x^2 = 8y$

- (i) Show that the equation of the tangent at *P* is $px y 2p^2 = 0$. 2
- (ii) Find the coordinates of the point of intersection T of the tangents at P and Q.
- (iii) If M is the midpoint of the chord PQ, show that TM is parallel to the **1** axis of the parabola.





In the diagram, AB is a diameter of the circle, and CB and ED are tangents to the circle. $\angle ECD = \theta$.

3

Copy or trace this diagram into your writing booklet. Prove that $\angle DEB = 2\theta$. 1



The diagram shows part of the graph of the function y = P(x) where P(x) 2 is an odd function.

Copy or trace the diagram into your writing booklet and complete the graph of y = P(x), given that it is an odd function.

(c) For the polynomial equation P(x) = 0 where $P(x) = x^3 - 5x + 3$, there is a root between x = 1 and x = 2.

(i) Determine if the root lies between 1 and 1.5 or between 1.5 and 2. **1**

(ii) Taking x = 1.5 as an approximation to the root, use Newton's method once to find a second approximation to the root.

(d) Using the Principle of Mathematical Induction, prove that, for all positive 4 integers n, $5^n + 2 \times 11^n$ is a multiple of 3.

(a)

3

volume of a gas is inversely proportional to its pressure. For a particular gas at a particular temperature, the pressure (*P* kilopascals) and its volume (*V*cm³) are related by the formula: PV = 3000

Boyle's Law in Physics states that, for a gas at constant temperature, the

If the volume of gas is increasing at a rate of 30cm³/minute, find the rate at which the pressure is decreasing when the volume is 100cm³.

(b) A tank contains a brine solution for curing hams. (Brine is salt dissolved in water.) Initially the tank contains 80kg of dissolved salt.

The amount of salt in the solution is known to change at a rate, in kg/minute,

given by:
$$\frac{dM}{dt} = -0.01(M - 50)$$
(i) Show that $M = 50 + Ae^{-0.01t}$ satisfies the equation. 1

| (ii) | Show that $A = 30$. | 1 |
|------|----------------------|---|
| | | |

- (iii) Find the amount of salt in the tank after 60 minutes. 1
- (iv) What is the least amount of salt that will remain in the solution? 1

(c) A particle moves in Simple Harmonic Motion with a period of 6 seconds, and an amplitude of 20cm.

| (i) | Write an equation for its motion in the form $x = A \sin nt$. | 2 |
|-------|--|---|
| (ii) | Find the maximum velocity of the particle. | 1 |
| (iii) | Find its distance from the centre of oscillation when its velocity is half its maximum velocity. | 2 |

QUESTION 7 (12 marks) Use a SEPARATE writing booklet.

(a) Find, as an integer, the coefficient of x^3 in the expansion of $\left(x - \frac{2}{x^2}\right)^9$. 2

Marks

(b) A particle is moving in a straight lie. Its velocity vm/s at position x metres is given by:

$$v = \frac{5}{x} \quad \text{for } x > 0.$$

Initially, x = 10.

- (i) Find the acceleration when x = 2. 3
- (ii) Find an expression for x in terms of t. 3
- (c) A particle is projected from ground level at an angle θ to the horizontal, with a speed of *V*. *g* is the acceleration due to gravity. Its position at time *t* is given by the equations:

$$x = Vt \cos \theta$$
, $y = Vt \sin \theta - \frac{1}{2}gt^2$

- (i) Find the maximum height reached, in terms of V and θ , in simplest form. 3
- (ii) What is the speed of the object at its maximum height? 1

End of paper

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STANDARD INTEGRALS

| $\int x^n dx$ | $=\frac{1}{n+1}x^{n+1}, \ n\neq -1; \ x\neq 0, \text{if } n<0$ |
|---------------------------------------|--|
| $\int \frac{1}{x} dx$ | $= \ln x, x > 0$ |
| $\int e^{ax} dx$ | $=\frac{1}{a}e^{ax}, a \neq 0$ |
| $\int \cos ax dx$ | $=\frac{1}{a}\sin ax, \ a\neq 0$ |
| $\int \sin ax dx$ | $=-\frac{1}{a}\cos ax, a \neq 0$ |
| $\int \sec^2 ax dx$ | $=\frac{1}{a}\tan ax, a \neq 0$ |
| $\int \sec ax \tan ax dx$ | $=\frac{1}{a}\sec ax, a \neq 0$ |
| $\int \frac{1}{a^2 + x^2} dx$ | $=\frac{1}{a}\tan^{-1}\frac{x}{a}, a\neq 0$ |
| $\int \frac{1}{\sqrt{a^2 - x^2}} dx$ | $=\sin^{-1}\frac{x}{a}, a > 0, -a < x < a$ |
| $\int \frac{1}{\sqrt{x^2 - a^2}} dx$ | $=\ln\left(x+\sqrt{x^2-a^2}\right), \ x>a>0$ |
| $\int \frac{1}{\sqrt{x^2 + a^2}} dx$ | $=\ln\left(x+\sqrt{x^2+a^2}\right)$ |

NOTE: $\ln x = \log_e x, x > 0$

Extension One Mathematics Trial HSC Examination 2006

- . Markers comments + Workes Solutions
- · Dot plats

Mathematics Extension 1: Question 1 Suggested Solutions Marks Marker's Comments Awarded (a) $x^{3}+27=(x+3)(x^{2}-3x+9)$ Well done 1 |x+2y|=4(b) poorly answered. x+2y=-4 or x+2y=4 All sorts of graphs were presented. 1. Mark awarded for cach. 2 $\begin{pmatrix} y_1 & y_2 \\ (2, -1) & (5, 3) \end{pmatrix}$ mi m (c) 3:-/ Mark for correct $\left(\frac{3\times 5+(-1)\times 2}{3+(-1)}, \frac{3\times 3+(-1)\times (-1)}{3+(-1)}\right)$ Substitution into formula 1 Marie for answes (62,5) 2 $\frac{3x-1}{x+2} > 4$ (d) 1 Mark multipying through 67 $(x+2)^{2}$: $(3x-1)(x+2) > 4(x+2)^{2}$ (x+2) $4(x+2)^2 - (3x-1)(x+2) < 0$ 1 Mark Factorisation (x+2)[4(x+2)-(3x-b)]<01 Marthe answer (x+2)(x+9)<0-9<×<-2 (e) y = 8-2x : grad = -2 $y = 3c^{2}$ $\frac{dy}{dx} = 2x$. grad = 4 at(2, 4). Generally well dore. $tan \phi = \left| \frac{m_1 - m_2}{1 + m_1 - m_2} \right|$ Some used an incorrect formula. $= \left| \frac{-2 - 4}{1 + (-2) \times 4} \right|$ 1 Mark Correct gradients $= \frac{6}{7}$ $\Theta = 40^{\circ}36' \text{ or } 41^{\circ}(\text{nearestategree})$ 2 Marks Substitution and Simple Ecolien in formula I purk answer

| Suggested Solutions (a) $\int \frac{x}{\sqrt{16-x^{4}}} dd = \frac{1}{2} \int \frac{2x doc}{\sqrt{16-x^{4}}} \qquad u = x^{2}$ $\frac{du}{dx} = 2x$ | Marks Awarded | Marker's Comments |
|---|------------------|---|
| (a) $\int \frac{x}{\sqrt{16-x^{4}}} dd = \frac{1}{2} \int \frac{2x doc}{\sqrt{16-x^{4}}} \qquad u = x^{2}$ $\frac{du}{dx} = 2x$ | Indiada | 1 · · · · |
| $= \frac{1}{2} \left(\frac{1}{1 - 1} \right) du = \frac{1}{2} \int \frac{1}{1 - 1} du$ | | Well done |
| $= \frac{1}{2} \sin^{-1} \frac{4}{4} + C$ = $\frac{1}{2} \sin^{-1} \frac{x^{2}}{4} + C$ (3) |) | |
| $(b) \int_{0}^{\frac{\pi}{4}} \sin^{2} 2x dx = \int_{0}^{\frac{\pi}{4}} \frac{1}{2} (1 - \cos 4x) dx$ $= \frac{1}{2} \left[x - \frac{1}{4} \sin 4x \right]_{0}^{\frac{\pi}{4}}$ $= \frac{1}{2} \left[\left(\frac{\pi}{4} - \frac{1}{4} \times 0 \right) - (0 - 0) \right]$ $= \frac{\pi}{4}$ | · · · | Those that started with the result $\sin^{e} x = \frac{1}{2} (1 - \cos 2x)$ did better than those that went |
| (c) $sin(2 \tan^{-1}\frac{2}{3})$ = $sin 2\theta$ $tan \theta = \frac{2}{3}$ | | straight into the expression integration |
| $= 2 \sin \theta \cos \theta$ $= 2 \times \frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}}$ $= \frac{12}{13}$ (3) | • | |
| (a) $f(x) = \frac{\pi}{2} + \tan^{-1}(x-i)$ (i) $-\frac{\pi}{2} < \tan^{-1}(x-i) < \frac{\pi}{2}$ (i) $0 < \frac{\pi}{2} + \tan^{-1}(x-i) < \pi$ | | i) well done |
| $(ii) \qquad $ | | ii) Some forgot to move the |
| $\frac{\pi}{a}$ | | curve across 1 unit \therefore had the y-intercept as $\frac{\pi}{2}$ |
| | | |
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Mathematics Extension 1: Question 3 Suggested Solutions Marks Marker's Comments Awarded COS Toc = (a) aposty answered. $\pi x = 2n\pi + \pi$ 1 more for each part x= 2n ± -2 to answer. $Sin(A+B) = Sin A \cos B + \cos A \sin B$ (Ъ) 6) Well onswered. sin 105°= sin (60°+45) = sin 60° cos45°+ cos60°sin45° Mark For rule 1 sugar of Valle = 孕×点+±×点 1 Mark for Simplification $= \frac{\sqrt{3}+1}{2\sqrt{2}} \quad OR \quad \frac{\sqrt{6}+\sqrt{2}}{4}$ 2 (c) y=secx. y↑ Many Sketches were Messy. 1 Some lost mark for >x 2年 Not showing asymptotes -1 2 (d) (i) 65in 0 - 8 cos 0 = R sin (0-2) = Rsin Ocosa - Rcos Osina a) Generally well done $R^{2}(\cos^{2}\alpha + \sin^{2}\alpha) = 6^{2} + 8^{2}$ · R cosal = 6 Mart for Rivin correct form. R sina = 8 R= 100 Tand = 6 R = 10Mark for correct angle (R > o)x = 53°08'(0R53°). ... 6 sin 0 - 8 cos 0 = 10 sin (0 - 53°08') 2 10 sin (0-53°08') = 4 (ii) posity onswered. sin (0-53°08')= 4 Many couldn't give 0-53°08'=23°35'or 156°25' both answers Q = 76°43' or 209°33' (2 (iii) Minimum value of -10 when poorly onswered 0-53°08'= 270° alarge number couldn't 1.e. & = 323°08' give the min- Jame and value for a when this occurred.

Mathematics Extension 1: Question 4-Suggested Solutions Marks Marker's Comments Awarded 8ay = x2 - 4ax - 20 a? (a) Many students (i) $x^2 - 4ax + 4a^2 = 8ay + 20a^2 + 4a^2$ didn't handle the algebra well. $(x-2a)^{2} = 8a(y+3a).$ or were confused Vertex is (za, -3a). 2 with working with the `a` (i) A = 2aDirectrix: y=-sa (aa,-3a) (b) $x^2 = 8y$ i.e. $y = \frac{x^2}{8}$ i) well done (i) $\frac{dy}{dx} = \frac{2x}{8} = \frac{2x + \mu P}{8} = P \text{ at } (4P, 2P^2)$ Tangent at P: $y - 2p^2 = p(x - 4p)$ $y - 2p^2 = px - 4p^2$ 2 $Px - y - 2P^2 = 0$ (ii) Tangent of P: $px-y = 2p^{2}$ ii) didn't have to (1)Tongent at 9: q'x - y' = 2q''(z)(1)-(2) (p-q) = 2q''(z)dérive tangent at Q. = 2(p-q)(p+q) a number of $\mathcal{L} = 2(P+q)$ students couldn't Subst. into(1): $y = p \times 2(p+q) - 2p^{2}$ = $2p^{2} + 2pq - 2p^{2}$ solve the simultaneous eqns = 2P9 3 T is [2(P+9), 2.P9] or didn't see that 2p2-292 (iii) x-coord of M = 4P+49 = 2(P+9) a(P+q)(P-q).Since T, M have the same x-coord, iii) many students then TM// y-axis (axis of Parabola) found gradient of TM and showed that (C) LADB = 90° (angle in it was undefined ... TM is a vertical a semi-circle] ∴LB]C = 90° line. Correct but more working than $\therefore LDBE = 90^{\circ} - \Theta$ (angle sum of ABDC) given explanation : LBDF= 90°-0 (base angles of isos ABDE · since EB=ED (tangents from c) well done by external pointare equal) I those that worked ... LBED = 180°-2(90°-0) (angle sum of) 4 BDE through it. *⇒ 20*

Mathematics Extension 1: Question 5 Suggested Solutions Marks Marker's Comments Awarded (a) Ч Generally were done (b). $P(x) = x^3 + 4x^2 + ax + b$ (i) P(i) = 0: i + 4 + a + b = 0: a + b = -5 (i) Well done. P(-2)=0: -8+16-2a+b=0: -2a+b=-8(2) (1) - (2): $3a = 3 \therefore a = 1, b = -6$ $(ii) (x-1)(x+2) (x+c) = 3c^{3} + 43c^{2} + 3c - 6$ well done : -2C=-6: c=3 Third factor is (x+3) OR Use sum of roots 1 $P(x) = 3c^{3} - 5x + 3$ (c)(i) P(t) = -t = P(x) = tFor mark needed to P(1.5) = 1.53-5×1.5+3 = -1.125 Show P(15)<0 P(2)>0 . Root lies between 1.5 and 2 ilies between 15 and 2. $(ii') x_2 = x_1 - \frac{P(x_1)}{P'(x_1)}$ $P'(x) = 3x^{2} - 3x^{2}$ P'(1.5-)=3×15-5 well done. = 1.5 - -1.125 = 1.75 = 2.14 (2dp) (d) Prove 5°+2×11° is a multiple of 3 Poorly arswered. When n=1, 5"+2×11"=5+2×11=27=3×9 1 Mark for praving true : it is true for n=1 Assume it is true for n=k. for nel i.e.assume 5"+2×11" = 3M (Minteger) 1 Mark for assuming When n= k+1 5 "+ 2 × 11"= 5 K+1 + 2 × 11 K+1 5 K+2x11 = 3M and = 5x5 K+2x11 x11 K $= 5 (\beta M - 2 \times 11^{k}) + 22 \times 11^{k}$ Substitution into TKH 1 Mark for Showing by assumption = ISM - 10 × 11 K + 22 × 11 K They is a multiple of 3 = 15-M + 12 × 11 K 1 Marte for Concluion. = 3(5-M+4×11 k) :. if it is true for n=k, then it is true Those that could not for n=K+1. Since it is true for n=1, it is true for prove it true for naked all positive integers n. war any awarded & Mark

| Mathematics Extension 1: Question 6 | | |
|--|------------------|---|
| Suggested Solutions | Marks Awarded | Marker's Comments |
| (a) $PV = 3000$ $P = 3000 V^{-1}$ $\frac{dP}{dV} = -3000 V^{-2} = -\frac{3000}{V^2} V^{-2}$ $\frac{dP}{dV} = \frac{dP}{dV} \times \frac{dV}{dV}$ When $V = 100$, $\frac{dV}{dt} = 30$ $\frac{dP}{dt} = -\frac{3000}{(100)^2} \times 30$ = -9 Pressure is decreasing at 9 kpc/minute. | م | Well done by those that attempted. Disappointing to see that a number of students didn't know where to start. |
| (b) $\frac{dM}{dt} = -0.01(M-50)$ (i) $M = 50 + Ae^{-0.07t}$ $\frac{dM}{dt} = -0.01Ae^{-0.07t}$ (i) $When t=0, M=80 \therefore 80 = 50 + Ae^{0}$ (ii) $When t=60, M = 50 + 30 \times e^{-0.01 \times 60}$ (iii) $When t=60, M = 50 + 30 \times e^{-0.01 \times 60}$ (iv) $Least amount of salt = 50 \text{ kg}$ (iv) $Least amount of salt = 50 \text{ kg}$ (iv) | | i) well done ii) well done iii) well done iv) Needed to know that as $t \rightarrow \infty$, $e^{-0.0it} \rightarrow 0$. |
| (c) (i) $x = A \sin nt$ $Period = \frac{2\pi}{n} = 6 \therefore n = \frac{2\pi}{6} = \frac{\pi}{3}$ $\therefore x = 20 \sin \frac{\pi}{3} t$ (i) $v = 20 \times \frac{\pi}{3} \cos \frac{\pi}{3} t$ Maximum velocity $= \frac{20\pi}{3} \operatorname{cm/sec}(1)$ (iii) $v^2 = n^2 (a^2 - xc^2)$ $(\frac{10\pi}{3})^2 = (\frac{\pi}{3})^2 (400 - x^2)$ $100 = 400 - x^2$ $x^2 = 300$ $Distance = 10\sqrt{3} \operatorname{cm}$ 2 $cos \frac{\pi}{3}t = \frac{1}{2} \therefore \frac{\pi}{3}t = \frac{\pi}{3} \therefore t = 1$ When $t = 1$, $x = 20 \sin \frac{\pi}{3}$ $= 20 \times \frac{\sqrt{3}}{2}$ $= 10/3 \sqrt{3}$ | | i) Some students incorrectly let n= 6 instead of <u>ett</u> = 6. ii) Many students found max. velocity by letting acceleration = 0. Easier & less working to consider amplitude of velocity function. iii) Most students used second nethod. First method produced |

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Mathematics Extension 1: Question 7:Suggested SolutionsMarksMarksMarker's Comments(a)
$$(x - \frac{2}{x^2})^q$$
MarksMarker's Comments(a) $(x - \frac{2}{x^2})^q$ Some residuated to(b) $(x = \frac{5}{x}) (x^2)^2$ Some residuated to(c) $a = \frac{4}{x^2} (4x^2)^3$ Coefficient of $x^3 = (\frac{2}{x})^2$ (b) $(x = \frac{5}{x}) (4x^2)^3$ Coefficient of $x^3 = (\frac{2}{x})^2$ (c) $a = \frac{4}{x^2} (4x^2)^3$ Coefficient of $x^3 = (\frac{2}{x})^2$ (c) $a = \frac{4}{x^2} (4x^2)^3$ Coefficient of $x^3 = (\frac{2}{x})^2$ (c) $a = \frac{4}{x^2} (4x^2)^3$ Coefficient of $x^3 = \frac{2}{x^3}$ (c) $x = \sqrt{x} (2x^2)^3$ Coefficient of $x^3 = \frac{2}{x^3}$ (d) $a = \frac{4}{x^2} (2x^2)^3$ Coefficient of $x^3 = \frac{2}{x^3}$ (i) $a = \frac{4}{x^2} (2x^2)^3$ Coefficient of $x^3 = \frac{2}{x^3}$ (ii) $g = \frac{2}{x^2} = \frac{2}{x^3} = \frac{2}{x^3}$ x^3 = \frac{2}{x^3}(iii) $g = \frac{2}{x^3} = \frac{2}{x^3} = \frac{2}{x^3} = \frac{2}{x^3}$ x^3 = \frac{2}{x^3} = \frac{2}{x^3}(i) $b = \sqrt{x} (x = 0)^2$ x^3 = \frac{2}{x^3} = \frac{2}{x

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