Student Number:______________

Roseville College

3 UNIT MATHEMATICS

Trial Higher School Certificate
1998

Time Allowed: 2 hours - five minutes for reading

Directions to Candidates

* Attempt all questions.
* All necessary working must be shown.
* Marks may be deducted for careless or badly arranged work.
* Show your candidate number on each page of your work.
* Begin each question on a new page.
* All questions are of equal value.

Question 1

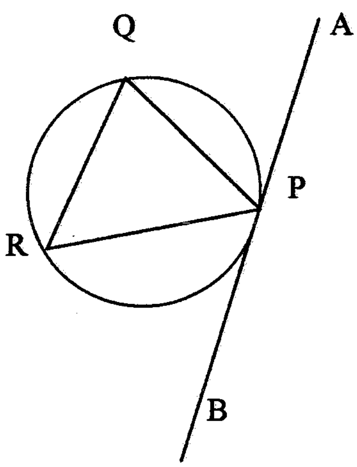
(a) (i) For what value of k is the polynomial $P(x) = x^3 + 2x^2 - x + k$ divisible by $(x - 2)$? (3)

(ii) Show that $P(x) = 0$ has only one root for that value of k.

(b) Find the derivative of $y = e^{\sin 3x}$. (2)

(c) If $f(x) = \frac{1}{x+3}$, find the inverse function, $f^{-1}(x)$. (2)

(d) (3)



Given that PQ = PR and AB is a tangent to the circle PQR at P, prove that RQ // BA.

(e) Use the substitution $u = 2x^2 - 5$ to find $\int \frac{x}{\sqrt{2x^2 - 5}}\, dx$ (2)

Question 2 (Start a new page)

(a) Solve $x-5<\dfrac{14}{x}$ (2)

(b) Differentiate $y=5\tan^{-1}\dfrac{x}{2}$ (2)

(c) Find the coefficient of x^3 in the expansion of $(3x+2)^7$ (2)

(d) Find the coordinates of the point which divides the interval joining A (3,-2) and B (-1,1) externally in the ratio 3:2. (2)

(e) Prove $\dfrac{\sin 2A}{1-\cos 2A}=\cot A$ and hence obtain an exact value for $\cot 67\frac{1}{2}^{0}$ in simplest surd form. (4)

Question 3 (Start a new page)

(a) If α,β,δ are the roots of $2x^3-x^2-x-2=0$, find the value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\delta}$. (2)

(b) Use one application of Newton's method to solve $f(x)=\cos x-\ln x$ given that there is a root near $x=1$. (2)

(c) If the volume of a cube is increasing at the rate of 25 mm^3/s, find the increase in its surface area when its side is 12 mm. (3)

(d) Sketch the graph of $y=3\cos^{-1}2x$ (at least one third of a page). Indicate the domain and range clearly on your axes. (2)

(e) The curves y = sin x and y = cos x intersect at $x=\frac{\pi}{4}$. If θ is the acute angle between the tangents to the curve y = sin x and y = cos x, at the point of intersection, find θ(to the nearest degree). (3)

Question 4 (Start a new page)

(a) $\int_2^4 \frac{x}{(3x-4)} dx$ using the substitution $u = 3x - 4$. (2)

(b) Solve the equation $\cos x - \sqrt{3}\sin x = 2$ for $0 \le x \le 2\pi$. (3)

(c) The velocity vcm/s of a particle moving in Simple Harmonic Motion along the x axis is given by $v^2 = 72 - 12x - 4x^2$. (4)

(i) Between which two values of x is the particle oscillating?

(ii) What is the amplitude of the motion?

(iii) Find the acceleration of the particle in terms of x.

(iv) Find the period of the oscillation.

(d) What is the general solution for $\tan\phi = -\sqrt{3}$? (2)

(e) Find $\lim_{x \to 0} \frac{\sin 2x}{4x}$ (1)

(b) The graph of the function $y = \log_e(x-1)$ meets the line $y = 2$ at P and the x axis at Q. From P perpendiculars are drawn to the x axis and y axis meeting them at R and S respectively. (5)

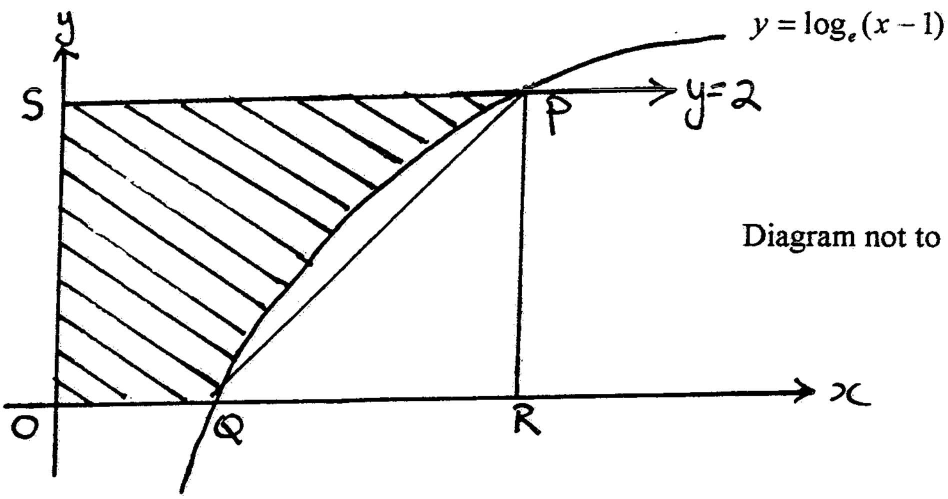


Diagram not to scale

(i) Show that the coordinates of P are $(e^2 + 1,\ 2)$ and write down the coordinates of the points R, S and Q.

(ii) Show that the shaded area enclosed by the arc PQ, the y axis and the lines $y = 2$ and $y = 0$ divides the rectangle OSPR into 2 portions of equal areas.

(iii) Show that the area enclosed by the arc QP and the straight line interval QP equals the area of triangle OSQ.

Question 7 (Start a new page)

(a) Mr Ryan hits a golf ball from a point O with an initial velocity of vm/s so that it rises at an angle of 30^0 to the horizontal. (7)

(i) Show that $x = \frac{\sqrt{3}}{2}vt$, and $y = -5t^2 + \frac{1}{2}vt$, where x and y are the horizontal and vertical displacements of the ball in metres from O, t seconds after the ball has been hit. Take g = -10m/s^2.

The ball lands on a horizontal green 24 metres below O, after a flight of 4 seconds.

(ii) Show that v = 28 m/s

(iii) Find the greatest height reached by the ball

(iv) Find the cartesian equation of the trajectory of the ball

(v) Find the horizontal distance that the ball travelled.

(b) See over for part (b)

Question 6 (Start a new page)

(a) Given that $(1+x)^{2n} = \sum_{k=0}^{2n} \binom{2n}{k} x^k$ (4)

(i) Show that $\sum_{k=0}^{2n} \binom{2n}{k} = 4^n$

(ii) By differentiating both sides show that $\sum_{k=1}^{2n} k\binom{2n}{k} = n4^n$

(b) Evaluate $\int_0^{\pi/6} \sin^5 x \cos x \, dx$ using the substation u = sin$x$ (2)

(c) A meteorite, soon after impact had a temperature of 2,520^{0}C, and cooled to 1,950^{0}C in 20 minutes when the surrounding temperature was -20^{0}C. How long would the meteorite take to cool to 0^{0}C? (Give your answer correct to the nearest minute). (3)

(d) In order to calculate the height of a mountain peak, a surveyor measured the angle of elevation from a certain stake and found it to be $18^0 40'$. He then walked 780 m over a level plain towards the mountain and set a second stake from which the angle of elevation was found to be $22^0 8'$. Find the height of the peak. (3)

Question 5 (Start a new page)

(a) P($2ap, ap^2$) is a point on the parabola $x^2 = 4ay$. Tangent PT is drawn at P. (6)

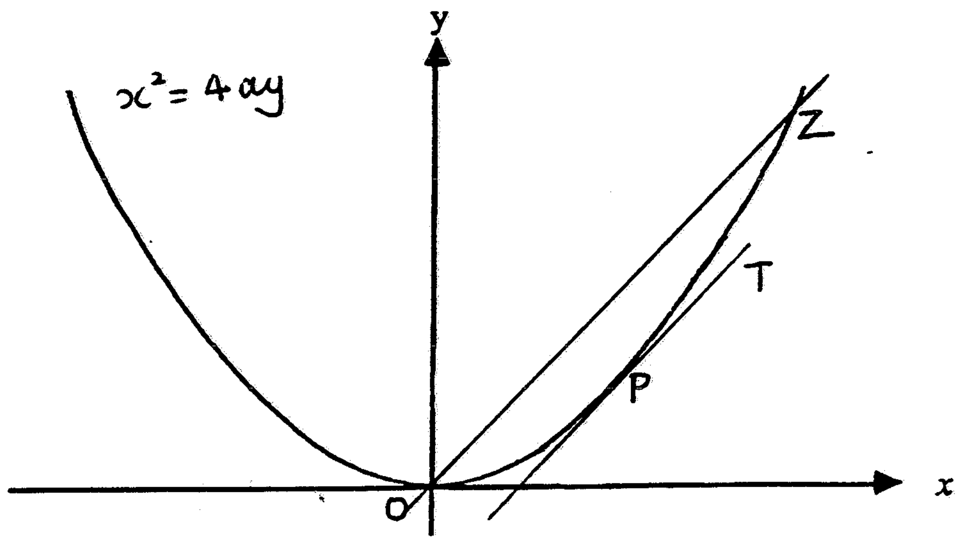
A straight line is drawn, parallel to this tangent and through the origin O. This cuts the parabola again at Z.

(i) What is the equation of the line OZ?

(ii) Show that Z is the point ($4ap$, $4ap^2$)

(iii) Find the coordinates of the point, M the midpoint of PZ

(iv) Find the equation of the locus of M as P moves around the parabola.



(b) Prove by Mathematical Induction that $8^n - 5^n$ is divisible by 3 for all positive integers n. (4)

(c) Find the term which is independent of x in the expansion of $\left(2x^3 + \frac{1}{3x^2}\right)^5$. (2)

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1a) i) $P(x) = x^3 + 2x^2 - x + k$

$P(2) = 8 + 8 - 2 + k = 0 \therefore \underline{k = -14}$

$$\begin{array}{r} x^2 + 4x + 7 \\ x-2 \,\big)\, x^3 + 2x^2 - x - 14 \\ x^3 - 2x^2 \\ 4x^2 - x \\ 4x^2 - 8x \\ 7x - 14 \\ 7x - 14 \end{array}$$

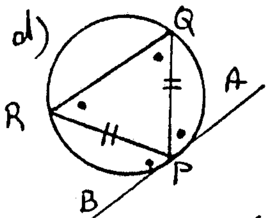
Δ of $x^2 + 4x + 7$
$= 16 - 28$
$= -ve$
$\therefore$ no real root

$\therefore P(x)$ has only $\underline{1 \text{ real root}}$.

b) $y = e^{\sin 3x}$ $\quad \underline{y' = 3\cos 3x\, e^{\sin 3x}}$

c) $y = \frac{1}{x+3}$ $\quad f^{-1}(x)$ $\quad x = \frac{1}{y+3}$

$\frac{1}{x} = y + 3 \quad \therefore \underline{y = \frac{1}{x} - 3}$

d) 

$\angle QPA = \angle PRQ = x$ (say) alt seg. theorem

$\angle PQR = \angle QRP = x$ isosceles Δ

$\therefore \angle QPA = \angle PQR = x$

$\therefore PQ \parallel BA$ (alt $\angle$s are equal)

e) $\int \frac{x}{\sqrt{2x^2 - 5}}\, dx$ $\quad u = 2x^2 - 5$, $\frac{du}{dx} = 4x$

$= \frac{1}{4} \int u^{-1/2}\, du = \frac{1}{4}\, 2u^{1/2} + C$

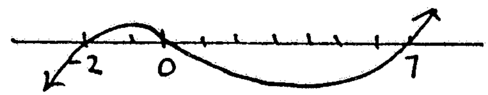
$= \underline{\frac{1}{2}\sqrt{2x^2 - 5} + C}$

2a) $x - 5 < \frac{14}{x}$, $\quad x \neq 0$

$x^2(x-5) < 14x$

$x^3 - 5x^2 - 14x < 0$

$x(x-7)(x+2) < 0$



$\underline{x < -2, \quad 0 < x < 7.}$

b) $y = 5\tan^{-1}\frac{x}{2}$ $\quad y' = \frac{10}{4 + x^2}$

c) $(3x+2)^7$ $\quad T_5 = {}^7C_4 (3x)^3 2^4$

coefficient $= \underline{15120}$

d) $A(3,-2)$ $\quad B(-1,1)$ $\quad 3:2$

$x = \frac{mx_2 + nx_1}{3-2} = \frac{3\cdot -1 + 3\cdot -2}{1} = -9$

$y = \frac{my_2 + ny_1}{3-1} = \frac{3\cdot 1 + -2\cdot -2}{1} = 7$

$\underline{\text{point} = (-9, 7)}$

e) Prove $\frac{\sin 2A}{1 - \cos 2A} = \cot 2A$

L.H.S $= \frac{2\sin A\cos A}{1 - (\cos^2 A - \sin^2 A)} = \frac{2\sin A\cos A}{1 - \cos^2 A + \sin^2 A}$

$= \frac{2\sin A\cos A}{2\sin^2 A} = \frac{\cos A}{\sin A} = \cot A = RHS$

$\cot 67\tfrac{1}{2}^\circ = \frac{\sin 135^\circ}{1 - \cos 135^\circ} = \frac{1}{\sqrt{2}} \Big/ 1 + \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} \Big/ \frac{\sqrt{2}+1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1}$

$\frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{1} = \underline{\sqrt{2}-1}$

3a) $2x^3 - x^2 - x - 2 = 0$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= -\frac{1}{2} \div 1 = \underline{-\frac{1}{2}}$

b) $a_2 = a_1 - \frac{f(a_1)}{f'(a_1)}$

$f(x) = \cos x - \ln x$

$f'(x) = -\sin x - \frac{1}{x}$.

$a_2 = 1 - \frac{\cos 1 - 0}{-\sin 1 - 1}$

$= 1 - \frac{0.5403023}{-1.841471}$

$\doteqdot 1.293408$

root $\doteqdot 1.29$ (2 d.p.)

c) $V = s^3$ $\quad \frac{dV}{ds} = 3s^2$. $\quad \frac{dV}{ds} \times \frac{ds}{dt} = 25 \text{ mm}^3/\text{s}$

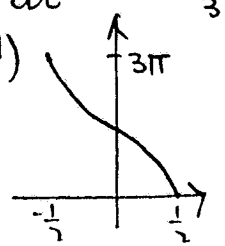
$A = 6s^2$ $\quad \frac{dA}{ds} = 12s$. $\quad \frac{dA}{ds} \times \frac{ds}{dt}$

$3s^2 \times \frac{ds}{dt} = 25 \quad \therefore \frac{ds}{dt} = 25 \div 3s^2$

$\frac{dA}{dt} = \frac{dA}{ds} \cdot \frac{ds}{dt} = 12s \times 25 \div 3s^2$

when $s = 12$.

$\frac{dA}{dt} = \frac{12 \times 12 \times 25}{3 \times 12 \times 12} = \underline{8\tfrac{1}{3} \text{ mm}^2/\text{s}}$

d) 

domain $-\frac{1}{2} \leq x \leq \frac{1}{2}$

range $0 \leq y \leq 3\pi$

3e $y = \sin x \quad y' = \cos x$

$y = \cos x \quad y' = -\sin x$

$\tan\theta = \dfrac{\frac{1}{\sqrt{2}} - -\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}}}$

$= \dfrac{1.41421356}{1-0.5}, \ \theta \doteqdot 71^\circ.$

4a) $\int_2^4 \dfrac{x}{3x-4}\,dx \qquad x=2, u=2 \quad x=4, u=8$

$= \frac{1}{3} \cdot \frac{1}{3} \int_2^8 \dfrac{u+4}{u}\,du \qquad u = 3x-4 \quad \dfrac{du}{dx} = 3.$

$= \frac{1}{9}\left[u + 4\ln u\right]_2^8 \qquad \dfrac{u+4}{3} = x$

$= \frac{1}{9}\left(\{8 + 4\ln 8\} - \{2 + 4\ln\}\right)$

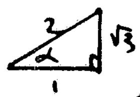
$= \frac{1}{9}\left\{\frac{1}{6} + 4\ln 4\right\} \doteqdot 1.28$ (2d.p.)

b) $\cos x - \sqrt{3}\sin x = 2$

$\cos\alpha\cos x - \sin\alpha\sin x = \frac{2}{2} = 1$

$\cos(x+\alpha) = 1$

$\cos\left(x + \frac{\pi}{3}\right) = 2\pi$



$\therefore \alpha = 60^\circ$

$x = 5\pi/3$

c) i) $v^2 = 72 - 12x - 4x^2$

$0 = 4(18 - 3x - x^2) = 4(6+x)(3-x)$

$\therefore x = -6$ and $+3$

ii) amplitude $= \dfrac{3 - -6}{2} = 4\frac{1}{2}$

iii) accel $= \dfrac{d}{dx}\left(\frac{1}{2}v^2\right) = \dfrac{d}{dx}(36 - 6x - 2x^2)$

$= -6 - 4x.$

iv) $-6 - 4x = -4\left(\frac{6}{4} + x\right) \therefore n = \sqrt{4} = 2$

period $= \dfrac{2\pi}{2} = \pi$ secs

d) $\tan\phi = -\sqrt{3} \quad \therefore \phi = -\pi/3$

$\therefore \phi = n\pi - \pi/3$

e) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{4x} = \lim\limits_{x\to 0} \frac{1}{2}\dfrac{\sin 2x}{2x}$

$= \frac{1}{2} \times 1 = \frac{1}{2}$

5a) i) $P(2ap, ap^2) \qquad x^2 = 4ay.$

$dy/dx = \dfrac{2x}{4a} \qquad m = \dfrac{2.2ap}{4a} = p.$

$\therefore$ gradient of tangent $= p$

$OQ = \quad y - 0 = p(x-0) \therefore y = px$

ii) $x^2 = 4ay \qquad y = px$

$x^2 = 4apx \qquad 0 = x^2 - 4apx$

$x(x - 4ap) \therefore x = 0$ or $4ap.$

$px = y \therefore x = P/y \qquad 4ay = y^2/p^2$

$4ap^2y = y^2 \quad \therefore y^2 - 4ap^2y = 0$

$y(y - 4ap^2) = 0 \therefore y = 0$ or $4ap^2.$

$\therefore Q = (4ap, 4ap^2)$

iii) $M = \left(\dfrac{2ap + 4ap}{2}, \dfrac{ap^2 + 4ap^2}{2}\right)$

$= \left(3ap, \frac{5}{2}ap^2\right)$

iv) $x = 3ap \quad \therefore p = x/3a.$

$y = \frac{5}{2}ap^2 \quad \therefore y = \frac{5}{2}a \cdot \dfrac{x^2}{9a^2}$

$y = \dfrac{5x^2}{18a}$ or $18ay = 5x^2$

5b) Prove $8^n - 5^n$ is divisible by 3 for all positive n.

① Test for $n=1$, $8^1 - 5^1 = 3$ which is divisible by 3

② Assume it is true for $n = k$ i.e. $8^k - 5^k = 3P$ where P is an integer

③ Test for $n = k+1$

$8^{k+1} - 5^{k+1} = 8.8^k - 5.5^k$

$= 8.8^k - 8.5^k + 3.5^k = 8(8^k - 5^k) + 3.5^k$

$= 8.3P + 3.5^k = 3(8P + 5^k)$ which is divisible by 3 $\therefore$ true for $n = k+1$ if true for $n = k$

④ Since it is true for $n=1$, then it is true for $n = 1+1 = 2$, $2+1 = 3$ etc for all positive n.

c) $\left(2x^3 + \dfrac{1}{3x^2}\right)^5 \quad T_{r+1} = {}^nC_r (2x)^{5-r}\left(\dfrac{1}{3x}\right)^n$

$(x^3)^{5-n} \cdot x^{-2n} = x^0$

$15 - 3n + -2n = 0 \qquad 15 - 5n = 0 \therefore n = 3$

$T_4 = {}^5C_3 (2x^3)^2 \left(\dfrac{1}{3x^2}\right)^3 = \dfrac{10 \times 4}{27} = \dfrac{40}{27}$

6a) $(1+x)^{2n} = \sum\limits_{k=0}^{2n} \dbinom{2n}{k} x^k$

i) Show that $\sum\limits_{k=0}^{2n} \dbinom{2n}{k} = 4^n$

Let $x = 1$

L.H.S. $= (1+1)^{2n} = 2^{2n} = 4^n$

RHS $= \sum\limits_{k=0}^{2n} \dbinom{2n}{k} 1^k = \sum\limits_{k=0}^{2n} \dbinom{2n}{k}$

$\therefore \sum\limits_{k=0}^{2n} \dbinom{2n}{k} = 4^n$

6a) ii) Differentiating

L.H.S. $2n(1+x)^{2n-1}$

RHS = $\binom{2n}{1} + 2\binom{2n}{2}x + 2\binom{2n}{3}x^2$

$+ \dots + 2n\binom{2n}{2n}x^{2n-1}$

Let $x = 1$

LHS = $2n(1+1)^{2n-1} = 2n(2)^{2n-1}$

$= n(2n)^{2n} = n4^n$

RHS. $\binom{2n}{1} + 2\binom{2n}{2}1 + 3\binom{2n}{3}1 + 2n\binom{2n}{2n}$

$= \sum_{k=1}^{2n} k\binom{2n}{k}$

$\therefore \sum_{k=1}^{2n} k\binom{2n}{k} = n4^n$

b) $\int_0^{\pi/6} \sin^5 \cos x\,dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$x = 0,\ \sin 0 = 0$

$x = \frac{\pi}{6}\ \sin\frac{\pi}{6} = \frac{1}{2}$

$\int_0^{1/2} u^5\,du$

$= \left[\frac{u^6}{6}\right]_0^{1/2} = \frac{1}{384}$

c) $T = -20 + Ae^{-kt}$

$2520 = -20 + Ae^0$

$2540 = A$

$T = -20 + 2540e^{-kt}$

$1950 = -20 + 2540e^{-20k}$

$1970 = 2540e^{-20k}$

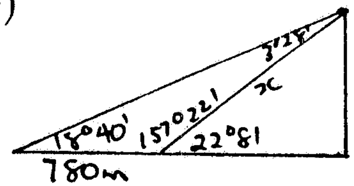
$\frac{1970}{2540} = e^{-20k}$

$\therefore k = 0.012706526$

$\ln\frac{20}{2540} = -kt$

$\therefore t = 381$ minutes

6d)



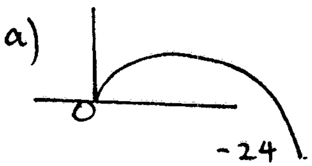
$\frac{x}{\sin 18°40'} = \frac{180}{\sin 3°28'}$

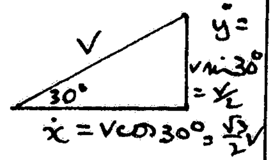
$x = \frac{780 \times \sin 18°40'}{\sin 3°28'} = 4128.6\text{m}$

$\sin 22°8' = \frac{h}{4128.6}$

$h = \sin 22°8' \times 4128.6 = 1555.5\text{m}$

7a)





i) $\ddot{x} = 0$

$\dot{x} = \int 0\,dt = 0 + c$

at $t = 0$, $\dot{x} = \frac{\sqrt{3}}{2}V$

$\therefore \dot{x} = \sqrt{3}/2$

$x = \int \frac{\sqrt{3}}{2} V\,dt = \frac{\sqrt{3}}{2}Vt + c_1$

at $t = 0$, $x = 0$ $\therefore c_1 = 0$

$x = \frac{\sqrt{3}}{2}Vt$

$\ddot{y} = -10 \therefore \dot{y} = \int -10\,dt = -10t + c_2$

at $t = 0$, $\dot{y} = \frac{V}{2} \therefore \dot{y} = -10t + \frac{V}{2}$

$y = \int -10t + \frac{V}{2}\,dt = -5t^2 + \frac{V}{2}t + c_3$

at $t = 0$, $y = 0 \therefore y = -5t^2 + \frac{V}{2}t$

ii) $y = -5t^2 + \frac{V}{2}t$ when $t = 4, y = -24$

$-24 = -5 \cdot 16 + \frac{4V}{2}$

$-24 = -80 + 2V$

$56 = 2V \therefore V = 28\text{m/s}$

iii) greatest height occurs when $\dot{y} = 0$

$\dot{y} = -10t + \frac{V}{2} = 0$

$-10t + \frac{28}{2} = 0$

$\therefore -10t = 14 \quad \therefore t = 1.4$ sec

$y = -5t^2 + 14t$

when $t = 1.4$ $y = -5(1.4)^2 + 14(1.4)$

$= 9.8\text{m}$

iv) $x = \frac{\sqrt{3}}{2} \cdot 28t$ ① $y = -5t^2 + 14t$ ②

from ① $t = \frac{x}{14\sqrt{3}}$ sub into ②

$y = -5\left(\frac{x}{14\sqrt{3}}\right)^2 + 14\left(\frac{x}{14\sqrt{3}}\right)$

$= -\frac{5x^2}{588} + \frac{\sqrt{3}x}{3}$.

v) $x = \frac{\sqrt{3}}{2}VT = \frac{\sqrt{3}}{2} \cdot 28 \cdot 4$

$\doteqdot 97\text{ m}$

7b) $y = \log_e(x-1)$

$2 = \log_e(x-1)$

$e^2 = x-1 \quad \therefore x = e+1 \therefore P(e+1, 2)$

$R = (e^2+1, 0)$, $S(0,2)$ $Q(2,0)$

ii) $y = \log_e(x-1)$

$e^y = x-1 \quad \therefore x = e^y+1$

$\int_0^2 (e^y+1)\,dy = \left[e^y + y\right]_0^2$

$= (e^2+2) - (e^0+0) = e^2+2-1 = (e^2+1)u^2$

Area rectangle OSPR $= 2 \times (e^2+1)\,u^2$

$\therefore$ shaded area $= \frac{1}{2}$ rectangle.

iii) $\Delta QRP = \frac{1}{2} \times 2\,(e^2+1-2) = e^2-1\,u^2$

Area of segment $= (e^2+1) - (e^2-1)$

$= 2\,u^2$

Area of $\Delta OSQ = \frac{1}{2} \times 2 \times 2 = 2u^2$.

$\therefore$ Area of segment $= \Delta OSQ = 2u^2$.

8 a) i) Area $\doteq \frac{2}{3}\left\{1 + 4\times\frac{\sqrt{3}}{2} + 0\right\}$ ✓

$= \frac{2}{3}\left(1 + 2\sqrt{3}\right)$ units^2 ✓

$\doteq 2.976$ units^2

ii) $V = \pi\int_{-2}^{2}\left(\frac{1}{2}\sqrt{4-x^2}\right)^2 dx$ ✓

$= 2\pi\int_0^2 \frac{4-x^2}{4}\,dx$

$= \frac{\pi}{2}\int_0^2 (4-x^2)\,dx$

$= \frac{\pi}{2}\left[4x - \frac{x^3}{3}\right]_0^2$ ✓

$= \frac{\pi}{2}\left(8 - \frac{8}{3} - (0-0)\right)$

$= \frac{8\pi}{3}$ units^3 ✓

b) $A = 80e^{-0.025t}$

i) $t = 10$, $A = 80e^{-0.25}$ ✓

$\doteq 62.3$ g

ii) Initial amt $= 80$ g

Half life $\rightarrow 40$ g

$40 = 80e^{-0.025t}$ ✓

$\frac{1}{2} = e^{-0.025t}$

$\ln\frac{1}{2} = -0.025t$ ✓

$t = \frac{\ln\frac{1}{2}}{-0.025}$ ✓

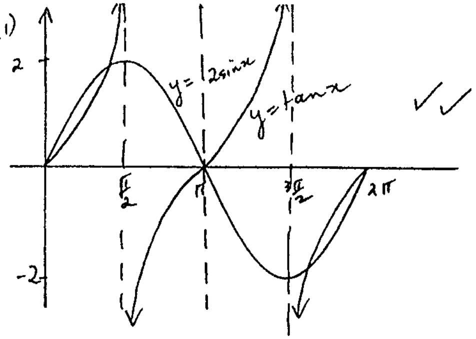
$\doteq 27.73$ years

iii) $\frac{dA}{dt} = 80\times -0.025e^{-0.025t}$

$= 2e^{-0.025t}$ ✓

$t = 8$ $\frac{dA}{dt} = 2e^{-0.2} \doteq 1.64$ g/year ✓

9 i)



✓✓

ii) There are 5 solutions to $2\sin x = \tan x$ from the graph ✓

iii) $2\sin x = \tan x$

$2\sin x = \frac{\sin x}{\cos x}$

$2\sin x\cos x = \sin x$ ✓

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

iv) $\sin x = 0$, $2\cos x - 1 = 0$

$x = 0, \pi, 2\pi$ $\cos x = \frac{1}{2}$ ✓

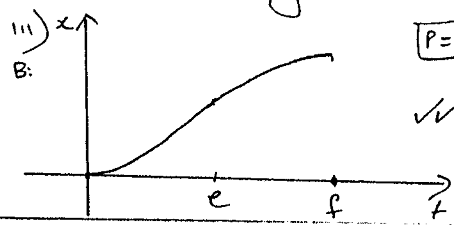
$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore$ Solns are $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$ ✓

b) i) A shows the greater acc^n since gradients of tangents (to vel) for A are always greater than gradients of tangents for B & $\frac{dv}{dt}$ = acceleration ✓✓

ii) Both A & B have the same velocity at $t = e$, however B is accelerating and A is decelerating. ✓✓

iii)



✓✓

P=6

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10 a) i) $OR = x$

$\therefore QR = e^{-x^2}$

$\therefore$ Area $PQRS = 2x\times e^{-x^2}$ ✓

$= 2xe^{-x^2}$

ii) $\frac{dA}{dx} = 2x\times -2xe^{-x^2} + 2e^{-x^2}$ ✓

$= -4x^2e^{-x^2} + 2e^{-x^2}$

$= 2e^{-x^2}(1 - 2x^2)$

for max Area, $\frac{dA}{dx} = 0$

$\therefore 2e^{-x^2}(1 - 2x^2) = 0$ ✓

$2e^{-x^2} \neq 0$, $1 - 2x^2 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$ ✓

$x = \pm\frac{1}{\sqrt{2}}$

But initially $x > 0$ $\therefore x = \frac{1}{\sqrt{2}}$

Check max

$\frac{d^2A}{dx^2} = 2e^{-x^2}(-4x) + -4xe^{-x^2}\times(1-2x^2)$

$= -8xe^{-x^2} - 4xe^{-x^2} + 8x^3e^{-x^2}$

$= 4xe^{-x^2}(2x^2 - 3)$

when $x = \frac{1}{\sqrt{2}}$, $2x^2 - 3 = 2\times\frac{1}{2} - 3$

$= -1 < 0$

$\therefore$ max $(4xe^{-x^2} > 0)$

$\therefore x = \frac{1}{\sqrt{2}}$ gives max area of PQRS ✓

b) i) $f(x) = 2 - \log_e x$ ✓

$f'(x) = -\frac{1}{x} < 0$ for al[l]

($\log_e x$ only exists for ...

$\therefore f(x)$ is always decrea[sing]

ii) x intercept, $f(x) = 0$

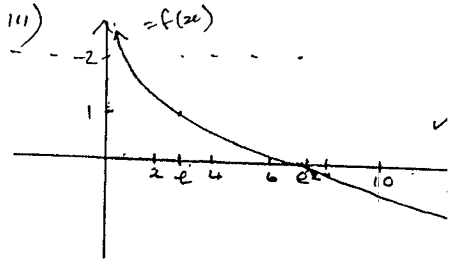
ie $0 = 2 - \ln x$ ✓

$\ln x = 2$

$x = e^2$ ✓

$\therefore x$ intercept $= (e^2, 0)$

iii)



iv) $A = \int_0^2 f(y)\,dy$

$y = 2 - \ln x$

$\ln x = 2 - y$

$x = e^{2-y}$

$\therefore A = \int_0^2 e^{2-y}\,dy$ ✓