Student number: ....

Roseville College

3 UNIT MATHEMATICS

Trial Higher School Certificate
1999

Time Allowed – 2 hours – five minutes for reading

Directions to Candidates

- Attempt all questions.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work.
- Show your candidate number on each page of your work.
- Begin each question on a new page.
- All questions are of equal value.

Marks **Question 1**

(2) **(a)** Divide the interval joining $A(-1,3)$ and $B(2,-3)$ internally in the ratio 1:2.

(2) **(b)** Evaluate $\int_0^1 \frac{dx}{1+x^2}$

(2) **(c)** Use the table of standard integrals to find $\int_0^2 \frac{4}{\sqrt{x^2+16}}$

(2) **(d)** Evaluate $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}(1)$

(2) **(e)** Evaluate $\lim_{x \to 0} \frac{\sin 4x}{3x}$

(2) **(f)** How many arrangements can be made from the letters of the word GOGGOMOBILE

Marks **Question 2**

(3) **(a)** It is known that $\ln x + \sin x \doteq 0$ has a root close to $x = 0.5$. Use one application of Newton's method to obtain a better approximation of the root.

(b) At the beginning of the summer season Barbie's pool is absolutely full. Due to use by her friends and evaporation, it loses 3% of its water each week. Assuming that no further rain falls during the season;

(3)

(i) What percentage remains after 4 weeks?

(ii) Barbie's filter system will not work when the pool is less than or equal to 75% full. Find approximately how many weeks before the pool filter system fails to work?

(3) **(c)** Determine (in radians) the acute angle between $x = 4$ and $2x - y - 5 = 0$

(3) **(d)** Find all solutions to $\tan^2\theta = \tan\theta$

Marks **Question 3**

(3) **(a)** Given that $2x^2 - 3x + 5 = P(x-1)(x-2) + Q(x-1) + R$ find the values of P, Q, R.

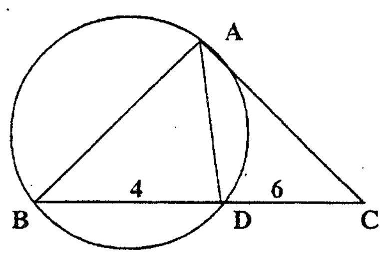
(b) If $\cos A = \frac{7}{9}$ and $\sin B = \frac{1}{3}$ where $0 \le A \le \frac{\Pi}{2}$ and $0 \le B \le \frac{\Pi}{2}$

(2) Find the value of $\tan(A+B)$ in simplest surd form.

(4) **(c)** PQ is a variable chord through the parabola $x^2 = 4ay$ and it subtends an angle of 90^0 at the vertex. Prove that PQ passes through a fixed point on the y axis.

(d) In the diagram, AC is a tangent to the circle and $\angle CAB$ is a rightangle.

(3)



(i) Show that ΔABD is similar to ΔCAD.

(ii) Calculate the length of AD and the radius of the circle.

Marks **Question 4**

(a) Show that the sum of the cubes of three consecutive integers; (n-1), n, (n+1) is $3n^3 + 6n$.

(b) Prove by mathematical induction that the sum of the cubes of three consecutive integers is divisible by 9.

(6)

(2) **(c)** Assume that Antartica can be approximately represented as a circle, and that its area is decreasing at the rate of 3000 square kilometres per year.

Find the rate at which the radius of the circle is decreasing when the length of the radius is 2100 kilometres.

(4) **(d)** A golfer plays a ball from a tee 5m above a level fairway. The ball leaves the tee with a speed of 48 ms^{-1} at an angle of elevation of 30°. Take $g = 10\,ms^{-2}$ to find out how far down the fairway the ball will land, giving your answers to the nearest metre. How far would the ball have carried had it been projected horizontally?

Marks **Question 5**

(4) **(a)** By using the substitution $x = u^2 - 1$ or otherwise prove that

$$\int_0^1 \frac{x\,dx}{\sqrt{1+x}} = \frac{4-2\sqrt{2}}{3}$$

(b) Consider the function $y = 2\cos^{-1}\frac{x}{3}$

(5)

(i) Sketch the graph of this function clearly showing the domain and range.

(ii) Find the angle θ, that the tangent to the curve

$y = 2\cos^{-1}\frac{x}{3}$ at $x = 0$ makes

with the positive direction of the x axis.

(3) **(c)** For the function $y = x^2 - 2x + 1$, find a suitable domain such that this function has an inverse. Find the equation of this inverse and state its range.

Marks **Question 6**

(a) Twelve girls decide to go to a *Maths* Festival. Only five girls are able to go by car and the rest have to walk to the Festival.

(4)
(i) How many different groups of five can be found to fill the car?
(ii) It is discovered that only 1 girl can drive and that the triplets Casey, Stacey and Macey, who cannot drive, must travel together. How many combinations are now able to fill the car?
(iii) Find the probability that the triplets will travel together, if no restrictions apply?

(2) (b) Show that $\frac{dv}{dt} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

(6) (c) A particle is moving on a straight line. At time t seconds it had displacement x metres from a fixed point 0 on the line, velocity $v\,m/s$ and acceleration $a\,m/s^2$. The particle starts from 0 and at time t seconds $v = (1-x)^2$

(i) Find an expression for a in terms of x.

(ii) Find an expression for x in terms of t.

(iii) Find the time taken for the particle to slow down to a speed of 1% of its initial speed.

Marks **Question 7**

(6) **(a)** A particle moves in a straight line according to the formula $x = 3\sin t + 4\cos t$ where x denotes its displacement in metres from the origin at time t seconds.

(i) Find an expression for acceleration in terms of t and show that the motion is simple harmonic.

(ii) Express $3\sin t + 4\cos t$ in the form $R\sin(t+\alpha)$ where R is a positive constant and α is in radians

(iii) Find the number of seconds that have elapsed when the particle is first at its maximum distance from the origin.

(6) **(b)** A mirror 3m high is placed on a wall with its base 1m above the level of an observer's eye if the observer stands xm from the wall, show that the angle of vision α subtended by the mirror is given

$$\alpha = \tan^{-1}\left(\frac{4}{x}\right) - \tan^{-1}\left(\frac{1}{x}\right)$$

Hence determine how far from the wall the observer should stand to maximise the angle of vision α.