Centre Number

Student Number

SCEGGS Darlinghurst

2010

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

Total marks – 84
Attempt Questions 1–7
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Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\lim_{x \to 0} \dfrac{\sin 5x}{2x}$ **1**

(b) Solve the inequality $\dfrac{2x-3}{x} \geq 1$ **3**

(c) Find the coordinates of $P(x, y)$, the point that divides the interval joining $A(4, 3)$ and $B(1, -1)$ externally in the ratio $3:2$. **2**

(d) Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt{2-x^2}}$ **2**

(e) (i) Show that $\tan x = \dfrac{\sin 2x}{1 + \cos 2x}$. **2**

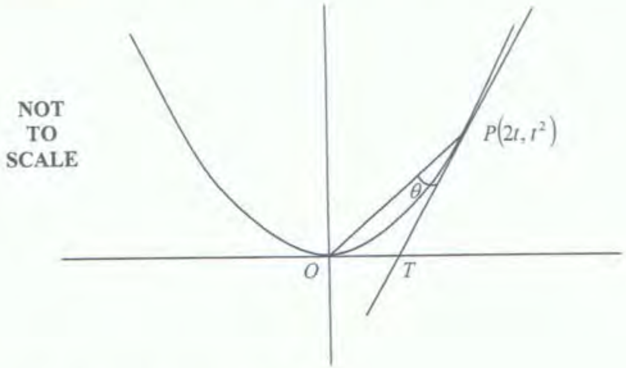
(ii) Hence evaluate $\tan\dfrac{\pi}{12}$ in simplest exact form. **2**

End of Question 1

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) $P(2t, t^2)$ is a point on the parabola $x^2 = 4y$ where $t > 0$.
The tangent to the parabola at P cuts the x-axis at T.
$\angle OPT = \theta$



(i) Find the gradients of OP and PT. **2**

(ii) Show that $\tan\theta = \dfrac{t}{t^2+2}$. **2**

(b) (i) Show that the function $f(x) = \dfrac{x-4}{x-2}$, $x \neq 2$ is increasing for all values of x in its domain. **1**

(ii) Sketch the graph of the function, showing clearly the coordinates of any points of intersection with the x-axis and y-axis, and the equations of any asymptotes. **2**

(iii) Find the inverse function, $f^{-1}(x)$, and state its range. **2**

Question 3 continues on page 5

Marks

Question 3 (continued)

(c) Let each different arrangement of all the letters of $G\,O\,O\,G\,L\,E\,P\,L\,E\,X$ be called a word.

(i) How many words are possible? **1**

(ii) If one of these words is chosen at random, what is the probability that all the vowels are together? **2**

End of Question 3

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Find the term independent of x in the expression $\left(\dfrac{1}{3x} - \dfrac{3}{2}x^2\right)^9$. **3**

(b) Given that a root of the equation $e^x - x - 2 = 0$ is close to $x = 1{\cdot}2$, use one application of Newton's Method of Approximation to find a second approximation of this root. (Answer to 2 decimal places.) **3**

(c) Find the exact volume of the solid formed when the region bounded by the x-axis and the curve $y = x\left(8 - x^3\right)^3$ between $x = 0$ and $x = 2$ is rotated about the x-axis. (You may need to use the substitution $u = 8 - x^3$.) **3**

(d) At a factory that produces Ipads it was found that on average 5% of Ipads produced have a fault. A batch of 15 Ipads is tested.

(i) What is the probability that there are exactly 2 Ipads containing faults in the batch of 15? (Answer to 2 decimal places.) **1**

(ii) What is the probability that at least 1 Ipad contains a fault? (Answer to 2 decimal places.) **2**

End of Question 4

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet.

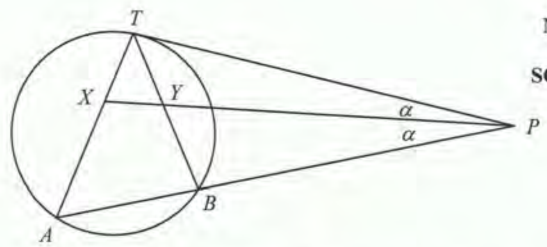
(a) On old 727 jet planes there are 56 rows of seats. Each row has three seats on each side of a central aisle. Three friends took a flight on a 727 jet plane with random seat allocation. Find the number of seating arrangements possible for the three friends if:

(i) all three friends must sit together on one side of the aisle. **1**

(ii) all three must sit in separate rows. **1**

(iii) no more than two can sit together. **1**

(b)



NOT TO SCALE

In the diagram above the tangent at T on the circle meets a chord AB produced to P. The bisector of $\angle TPA$ meets TA and TB at X and Y respectively.

(i) Give a reason why $\angle PTB = \angle TAB$. **1**

(ii) Prove $TX = TY$. **2**

(iii) Prove $\dfrac{TX}{AX} = \dfrac{TP}{AP}$. **2**

Question 5 continues on page 8

Marks

Question 5 (continued)

(c) (i) Differentiate $y = \tan^{-1}\frac{1}{x}$, $x \neq 0$ and hence show that **2**

$$\frac{d}{dx}\left(\tan^{-1} x + \tan^{-1}\frac{1}{x}\right) = 0$$

(ii) Sketch the curve $y = \tan^{-1} x + \tan^{-1}\frac{1}{x}$ **2**

End of Question 5

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the binomial expansion

$$1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{n}x^n = (1+x)^n$$

(i) Show that $1 - \binom{n}{1} + \binom{n}{2} - \ldots + (-1)^n\binom{n}{n} = 0$ **1**

(ii) Show that $1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \ldots + (-1)^n \frac{1}{n+1}\binom{n}{n} = \frac{1}{n+1}$ **3**

(b) $P(2at, at^2)$ is a point on the parabola $x^2 = 4ay$ with focus $S(0, a)$.

(i) Derive the equation of the normal to the parabola at P. **2**

(ii) The normal meets the y-axis at G. Show that the coordinates of G are $(0, 2a + at^2)$. **1**

(iii) Find the length of GP and PS in terms of a and t. **2**

(iv) Given that ΔSPG is equilateral, prove there are two positions of P and give the coordinates of these points in terms of a. **3**

End of Question 6

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Suppose $(5+2x)^{12} = \sum_{k=0}^{12} t_k\, x^k$.

(i) Use the binomial theorem to write an expression for t_k where $0 \le k \le 12$. **1**

(ii) Show that $\dfrac{t_{k+1}}{t_k} = \dfrac{2(12-k)}{5(k+1)}$. **1**

(iii) Hence or otherwise, find the greatest coefficient. **2**

(b) Find the general solution to $\sin 2\theta + \sqrt{3}\cos 2\theta = 0$. **2**

(c) (i) Show that $\sin A + \cos A = \sqrt{2}\sin\left(A + \dfrac{\pi}{4}\right)$. **2**

(ii) Prove that the derivative of $y = e^x \sin x$ is given by **1**

$\dfrac{dy}{dx} = \sqrt{2}\, e^x \sin\left(x + \dfrac{\pi}{4}\right)$.

(iii) Given the function $y = e^x \sin x$, prove by mathematical induction that the nth derivative of y for a positive integer n is: **3**

$$\frac{d^n y}{dx^n} = \left(\sqrt{2}\right)^n e^x \sin\left(x + \frac{n\pi}{4}\right)$$

$$\left(\textit{Note}: \frac{d}{dx}\left(\frac{d^k y}{dx^k}\right) = \frac{d^{k+1}y}{dx^{k+1}}\right)$$

End of paper

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Question 1

a) $\lim\limits_{x \to 0} \dfrac{\sin 5x}{5x} \times \dfrac{5}{2}$

$= 1 \times \frac{5}{2}$

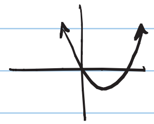
$= \frac{5}{2}$

b) $x(2x-3) \geq x^2$

$2x^2 - 3x \geq x^2$

$x^2 - 3x \geq 0$

$x(x-3) \geq 0$



R 3   Com 3

$\therefore \quad x \leq 0, \; x \geq 3 \quad$ but $x \neq 0$

$\therefore$ solution is $\quad x < 0, \; x \geq 3$

c) $A(4,3) \qquad B(1,-1)$

$-3 : 2$

$x = \dfrac{-3 \times 1 + 2 \times 4}{-3+2} \qquad y = \dfrac{-3 \times -1 + 2 \times 3}{-3+2}$

$x = -5 \qquad y = -9$

d) $\displaystyle\int_0^1 \frac{dx}{\sqrt{2 - x^2}} = \left[\sin^{-1} \frac{x}{\sqrt{2}}\right]_0^1$

Ca 2

$= \sin^{-1} \frac{1}{\sqrt{2}} - \sin^{-1} 0$

$= \frac{\pi}{4}$

e) i) Show $\tan x = \dfrac{\sin 2x}{1 + \cos 2x}$

$RHS = \dfrac{2 \sin x \cos x}{1 + 2\cos^2 x - 1}$

R 2

$= \dfrac{2 \sin x}{2 \cos x}$

$= \tan x$

$= LHS$

ii) $\tan \frac{\pi}{12} = \dfrac{\sin \frac{\pi}{6}}{1 + \cos \frac{\pi}{6}}$

$= \frac{1}{2} \div \left(1 + \frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2} \times \frac{2}{2 + \sqrt{3}}$

$= \dfrac{1}{2 + \sqrt{3}}$

Question 2.

a) $\angle BOD = 2\angle BAD$ ($\angle$ at the centre is twice the angle at the circumference on the same arc)

$= 2x$

$\angle BCD = \angle BOD$ (given)

$= 2x$

$\angle BAD + \angle BCD = 180°$ (opposite angles in a cyclic quad are supplementary)

$x + 2x = 180°$

$3x = 180°$

$x = 60°$

b) $P(-1) = -a + b + 19 - 15$

$0 = -a + b + 4$

$a = b + 4$ ①

$P(3) = 27a + 9b - 57 - 15$

$0 = 27a + 9b - 72$

$0 = 3a + b - 8$ ②

sub ① into ②

$0 = 3(b+4) + b - 8$

$0 = 3b + 12 + b - 8$

$4b = -4$

$b = -1$ $\therefore$ $a = 3$

c) $3^{2n} - 1$

for $n = 1$

$3^{2n} - 1 = 9 - 1$

$= 8$ which is divisible by 8 $\therefore$ true for $n = 1$

assume true for $n = k$

ie $3^{2k} - 1 = 8p$ (p being some positive integer)

$3^{2k} = 8p + 1$

RTP for $n = k+1$

$3^{2n} - 1 = 3^{2k+2} - 1$

$= 3^{2k} \times 3^2 - 1$

$= (8p + 1) \times 9 - 1$ (using assumption)

$= 72p + 9 - 1$

$= 72p + 8$

$= 8(9p + 1)$

divisible by 8 since p is an integer

true for $n = k$ and for $n = k+1$, since also true for $n = 1$ it follows that it is true for $n = 2, 3$ etc $\therefore$ true for all positive integers

d) $u = x + 1$

$\frac{du}{dx} = 1$

$du = dx$

$$\int \frac{3x+1}{\sqrt{x+1}}\, dx$$

$$= \int \frac{3(u-1)+1}{u^{\frac{1}{2}}}\, du$$

$$= \int \frac{3u - 2}{u^{\frac{1}{2}}}\, du$$

$$= \int 3u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\, du$$

$$= \frac{2}{3} \times 3u^{\frac{3}{2}} - \frac{2}{1} \times 2u^{\frac{1}{2}}$$

$$= 2u^{\frac{3}{2}} - 4u^{\frac{1}{2}}$$

$$= 2u^{\frac{1}{2}}(u - 2)$$

$$= 2\sqrt{x+1}\,(x - 1) + C$$

Question 3

a) $P(2t, t^2)$ $x^2 = 4y$ $t > 0$

i) $M_{OP} = \frac{t^2}{2t}$

$= \frac{t}{2}$

$M_{PT} = \frac{dy}{dx}$

$= \frac{2x}{4}$

$= \frac{x}{2}$ at $x = 2t$

$= t$

ii) $\angle$ between two lines: $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$

$\therefore \tan\theta = \frac{t - \frac{t}{2}}{1 + t \times \frac{t}{2}}$ $t > 0$

$= \frac{2t - t}{2} \div \frac{2 + t^2}{2}$

$= \frac{2t - t}{2 + t^2}$

$= \frac{t}{t^2 + 2}$

b) i) $f(x) = \frac{x-4}{x-2}$, $x \neq 2$

$f'(x) = \frac{(x-2)-(x-4)}{(x-2)^2}$ $u = x-4$ $v = x-2$

$u' = 1$ $v' = 1$

$f'(x) = \frac{2}{(x-2)^2}$

since $\frac{2}{(x-2)^2} > 0$ $f'(x) > 0$ $\therefore f(x)$ is increasing

ii) points of intersection: x intercept $= 4$

y intercept $= 2$

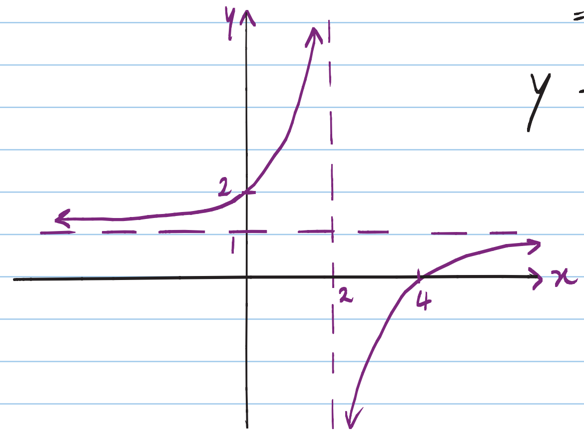
asymptotes: vertical when $x = 2$

horizontal when $x \to \infty$

$\lim_{x \to \infty} \frac{\frac{x}{x} - \frac{4}{x}}{\frac{x}{x} - \frac{2}{x}}$

$= \frac{1-0}{1-0}$

$y = 1$



iii) $f^{-1}(x)$: $x = \frac{y-4}{y-2}$

$xy - 2x = y - 4$

$xy - y = 2x - 4$

$y(x-1) = 2(x-2)$

$y = \frac{2(x-2)}{x-1}$

Range: all real y, $y \neq 2$

c)
$$\left.\begin{array}{l} C \times 2 \\ O \times 2 \\ L \times 2 \\ E \times 2 \\ P \\ X \end{array}\right\} \text{10 letters}$$

i) $\dfrac{10!}{2\times2\times2\times2} = 226800$

ii) $\dfrac{7!}{2\times2} \times \dfrac{4!}{2\times2}$

$= 1260 \times 6$

$= 7560$

Question 4

a) $\left(\frac{1}{3x} - \frac{3}{2}x^2\right)^9$

General Term : $\binom{9}{k}\left(\frac{1}{3}x^{-1}\right)^{9-k}\left(-\frac{3}{2}x^2\right)^k$ ✓

Find k : $x^{-9+k} \times x^{2k} = x^0$

$-9+k+2k = 0$

$3k = 9$

$k = 3$ ✓

Independant Term: $\binom{9}{3}\left(\frac{1}{3}x^{-1}\right)^6\left(-\frac{3}{2}x^2\right)^3$

$= 84 \times \frac{1}{729} \times -\frac{27}{8}$

$= -\frac{7}{18}$ ✓

b) $e^x - x - 2 = 0$ $\quad x = 1.2$

$f(1.2) = e^{1.2} - 1.2 - 2$

$= 0.1201$

$f'(x) = e^x - 1$

$f'(1.2) = e^{1.2} - 1$

$= 2.3201$

$x_1 = 1.2 - \frac{0.1201}{2.3201}$

$= 1.15$

c) $V = \pi\int_0^2 y^2\,dx$

$y = x(8-x^3)^3$

$y^2 = x^2(8-x^3)^6$

$u = 8 - x^3$

$\frac{du}{dx} = -3x^2$

$dx = \frac{du}{-3x^2}$

when $x = 0$, $u = 8$

$x = 2$, $u = 0$ ✓

$= \pi\int_0^2 x^2(8-x^3)^6\,dx$

$= \pi\int_8^0 u^6 x^2 \times \frac{du}{-3x^2}$

$= -\frac{\pi}{3}\int_8^0 u^6\,du$ ✓

$$= \frac{-\pi}{3}\left[\frac{u^7}{7}\right]_8^0$$

$$= -\frac{\pi}{3}\left(0 - \frac{8^7}{7}\right)$$

$$= \frac{8^7\pi}{21} \quad \text{or} \quad \frac{2097152\pi}{21}$$

d) Good Faulty $n=15$

0.95 0.05

i) 2 faults $= \binom{15}{2} 0.95^{13} \times 0.05^2$

$= 0.13$

ii) at least 1 fault $= 1 -$ no faults

$= 1 - \binom{15}{0} 0.95^{15}$

$= 0.54$

Question 5

a) $56 \times 6 = 336$ seats

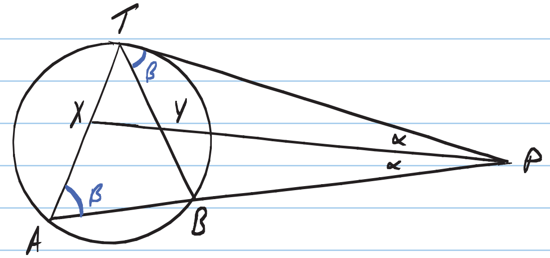
i) $336 \times 2 \times 1 = 672$

ii) $336 \times 330 \times 324 = 35925120$

iii) no more than 2 together = no restriction − 3 together

$= (336 \times 335 \times 334 - 672)$

$= 37594368$



b) i) $\angle PTB = \angle TAB$ (angle between a tangent and chord is equal to the angle in the alternate segment)

ii) let $\angle PTB = \angle TAB = \beta$

$\angle TXP = \alpha + \beta$ (exterior $\angle$ of Δ = sum opp interior $\angle$s)

$\angle TYX = \alpha + \beta$ (" " " ")

$\therefore \angle TXP = \angle TYX$ $\therefore \Delta TXY$ is isosceles

$TX = TY$ (sides opp = angles in an isosceles Δ are =)

iii) in ΔTYP and ΔAXP

$\angle TPY = \angle APX$ (given)

$\angle PTY = \angle PAX$ (proven in (i))

$\therefore \Delta TYP \,|||\, \Delta AXP$ (equiangular)

since corresponding sides in similar Δs are in proportion $\frac{TX}{AX} = \frac{TP}{AP}$

c) i) $y = \tan^{-1}\frac{1}{x}$

$y = \tan^{-1} x^{-1}$

$\frac{dy}{dx} = \frac{-x^{-2}}{1^2 + x^{-2}}$

$= -\frac{1}{x^2} \times \frac{1}{\frac{x^2+1}{x^2}}$

$= -\frac{1}{x^2} \times \frac{x^2}{x^2+1}$

$= \frac{-1}{x^2+1}$

also if $y = \tan^{-1} x$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

R 2

$\frac{d}{dx}\left(\tan^{-1}x + \tan^{-1}\frac{1}{x}\right) = \frac{d}{dx}\left(\tan^{-1}x\right) + \frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)$

$= \frac{1}{1+x^2} - \frac{1}{1+x^2}$

$= 0$

ii) if $\frac{d}{dx} = 0$ the curve is horizontal for all x in the domain

also $x \neq 0$

test a point when $x > 0$

ie let $x = 1$ $\quad y = \tan^{-1} 1$

$y = \frac{\pi}{4}$

test a point when $x < 0$

$y = \tan^{-1}(-1)$

$y = \frac{-\pi}{4}$

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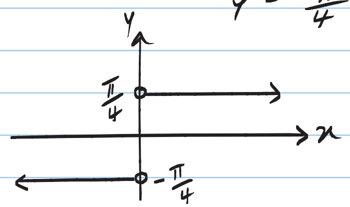


Question 6

a) $1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \dots + \binom{n}{n}x^n = (1+x)^n$

i) substitute $x = -1$

$1 + -\binom{n}{1} + \binom{n}{2} + -\binom{n}{3} + \dots + \binom{n}{n}(-1)^n = (1-1)^n$

$1 - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots + (-1)^n\binom{n}{n} = 0$

ii) Integrate original

$x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1} = \frac{1}{n+1}(1+x)^{n+1} + C$

let $x = 0$, $\quad 0 = \frac{1}{n+1} + C$

$C = -\frac{1}{n+1}$

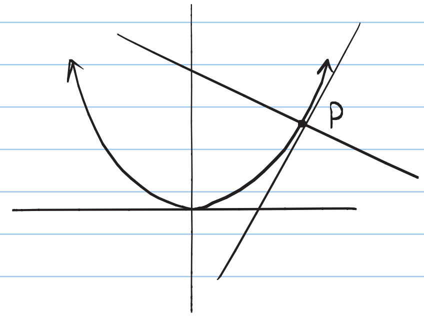
sub $x = -1$

$-1 + \frac{1}{2}\binom{n}{1} - \frac{1}{3}\binom{n}{2} + \dots + \frac{1}{n+1}\binom{n}{n}(-1)^{n+1} = 0 - \frac{1}{n+1}$

$-1 + \frac{1}{2}\binom{n}{1} - \frac{1}{3}\binom{n}{2} + \dots + \frac{1}{n+1}\binom{n}{n} \times (-1)^n \times (-1)^1 = -\frac{1}{n+1}$

$1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \dots + (-1)^n \frac{1}{n+1}\binom{n}{n} = \frac{1}{n+1}$

b) $P(2at, at^2)$ $\quad x^2 = 4ay$ $\quad S(0, a)$



i) $y = \frac{x^2}{4a}$

$y' = \frac{x}{2a}$

$M_T = t \quad M_N = -\frac{1}{t}$

Eqn: $y - at^2 = -\frac{1}{t}(x - 2at)$

$yt - at^3 = -x + 2at$

$x + yt = 2at + at^3$

ii) G lies on y-axis $\therefore x = 0$

$0 + yt = 2at + at^3$

$y = 2a + at^2 \quad \therefore G(0, 2a + at^2)$

iii) $GP = \sqrt{(2at)^2 + (at^2 - 2a - at^2)^2}$

$= \sqrt{4a^2t^2 + 4a^2}$

$= 2a\sqrt{t^2+1}$

$PS = \sqrt{(2at)^2 + (at^2 - a)^2}$

$= \sqrt{4a^2t^2 + a^2t^4 - 2a^2t^2 + a^2}$

$= a\sqrt{2t^2 + t^4 + 1} \qquad (t^2+1)^2$

$= a\sqrt{(t^2+1)^2}$

$= a(t^2+1)$

iv) If ΔSPG is equilateral $GP = PS$

$2a\sqrt{t^2+1} = a(t^2+1)$

$4(t^2+1) = (t^2+1)^2$

$0 = (t^2+1)^2 - 4(t^2+1)$

$0 = (t^2+1)(t^2+1-4)$

$0 = (t^2+1)(t^2-3)$

$\therefore t^2 = -1$, $t^2 = 3$

no soln. $\quad t = \pm\sqrt{3}$

$\therefore$ point $(2\sqrt{3}a, 3a)$ or $(-2\sqrt{3}a, 3a)$

Question 7.

a) $(5+2x)^{12} = \sum_{k=0}^{12} T_k$

i) $t_k = \binom{12}{k} 5^{12-k} 2^k \cancel{x^k}$

ii) $t_{k+1} = \binom{12}{k+1} 5^{11-k} 2^{k+1} \cancel{x^{k+1}}$

$\frac{t_{k+1}}{t_k} = \left(\frac{12!}{(k+1)!(11-k)!} \times 5^{11-k} \times 2^{k+1}\right) \div \frac{12!}{k!(12-k)!} \times 5^{12-k} \times 2^k$

$= \frac{\cancel{12!}}{(k+1)\cancel{k!}\cancel{(11-k)!}} \times \frac{\cancel{k!}(12-k)\cancel{(11-k)!}}{\cancel{12!}} \times 5^{-1} \times 2^1$

$= \frac{2(12-k)}{5(k+1)}$

iii) $\frac{2(12-k)}{5(k+1)} \geq 1 \qquad 0 \leq k \leq 12$

R 2

$24 - 2k \geq 5k + 5$

$19 \geq 7k$

$2.7 \geq k$

$\therefore k = 2$

greatest coefficient $(t_{k+1}) = \binom{12}{3} 5^9 2^3$

$= 3\,437\,500\,000$

b) $\sin 2\theta + \sqrt{3}\cos 2\theta = 0$

$\tan 2\theta + \sqrt{3} = 0$

$\tan 2\theta = -\sqrt{3} \qquad Q2, Q4$

related angle $= \frac{\pi}{3}$

Q2: $2\theta = \pi - \frac{\pi}{3}, \ 2\pi + \pi - \frac{\pi}{3}, \ 4\pi + \pi - \frac{\pi}{3} \ \ldots$

Q4: $2\theta = 2\pi - \frac{\pi}{3}, \; 2\pi + 2\pi - \frac{\pi}{3}, \; 4\pi + 2\pi - \frac{\pi}{3}$

General Soln: $2\theta = n\pi - \frac{\pi}{3}$ ✓

$\theta = \frac{n}{2}\pi - \frac{\pi}{6}$ ✓

[OR by Formula $2\theta = n\pi + \tan^{-1}(-\sqrt{3})$

$2\theta = n\pi - \frac{\pi}{3}$

$\theta = \frac{n}{2}\pi - \frac{\pi}{6}$]

c) i) $\sin A + \cos A = \sqrt{2}\sin\left(A + \frac{\pi}{4}\right)$

$RHS = \sqrt{2}\left(\sin A \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos A\right)$

$= \sqrt{2}\left(\sin A \times \frac{1}{\sqrt{2}} + \cos A \times \frac{1}{\sqrt{2}}\right)$

$= \sin A + \cos A$

$= LHS$

ii) $y = e^x \sin x$ $\quad u = e^x \quad v = \sin x$

$u' = e^x \quad v' = \cos x$

$\frac{dy}{dx} = e^x \sin x + e^x \cos x$

$= e^x(\sin x + \cos x)$

$= e^x\left[\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right]$ (from (i))

$= \sqrt{2}e^x \sin\left(x + \frac{\pi}{4}\right)$

iii) $y = e^x \sin x$ $\quad \frac{d^n y}{dx^n} = \sqrt{2}^n e^x \sin\left(x + \frac{\pi}{4}\right)$

- for $n = 1$

$\frac{dy}{dx} = \sqrt{2}e^x \sin\left(x + \frac{\pi}{4}\right)$

true as proven in (ii)

R 3

- assume true for $n = k$

$\frac{d^k y}{dx^k} = \sqrt{2}^k e^x \sin\left(x + \frac{k\pi}{4}\right)$

- RTP for $n = k+1$

$\frac{d^{k+1}y}{dx^{k+1}} = \sqrt{2}^{k+1} e^x \sin\left(x + \frac{\pi(k+1)}{4}\right)$

since $\frac{d^{k+1}y}{dx^{k+1}} = \frac{d}{dx}\left(\frac{d^k y}{dx^{k+1}}\right)$

from assumption

$\frac{d}{dx}\left(\frac{d^k y}{dx^k}\right) = \frac{d}{dx}\left[\sqrt{2}^k e^x \sin\left(x + \frac{k\pi}{4}\right)\right]$ $\quad u = e^x \quad v = \sin\left(x + \frac{k\pi}{4}\right)$

$u' = e^x \quad v' = \cos\left(x + \frac{k\pi}{4}\right)$

$= \sqrt{2}^k\left[e^x \sin\left(x + \frac{k\pi}{4}\right) + e^x \cos\left(x + \frac{k\pi}{4}\right)\right]$

$= \sqrt{2}^k e^x\left[\sin\left(x + \frac{k\pi}{4}\right) + \cos\left(x + \frac{k\pi}{4}\right)\right]$

from (ii)

$= \sqrt{2}^k e^x\left[\sqrt{2}\sin\left(\left(x + \frac{k\pi}{4}\right) + \frac{\pi}{4}\right)\right]$

$= \sqrt{2}^{k+1} e^x \sin\left(x + \frac{(k+1)\pi}{4}\right)$